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Study of Topological Analogies of Perfect difference Network and Complete Graph

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Abstract - *In this paper we have used the topological properties of complete graph and Perfect Difference Network of ($\delta^2+\delta+1$) nodes. We have shown how the Complete Graph can be derived from the system of Perfect Difference Set (PDS) by the union of Perfect Difference Network and the graph of missing links. We have also presented the formulas that derive the diagonal links, circular links and bidirectional links of Perfect difference Network (PDN).*

Key Words: **Perfect Difference Network (PDN), Perfect Difference set (PDS), Perfect Difference Graph (PDG), Complete Graph, Galois Field (GF).**

1. INTRODUCTION

A study of topological properties of Complete Graph and Perfect Difference Network (PDN) of ($\delta^2+\delta+1$) nodes. The calculation of total number of diagonal links, Circular links and Bidirectional links in a Perfect Difference Network (PDN) is done on the basis of mathematical procedures which are presented in the form of lemmas.

1.1 Perfect Difference Set

The Perfect Difference Sets were fist discussed by J. Singers in 1938 in terms of points and lines in a projective plane of a Galois Field (GF) [1], [2].

Definitions 1: Perfect Difference Set-: If the set S of $\delta+1$ distinct integers S_0,S_1,...Sδ has the property that the $\delta^2+\delta$ difference S_i - S_j ($0 \leq i, j \leq \delta$, $i \neq j$) are distinct modulo $\delta^2+\delta+1$, S is called a Perfect Difference Set mod $\delta^2+\delta+1$.

The existence of Perfect Difference sets seems intuitively improbable, at any rate for large δ, but in 1938 J.Singer proved that whenever δ is a prime or power of prime ,say δ =P^n , a Perfect Difference Set mod $P^{2n}+P^n+1$ exists.[3],[4],[15].
From now we on, let δ denote P^n and we write that n= $\delta^2+\delta+1$, = $P^{2n}+P^n+1$.
S={s:$|s_i-s_j|$ mod n ,where $0 \leq$ i, j$\leq \delta$,i≠j), δ is a prime or power of prime and n= $\delta^2+\delta+1$}[1].

1.2 Perfect Difference Network

Perfect Difference Network architecture, based on a PDS is designed where each i^{th} node is connected via direct links to node i±1 and i±S_j (mod n), for 2≤ j≤ δ. Each link is bidirectional and the preceding connectivity leads to a chordal ring of δ in –degree and δ out-degree (total degree of any node d(v)= 2δ) and diameter D=2 [6],[8]. PDN has already been studied for, high performance communication and parallel processing network [8] and some topological properties of PDNs and parallel algorithms [14],[9],[11],[3], were suggested. It was shown that an n-node PDN can emulate the complete network with optimal slow down and balanced message traffic.

Although other interconnection architecture with topological and performance characteristics similar to PDNs exist, we view PDNs as worthy additions to the repertoire of computer system designers.

Alternative network topologies offer additional design points that can be exploited to accommodate the needs of new and emerging technologies. Further study is needed to resolve some open questions and to cost /performance comparisons for PDNs.

1.3 Perfect Difference Graph

1 PDGs [6], based on the mathematical notion of perfect difference sets (PDSs),are undirected graphs of degree d=2δ (where δ is the number of elements in the PDS) and diameter D=2.

2 Definition 2: A PDG is an undirected interconnection graph with n=$\delta^2+\delta+1$ vertices, numbered 0 to n-1. In the PDG, each vertex "i" is connected via undirected edges to vertices (i±S_j) (mod n) for 1≤ j≤ δ, where S_j is an element of PDS {S_1,S_2,....,S_j }of order δ [4][5].

1.4 Complete Graph

A complete Graph is a simple graph G=(V,E) where for all vertices v_i ,v_j € V , $v_i \neq v_j$,there exits an edge (v_i, v_j) [13][7]. In other words, in a Complete Graph every vertex is connected to every other vertex i.e. every pair of different vertices is adjacent.

Properties of Complete graph-

1. A complete digraph is directed graph in which every pair of distinct vertices is connected by a pair of unique edge.
2. The complete graph on n vertices is denoted by K_n.
3. K_n has n(n-1)/2 edges and is a regular graph of degree n-1.
4. The complement graph of a complete graph is an empty graph.
5. The degree of every vertex is n-1.
6. A complete graph is sometimes also referred to as a universal graph or a clique.

2. STUDY OF COMPLETE GRAPH AND PERFECT DIFFERNCE NETWORK

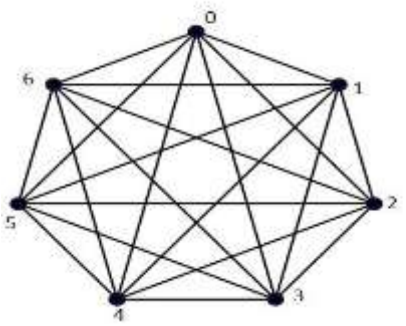

Fig -1: Complete Graph of $\delta^2+\delta+1$

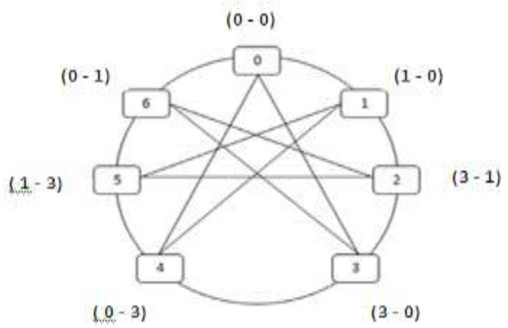

Fig -2: Perfect Difference Network of $\delta^2+\delta+1$

Lemma 1:- Total number of missing links in a complete graph which we have derived from the set of PDS will be β ($\delta^2+\delta+1$).

Proof: We have considered that a Perfect Difference Set (PDS) is of order n=$\delta^2+\delta+1$ and δ is a prime or power of prime which gives following properties.

1. Total number of circular links in a Perfect Difference network is $\delta^2+\delta+1$.
2. Total number of diagonal links in PDN is β ($\delta^2+\delta+1$) where β is a constant.
3. Total number of non-zero elements in PDS is δ.
4. Total number of elements in a PDS is $\delta+1$.
5. Total numbers of bidirectional links in a PDN is ($\delta^2+\delta+1$) (2 δ) and bidirectional links are even.

Therefore β ($\delta^2+\delta+1$) is also equivalent to $=\frac{(\delta^2+\delta+1)(\delta^2-\delta)}{2}$

So, we can say that

$$\beta=\frac{(\delta^2-\delta)}{2}$$

Hence the total missing links in Complete graph is β ($\delta^2+\delta+1$).

Lemma 2:- Total number of links in a complete graph derived from the system of PDS of $(\delta^2+\delta+1)(\delta^2+\delta)/_2$.

1. Total number of circular links in a PDN is $\delta^2+\delta+1$.
2. Total number of Diagonal links in a PDN = total links in a PDN – total circular Links in PDN
$$=\frac{(2\delta)(\delta^2+\delta+1)}{2}-(\delta^2+\delta+1)$$
$$=\delta(\delta^2+\delta+1)-(\delta^2+\delta+1)$$
$$=(\delta^3+\delta^2+\delta)-(\delta^2+\delta+1)$$
$$=\delta^3+\delta^2+\delta-\delta^2-\delta-1$$
$$=\delta^3-1$$
3. Total number of links in PDN =–total circular Links in PDN + total diagonal links in PDN
$$=(\delta^2+\delta+1)+(\delta^3-1)$$
$$=(\delta^3+\delta^2+\delta)$$
This equation is also equivalent to
$$=\frac{(2\delta)(\delta^2+\delta+1)}{2}$$

Proof:- Total links = Total no. of Circular links + Total number of Diagonal links + Total number of missing links in a complete graph.

$$=(\delta^2+\delta+1)+(\delta^3-1)+\frac{(\delta^2+\delta+1)(\delta^2-\delta)}{2}$$

$$=(\delta^3+\delta^2+\delta)+\frac{(\delta^2+\delta+1)(\delta^2-\delta)}{2}$$

$$=\delta(\delta^2+\delta+1)+\frac{(\delta^2+\delta+1)(\delta^2-\delta)}{2}$$

$$=\frac{2\delta(\delta^2+\delta+1)+(\delta^2+\delta+1)(\delta^2-\delta)}{2}$$

$$=\frac{(\delta^2+\delta+1)(2\delta+\delta^2-\delta)}{2}$$

$$=\frac{(\delta^2+\delta+1)(\delta^2+\delta)}{2}$$

Lemma3:- The complete graph can be derived from the system of PDS by the union of PDN and the graph of missing links i.e. (Complete graph = PDN + missing links of a graph).

Proof: - The total number of links in a complete graph from the system of perfect difference set can be derived as

$$=\frac{(\delta^2+\delta+1)(\delta^2+\delta)}{2} \quad \text{.....................................(i)}$$

In addition, the total number of links in a PDN can be derived from the half of the multiplication of set of PDS as on double of elements of the set of non-zero element i.e. $(2\,\delta)$ with $(\delta^2+\delta+1)$ i.e. $\frac{(2\delta)(\delta^2+\delta+1)}{2}$

Now the difference will be

$$=\frac{(\delta^2+\delta+1)\,(\delta^2+\delta)}{2}-\frac{2\delta(\delta^2+\delta+1)}{2}$$

$$=\frac{(\delta^2+\delta+1)\,(\delta^2+\delta)-(2\delta)(\delta^2+\delta+1)}{2}$$

$$=\frac{(\delta^2+\delta+1)\,(\delta^2+\delta-2\delta)}{2}$$

$$=\frac{(\delta^2+\delta+1)(\delta^2-\delta)}{2}$$

The missing links of complete graph is $\frac{(\delta^2+\delta+1)(\delta^2-\delta)}{2}$.

Hence it is proved that the Complete graph = PDN + missing links of graph.

3. CONCLUSION

Analogous study of Complete Graph and Perfect Difference Network is done on the basis of topological properties. We have found that the Complete Graph can be derived from the system of Perfect Difference Set (PDS) by the union of Perfect Difference Network (PDN) and the graph of missing links. We have explored that the Complete graph = PDN + missing links of a graph.

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