International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 06 Issue: 06 | June 2019 www.irjet.net p-ISSN: 2395-0072

On the Analysis of Shortest Queue with Catastrophes and Restricted Capacities

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Abstract - *In this paper we present a catastrophe on a system consisting of two parallel queues with jockeying and limited capacities. The capacity of each queue is restricted to L including that was being served. Customers arrive according to a Poisson process and on arrival; they join the shortest feasible queue. Where we study the impact of catastrophe on the system and derive some important performance measures.*

Key Words: **Steady state solution, Shortest queue, Jockeying, Catastrophes, Single server queue.**

1. INTRODUCTION

System in this paper consisting of two parallel servers with different service rates With a jockeying system, Jobs arrive according to a Poisson .Jockeying can be described as the movement of a waiting customer from one queue to another of shorter length or which appears to be moving faster, etc. in anticipation of a shorter delay. On arrival a job joins the shortest queue as and in case both queues have equal lengths, he joins the first queue with probability α and the second one with probability β . Haight [14] originally introduced the problem Kingman [16] and Flatto and McKean [11] treated the symmetric problem by using a generating function analysis. They showed that the generating function for the equilibrium distribution of the lengths of the two queues is a meromorphic function. Then by the decomposition of the generating function into partial fractions, it follows that the equilibrium probabilities can be represented by an infinite sum of product form solutions. However, the decomposition leads to cumbersome formulae for the equilibrium probabilities and the method does not seem to be generalizable to the asymmetric problem. Cohen and Boxma [6] and Fayolle and Iasnogorodski [12] studied the asymmetric shortest queue problem and showed that the analysis of the asymmetric shortest queue problem can be reduced to a simultaneous boundary value problem in two unknowns. The resulting boundary value problem however, is not of a standard type and further research remains to be done here.For the numerical approach, see Aden et al. [1–3] showed for the symmetric shortest queue problemthat the steady state distribution of the queue lengths of the two queues can be found in an elementaryway directly from the equilibrium equations. They showed the compensation procedure can be easily extendedto the asymmetric shortest queue problem. Moreover, they extended the compensation method to the case oftwo identical Erlange-k servers and shortest expected delay routing. Conolly [8] discussed the finite waiting room version of the shortest queue problemand showed that this problem can be solved efficiently, essentially by dimension reduction. Zhao and Grassman [21] obtained explicitsolutions for the equilibrium probabilities, the expected customers, and the expected waiting time of a customerin the system. Van Houtum et al. [19] study a production system consisting of a group of parallel machines producing multiple job types. On arrival, a job joins the shortest queue among all queues capable of serving that job. They formulate anumerical approach to determine the upper and lower bounds for the mean waiting time. For further overviewof solution methods of the performance analysis of parallel and distributed system see [5] Recently, Yao and Knessl [20] considered two parallel M/M/1 queues. Tarabia [17,18] study a system consisting of two parallel queues with jockeying and different server's rates. In recent times, a new class of queuing systems with Catastrophes has been introduced. These catastrophes may occur indiscriminately, killing all customers and temporarily disrupting the service facility until new customers arrive. The disasters may come from outside the system or from another service station. have been investigated by Boucherie and Boxma [7], Jain and Sigman [15] and Dudin and Nishimura [10]. The notion of catastrophes occurring at random, leading to annihilation of all the customers there and the momentary inactivation of the service facilities until a new arrival of customers is not uncommon in many practical situations. The catastrophes may come either from outside the system or from another service station. Comprehensive treatment of queueing models with catastrophes can be found in Gelenbe and Pujolle [13], Chao et al. [9] and Artalejo [4] a new class of queueing systems with catastrophes. Queueing models with catastrophes and breakdown are extensively studied as mentioned above, no work has been found in the literature which studies queueing systems taking together the above mentioned features. Based on this observation, In this paper we have been studying parallel lines with jockeying, limited capabilities and disasters.

The results of this paper are organized as follows: In Section 2, we describe the model and we formulate its difference equations. Section 3, the solution for the stationary distribution of these equations is given in simple matrix form. In Section 4, we present some important performance measures our model are given. In Section 5, we compute the mean of queue. in Section 6, some other numerical results are presented .

2. Model Description

In this model the two queues in parallel with jockeying and the capabilities are constrained with Catastrophes and the customer access rate is the Poisson process with the arrival rate λ . The queueing system consists of two parallel servers with different rates μ_1 and μ_2, respectively.

Where the client organizes the next to the short queue, if the two queues are equal it chooses the first column with probability α or second column with probability of β , where $\alpha+\beta=1$.

With disasters occurring on this system as an independent Poisson process with parameter v and inactivate the server upon arrival as shown in Figure (1).

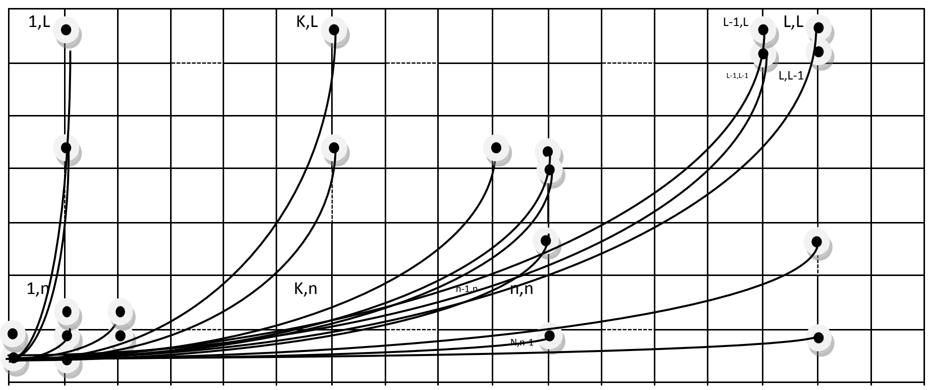

Chart-1: Shows the catastrophes of the system

The capacity of each queue is restricted to *L* including the one being served. The moment the server becomes idle and there is a customer waiting in the other queue, the customer immediately following the customer who is receiving service at that counter is transferred to the idle server's queue. Let N_1 and N_2 be, respectively, the number of customers in each queue. Since all distributions are exponential, the stochastic process *(N_1, N_2)* is a Markov chain with state space {0,1,2,…,L}x{0,1,2,…,L} and we can write $P_{i,j}=\Pr(N_1=i, N_2=j)$ for the steady state probability. From the above assumptions, the probability $P_{i,j}$ satisfies the following system of difference equations:

$$\mu_1 P_{1,0}+\mu_2 P_{0,1}+\nu-(\lambda+\nu)P_{0,0}=0 \tag{1}$$

$$\beta\lambda P_{0,0}+\mu_1 P_{1,1}-(\lambda+\mu_2+\nu)P_{0,1}=0 \tag{2}$$

$$\alpha\lambda P_{0,0}+\mu_2 P_{1,1}-(\lambda+\mu_1+\nu)P_{1,0}=0 \tag{3}$$

$$\lambda(P_{0,1}+P_{1,0})+\mu_1 P_{2,1}+\mu_2 P_{1,2}-(\lambda+\mu_1+\mu_2+\nu)P_{1,1}=0 \tag{4}$$

$$\lambda\beta P_{1,1}+\mu_1 P_{2,2}+\mu_1 P_{1,3}+\mu_2 P_{1,3}-(\lambda+\mu_1+\mu_2+\nu)P_{1,2}=0 \tag{5}$$

$$\lambda\alpha P_{1,1}+\mu_2 P_{2,2}+\mu_1 P_{3,1}+\mu_2 P_{3,1}-(\lambda+\mu_1+\mu_2+\nu)P_{2,1}=0 \tag{6}$$

$$\lambda(P_{1,2}+P_{2,1})+\mu_1 P_{3,2}(t)+\mu_2 P_{2,3}-(\lambda+\mu_1+\mu_2+\nu)P_{2,2}=0 \tag{7}$$

$$(\mu_1+\mu_2)P_{1,n+1}+\mu_1 P_{2,n}-(\lambda+\mu_1+\mu_2+\nu)P_{1,n}=0, \qquad 3\le n\le L-1 \tag{8}$$

$$\mu_2 P_{n,2}+\mu_1 P_{n+1,1}+\mu_2 P_{n+1,1}-(\lambda+\mu_1+\mu_2+\nu)P_{n,1}=0 \qquad 3\le n\le L-1 \tag{9}$$

$$\lambda P_{k-1,n}+\mu_1 P_{k+1,n}+\mu_2 P_{k,n+1}-(\lambda+\mu_1+\mu_2+\nu)P_{k,n}=0 \qquad 2\le k\le n-2 \tag{10}$$

$$\lambda P_{n,k-1}+\mu_1 P_{n+1,k}+\mu_2 P_{n,k+1}-(\lambda+\mu_1+\mu_2+\nu)P_{n,k}=0, \qquad 2\le k\le n-2 \tag{11}$$

$$\lambda P_{n-2,n}+\beta\lambda P_{n-1,n-1}+\mu_1 P_{n,n}+\mu_2 P_{n-1,n+1}-(\lambda+\mu_1+\mu_2+\nu)P_{n-1,n}=0 \tag{12}$$

$$\alpha\lambda P_{n-1,n-1}+\lambda P_{n,n-2}+\mu_1 P_{n+1,n-1}+\mu_2 P_{n,n}-(\lambda+\mu_1+\mu_2+\nu)P_{n,n-1}=0 \tag{13}$$

$$\lambda P_{n-1,n}+\lambda P_{n,n-1}+\mu_1 P_{n+1,1}+\mu_2 P_{n,,n+1}-(\lambda+\mu_1+\mu_2+\nu)P_{n,n}=0 \tag{14}$$

$$\mu_1 P_{2,L}-(\lambda+\mu_1+\mu_2+\nu)P_{1,L}=0 \tag{15}$$

$$\mu_2 P_{L,2}-(\lambda+\mu_1+\mu_2+\nu)P_{L,1}=0 \tag{16}$$

$$\lambda P_{k-1,L}+\mu_1 P_{k+1,L}-(\lambda+\mu_1+\mu_2+\nu)P_{k,L}=0, \qquad 2\le k\le L-2 \tag{17}$$

$$\lambda P_{L,k-1}+\mu_2 P_{L,k+1}-(\lambda+\mu_1+\mu_2+\nu)P_{L,k}=0, \qquad 2\le k\le L-2 \tag{18}$$

$$\lambda P_{L-2,L}+\beta\lambda P_{L-1,L-1}+\mu_1 P_{L,L}-(\lambda+\mu_1+\mu_2+\nu)P_{L-1,L}=0 \tag{19}$$

$$\lambda P_{L,L-2}+\alpha\lambda P_{L-1,L-1}+\mu_2 P_{L,L}-(\lambda+\mu_1+\mu_2+\nu)P_{L,L-1}=0 \tag{20}$$

$$\lambda P_{L-1,L}+\lambda P_{L,L-1}-(\mu_1+\mu_2+\nu)P_{L,L}=0 \tag{21}$$

Define the column vector P_k of $2k-1$ elements as

$P_k=(P_{1,k},P_{k,1},P_{2,k},P_{k,2},\ldots,P_{k-1,k},P_{k,k-1},P_{k,k})^t, \qquad k=1,2,3,\ldots,L,$

and $P_k^+=(P_{0,1},P_{1,0},P_k,P_{k,2})^t, \qquad k=1,2,3,\ldots,L.$

Also, let $\rho=\dfrac{\lambda}{\mu_1+\mu_2},\omega=\dfrac{\nu}{\mu_1+\mu_2},\gamma_1=\dfrac{\mu_1}{\mu_1+\mu_2} and\, \gamma_2=\dfrac{\mu_2}{\mu_1+\mu_2}.$

Ignoring the Eqs. (1)–(4) the above system (5)–(7) can be rewritten in the following block-matrix form:

$$\mathrm{A}_1 P_1+\mathrm{B}_2 P_2+C_3 P_3=0,$$

Where

$$\mathrm{A}_1=\begin{pmatrix}\beta\rho\\ \alpha\rho\\ 0\end{pmatrix} \quad , \quad \mathrm{B}_2=\begin{pmatrix}-(1+\rho+\omega) & 0 & \gamma_1\\ 0 & -(1+\rho+\omega) & \gamma_2\\ \rho & \rho & -(1+\rho+\omega)\end{pmatrix} \quad and \quad C_3=\begin{pmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & \gamma_2 & \gamma_1 & 0\end{pmatrix}$$

In general, the above system (5)-(21) can be rewritten as :

$$\mathrm{A}_{k-1}P_{k-1}+\mathrm{B}_kP_k+C_{k+1}P_{k+1}=0, \qquad k=2,3,4,\ldots,L-1, \tag{22}$$

$$\mathrm{A}_{L-1}P_{L-1}+\mathrm{B}_LP_L=0, \qquad k=L \tag{23}$$

And the normalizing equation :

$$\sum_{i=0}^{L}\sum_{j=0}^{L}P_{ij}=1,$$

where

$$\mathrm{A}_k=(a_{ij})_{(2k+1)\times(2k-1)}, \qquad k=1,2,\ldots,L-1,$$

with $a_{2k-1,2k-1}=\beta\rho, \quad a_{2k,2k-1}=\alpha\rho$ and $a_{i,j}=0$ otherwise :

$$\mathrm{B}_k=(b_{i,j})_{(2k-1)\times(2k-1)}, \qquad k=2,3,\ldots L,$$

with :

$$b_{2k-1,2k-2}=b_{2k-1,2k-3}=\rho, \quad b_{2k-3,2k-1}=\gamma_1, \quad b_{2k-2,2k-1}=\gamma_2, \quad k=2,3,\ldots,L,$$

$$b_{i,i}=-(1+\rho+\omega), \quad i=1,2,3,\ldots,2k-2 \quad b_{i+2,i}=\rho, \quad i=1,2,3,\ldots,2k-4$$

$$b_{2i-1,2i+1}=\gamma_1 \quad , \quad b_{2i,2i+2}=\gamma_2, \quad i=1,2,3,\ldots,k-2,$$

$$b_{2k-1,2k-1}=-(1+\rho+\omega), \quad k\neq L,$$

$$b_{2L-1,2L-1}=-1,$$

and $b_{i,j}=0$ otherwise. Finally

$$C_k=(c_{i,j})_{(2k-3)\times(2k-1)}, \qquad k=3,4,\ldots,L,$$

with:

$$c_{1,1}=c_{2,2}=1, \quad c_{2k-3,2k-2}=\gamma_1, \quad c_{2k-3,2k-3}=\gamma_2, \quad k=3,4,\ldots,L,$$

$c_{2i,2i}=\gamma_1, \quad c_{2i-1,2i-1}=\gamma_2, \quad i=2,3,\ldots,k-2,$ and $c_{i,j}=0$, otherwise.

3. Theoretical Results :

In this section, we formulate the solution for the stationary distribution of the system (1)–(21). Clearly, using the Eqs. (22) and (23) the stationary distribution of the given Markov process has a matrix solution and can be obtained recursively by solving the Eq. (22). More specifically, we have first to prove the following lemma which helps us in obtaining the stationary solution of the above system.

Lemma 1. The inverses of $\mathrm{B}_k, k = 2,3,\ldots,L$ are exist and their determinants can be computing by

$$\det(\mathrm{B}_k) = \left(-s + \frac{\gamma_1 \rho}{d_{k-1}} + \frac{\gamma_2 \rho}{e_{k-1}} \right) \prod_{i=1}^{k-1} d_i e_i, \qquad k = 2,3,\ldots,L,$$

where

$$d_1 = -(1+\rho+\omega), \qquad d_i = (1+\rho+\omega) - \frac{\gamma_1 \rho}{d_{i-1}}, \qquad i = 2,3,\ldots,k-1$$

$$e_1 = -(1+\rho+\omega), \qquad e_i = (1+\rho+\omega) - \frac{\gamma_2 \rho}{e_{i-1}}, \qquad i = 2,3,\ldots,k-1$$

$$s = (1+\rho+\omega) \text{ for } k \neq L \text{ and } s = 1 \text{ for } k = L$$

Proof. We prove the lemma by calculating the inverse B_k^{-1}. The idea of method for evaluating the determinant of B_k is to reduce the given matrix to upper triangular matrix form by elementary row operations, we obtain :

$$\mathrm{Det(B_k)=}\begin{vmatrix}
-d_1 & 0 & \gamma_1 & 0 & . & .. & 0 & 0 \\
0 & -e_1 & 0 & \gamma_2 & 0 & .. & 0 & 0 \\
 & 0 & -d_2 & 0 & \gamma_1 & 0 & .. & . \\
 & & 0 & -e_2 & 0 & \gamma_2 & & . \\
 & & & 0 & -d_3 & & & . \\
 & & & & 0 & -e_3 & & . \\
 & & & & & 0 & 0 & \\
 & & & & & & e_{k-1} & \gamma_2 \\
0 & .. & .. & .. & 0 & 0 & 0 & \left[-s + \frac{\gamma_1 \rho}{d_{k-1}} + \frac{\gamma_2 \rho}{e_{k-1}} \right]
\end{vmatrix}$$

Where

$$d_1 = -(1+\rho+\omega), \qquad d_i = (1+\rho+\omega) - \frac{\gamma_1 \rho}{d_{i-1}}, \qquad i = 2,3,\ldots,k-1,$$

$$e_1 = -(1+\rho+\omega), \qquad e_i = (1+\rho+\omega) - \frac{\gamma_2 \rho}{e_{i-1}}, \qquad i = 2,3,\ldots,k-1,$$

$$s = (1+\rho+\omega) \text{ for } k \neq L \text{ and } s = 1 \text{ for } k = L,$$

simply we have

$$\det(\mathrm{B}_k)=\left(-s+\frac{\gamma_1\rho}{d_{k-1}}+\frac{\gamma_2\rho}{e_{k-1}}\right)\prod_{i=1}^{k-1}d_i e_i .$$

Clearly.det $(\mathrm{B}_k)\neq 0$ for any value of ρ, then B_k is invertible for $k=2,3,\ldots,L$. This completes the proof .

Theorem 1.

Let P_k^+ be the unique positive solution of the system (1)–(21), then

$$P_k=(-1)^{k+1}R_k\mathrm{A}_{k-1}R_{k-1}\mathrm{A}_{k-2}\ldots R_2\mathrm{A}_1 p_{11}, \qquad k=1,2,3,\ldots,L \tag{24}$$

$$P_{1,0}=\frac{[\rho_1(\lambda+(\mu_1+\mu_2)\alpha)+\omega_1(2\lambda+\mu_2+\nu)]P_{0,0}-\omega_1(\lambda+\mu_2+\nu)}{(2\lambda+\mu_1+\mu_2+2\nu)} \tag{25}$$

$$P_{0,1}=\frac{[\rho_2(\lambda+\beta(\mu_1+\mu_2))+\omega_2(2\lambda+\mu_1+\nu)]P_{0,0}-\omega_2(\lambda+\mu_1+\nu)}{(2\lambda+\mu_1+\mu_2+2\nu)} \tag{26}$$

$$P_{1,1}=\frac{\left[\rho_1\rho_2\left(\beta\mu_1+\alpha\mu_2+\lambda+3\nu\right)+\rho_1\omega_2(\beta\mu_1+\alpha\mu_2+\mu_1+\mu_2+3\nu)+\omega_1\omega_2(\mu_1+\mu_2+\nu)+\nu\right]P_{0,0}}{(2\lambda+\mu_1+\mu_2+2\nu)}$$
$$-\frac{\rho_1\omega_2(\lambda+\mu_1+\mu_2+2\nu)+\omega_1\omega_2(\mu_1+\mu_2+\nu)+\nu}{(2\lambda+\mu_1+\mu_2+2\nu)} \tag{27}$$

With

$$P_{0,0}=\begin{cases}\dfrac{\begin{array}{c}(1-\rho)\left[(2\rho+2\omega+1)+\omega_2(\rho+\gamma_1+\omega)+\omega_1(\rho+\gamma_2+\omega)\right]\\+(\rho+\omega)(1-\rho^{2L-2})\left[\rho_1\omega_2(\rho+2\omega+1)+\omega_1\omega_2(\omega+1)-\omega\right]\end{array}}{\begin{array}{c}(1-\rho)\left[(2\rho+2\omega+1)+\rho_2(\rho+\beta)+\omega_2(2\rho+\gamma_1+\omega)+\rho_1(\rho+\alpha)+\omega_1(2\rho+\gamma_2+\omega)\right]\\+(\rho+\omega)(1-\rho^{2L-2})\left[\rho_1\rho_2(\beta\gamma_1+\alpha\gamma_2+\rho+3\omega)+\rho_1\omega_2(\beta\gamma_1+\alpha\gamma_2+3\omega+1)+\omega_1\omega_2(\omega+1)+\omega\right]\end{array}}, & \rho\neq 1\\[2ex] \dfrac{\left[(3+2\omega)+\omega_2(1+\gamma_1+\omega)+\omega_1(1+\gamma_2+\omega)\right]+\left[\rho_1\omega_2(2+2\omega)+\omega_1\omega_2(1+\omega)-\omega\right](2L-2)(1+\omega)}{\begin{array}{c}\left[(3+2\omega)+\rho_2(1+\beta)+\omega_2(2+\gamma_1+\omega)+\rho_1(1+\alpha)+\omega_1(2+\gamma_2+\omega)\right]\\+\left[(\rho_1\rho_2+\rho_1\omega_2)(\beta\gamma_1+\alpha\gamma_2+3\omega+1)+\omega_1\omega_2(1+\omega)+\omega\right](2L-2)(1+\omega)\end{array}}, & \rho=1\end{cases} \tag{28}$$

$$\rho_1=\frac{\lambda}{\mu_1},\rho_2=\frac{\lambda}{\mu_2},\omega_1=\frac{\nu}{\mu_1},\omega_2=\frac{\nu}{\mu_2},R_1=A_0=1,R_L=B_L^{-1},\gamma_1=\frac{\mu_1}{\mu_1+\mu_2},\gamma_2=\frac{\mu_2}{\mu_1+\mu_2}\ and$$
$$R_k=[B_L-C_{k+1}R_{k+1}A_k]^{-1}, k=L-1,L-2,\ldots,3,2.$$

Proof. From Lemma 1, we have proved that matrix B_k is invertible. Now using Eq. (23) we obtain

$$P_L=-R_L\mathrm{A}_{L-1}P_{L-1}, \qquad k=L, \tag{29}$$

where $R_L=\mathrm{B}_L^{-1}$ Hence $R_{L-1},R_{L-2},\ldots,R_2$ can be solved recursively by the following formula:

$$R_k = \left[B_k - C_{k+1} R_{k+1} A_k \right]^{-1}, \qquad k = L-1, L-2, \ldots, 3, 2$$

Furthermore, from Eq. (22) we get

$$P_k = -R_k A_{k-1} P_{k-1}, \qquad k = 2, 3, \ldots, L-1 \tag{30}$$

Hence $P_{L-1}, P_{L-2}, P_{L-3}, \ldots, P_1$ can be obtained recursively using (29) and (30) in terms of $P_{1,1}$, we get

$$P_k = (-1)^{k+1} R_k A_{k-1} R_{k-1} A_{k-2} \ldots R_2 A_1 P_{11}, \qquad k = 1, 2, 3, \ldots, L, \tag{31}$$

Now by simple algebraic manipulations, Eqs.(25)–(27) can be obtained by solving the system (1)–(3) in terms of $P_{0,0}$. Finally to complete our proof, let us assume that $N = N_1 + N_2$, be the number of customers in the system and let $g_k = P_r(N = k)$. Now, by induction on k we need to show that $g_k = (\rho + \omega)\rho^{k-3} g_2, \quad k = 3, 4, \ldots, 2L$. The following cases arise: for the simple case, using Eq. (4), we get $g_3 = \rho g_2$.. Similarly, working on Eqs.(5) and (6),we obtain

$$(\lambda + \mu_1 + \mu_2 + \nu)(P_{2,1} + P_{1,2}) = \lambda P_{1,1} + (\mu_1 + \mu_2)(P_{2,2} + P_{1,3} + P_{3,1}),$$

$$g_4 = (1 + \rho + \omega) g_3 - \rho g_2$$

$$g_4 = (\rho + \omega)^2 g_2 + \omega g_2$$

In general for $k = 2r$, , replacing n by $(2r-1), (2r-2), (2r-3), \ldots, r$ in the Eqs. (8)–(14) respectively and add, we have

$$(\lambda + \mu_1 + \mu_2 + \nu)\left[\sum_{i=1}^{r-1} P_{i,2r-1} + \sum_{i=1}^{r-1} P_{2r-i,i} + P_{r,r} \right] = \lambda \left[\sum_{i=1}^{r} P_{i,2r-i-1} + \sum_{i=1}^{r} P_{2r-i-1,i} \right] + (\mu_1 + \mu_2) \sum_{i=2}^{2} P_{i,2r-i+1}$$

$$+ (\mu_1 + \mu_2) \sum_{i=2}^{2} P_{2r-i+1,i} + (P_{1,2r} + P_{2r-1})$$

$$g_{2r+1} = (1 + \rho + \omega) g_{2r} - \rho g_{2r-1}, \qquad r = 3, 4, \ldots, L-1$$

Working likewise for the case $k = 2r+1$, by replacing n by $2r, (2r-1), (2r-2), \ldots, r+1$ one can easily establish the following relation:

$$g_{2r+2} = (1 + \rho + \omega) g_{2r+1} - \rho g_{2r}, \qquad r = 3, 4, \ldots, L-1$$

In general, we can write

$$g_k = (1 + \rho + \omega) g_{k-1} - \rho g_{k-2}, \qquad k = 2, 3, 4, \ldots, 2L \tag{32}$$

Clearly, the case for $k = 2L$ can be easily obtained using Eqs. (19) and(20). Now let the relation $g_k = (\rho + \omega)\rho^{k-3} g_2$, is true for some $m \le k$ (say). On making use of Eq. (30), we get

$g_{m+1} = (1 + \rho + \omega) g_m - \rho g_{m-1} = \left[(1 + \rho + \omega)(\rho + \omega)^{m-2} - (\rho + \omega)(\rho + \omega)^{m-3} \right] g_2 = (\rho + \omega)^{m-1} g_2$ Hence the relation $g_k = (\rho + \omega)\rho^{k-3} g_2$, is true for all $k = 3, 4, \ldots, 2L$. Using the normalizing equation $\sum_{k=0}^{2L} g_k = 1$, we get

$$g_0 + g_1 + g_2 (\rho + \omega) \sum_{k=3}^{2L} \rho^{k-3} = 1 \tag{33}$$

Using the facts $g_0 = P_{0,0}$, $\quad g_1 = P_{0,1} + P_{1,0}$, $\quad g_2 = P_{1,1}$ and using Eqs. (25)–(27)into the last one, the theorem gets fully established.

4. Particular case

Furthermore, another interesting result can be obtained for finite waiting space queueing system by putting :

$\alpha = \beta, \mu_1 = \mu_2 = \mu$ and $\rho = \dfrac{\lambda}{2\mu}$, we get :

$$P_{1,0} = (\rho + \omega) P_{0,0} - \omega, P_{0,1} = (\rho + \omega) P_{0,0} - \omega, P_{1,1} = \left[2(\rho + \omega)^2 + \omega \right] P_{0,0} - 2\omega(\rho + 2\omega + 1)$$

$$g_n = \begin{cases} \dfrac{(1-\rho)(1+2\omega)+(\rho+\omega)(2\rho\omega+4\omega^2+\omega)(1-\rho^{2L-2})}{(1-\rho)(1+2(\rho+\omega))+(\rho+\omega)(2(\rho+\omega)^2+\omega)(1-\rho^{2L-2})}, & n=0, \quad \rho\neq 1, \\ \dfrac{(1+2\omega)+(2L-2)(1+\omega)(3\omega+4\omega^2)}{(3+2\omega)+(2L-2)(1+\omega)(2(1+\omega)^2+\omega)}, & n=0, \quad \rho=1, \\ \left[2(\rho+\omega)^n+\omega\right]P_{0,0}-2\omega(\rho+2\omega+1), & 1\leq n\leq 2L, \end{cases} \tag{34}$$

5. Mean of the queue

We can define the mean of the total of costumers N is the system as:

$$E(N)=\sum_{n=0}^{2L} n g_n = \sum_{i=0}^{L}\sum_{j=0}^{L}(i+j)p_{i,j}$$

$$E(N)=\frac{(\rho+\omega)P_{1,1}}{\rho^2}\left[\frac{(1-\rho^{2L-1})-(2L-1)(\rho^{2L-2}-\rho^{2L-1})}{(1-\rho)^2}\right] \tag{35}$$

6. Numerical results

In this section we not try to provide a numerical analysis of the impact of Catastrophes on some of the important performance measures of this system. The computational results obtained by employing the above technique are discussed through tables and graphs. For different values of parameters $\alpha, \beta, \lambda, \nu, \mu_1$ and μ_2, we conducted several calculations on $P_{0,0}$ with the same parameters according to ν values, as shown in Tables.\

Table 1

Effect of ν on $E(N)$, this table has been generated using : $\mu_1 = 0.4, \quad \mu_2 = 0.6, \quad \alpha = 0.4, \quad \beta = 0.6, \quad L = 25$

Table -1: Effect of ν on $E(N)$

λ	0.25	0.50	0.75
ν	$E(N)$	$E(N)$	$E(N)$
0.1	0.671019	1.513310	3.424730
0.25	0.652010	1.314780	2.663630
0.50	0.643295	0.996633	1.844020
0.75	0.530110	0.754861	1.346920
0.95	0.444996	0.612408	1.080010

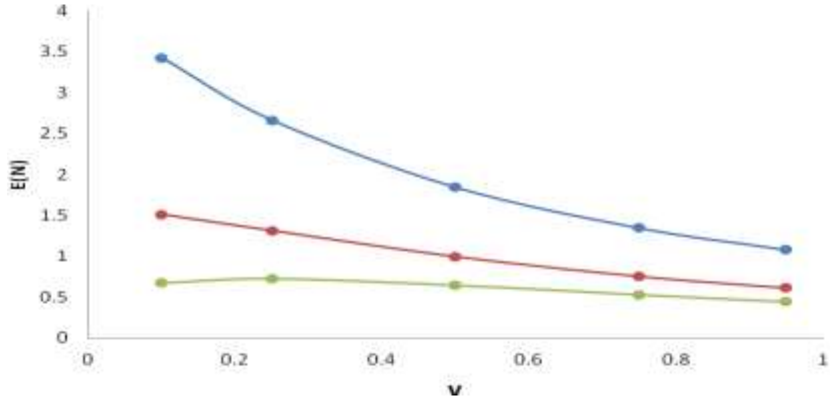

Chart -2: Effect of v on E(N)

For the sake of clarity, We compared between our new model, and the Tarabia model [17] in terms of the λ effect on E(N)when ν = 0. As shown below:

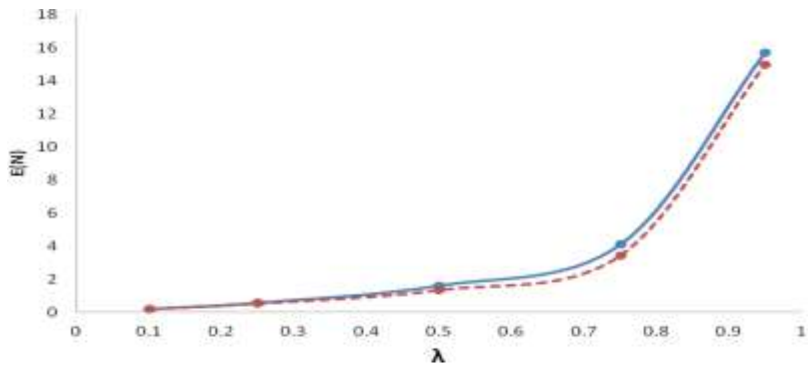

Chart -3 Comparison between our new model, and the Tarabia model [17] in terms of the λ effect on E(N)when $\nu = 0$

Table -2: Effect of μ_1 on Performance measures, this table has been generated using :

$\mu_2 = 0.6, \quad \lambda = 0.1, \quad \nu = 0.1, \quad \alpha = 0.4, \quad \beta = 0.6, \quad L = 25$

Table -2: Effect of μ_1

μ_1	$P_{0,0}$	$P_{1,0}$	$P_{0,1}$	$P_{1,1}$	$E(N)$
0.1	0.765306	0.260668	0.044991	0.030612	0.583333
0.25	0.827879	0.144031	0.049280	0.019505	0.416054
0.4	0.854965	0.101418	0.050709	0.013886	0.342873
0.6	0.873606	0.073188	0.051348	0.010223	0.291991
0.75	0.881949	0.060673	0.051476	0.008542	0.269012
0.95	0.889409	0.049477	0.051465	0.007010	0.248321

With increase in the value of μ_1 increases $P_{0,0}$ and $P_{0,1}$ while $E(N)$, $P_{1,0}$ and $P_{1,1}$ decreases.

Table -3: Effect of μ_2 on Performance measures, this table has been generated using :

$\mu_1 = 0.4, \quad \lambda = 0.1, \quad \nu = 0.1, \quad \alpha = 0.4, \quad \beta = 0.6, \quad L = 25$

Table -3: Effect of μ_2

μ_2	$P_{0,0}$	$P_{1,0}$	$P_{0,1}$	$P_{1,1}$	$E(N)$
0.1	0.601135	0.092584	0.483638	0.453922	33.62390
0.25	0.646894	0.113879	0.223445	0.238133	14.24730
0.4	0.675487	0.130269	0.148282	0.176705	8.998890
0.6	0.698781	0.146498	0.101623	0.136538	5.899780
0.75	0.709860	0.155689	0.081567	0.117830	4.617070
0.95	0.719766	0.165223	0.064030	0.100134	3.525510

With increase in the value of μ_2 increases $P_{0,0}$ and $P_{1,0}$ while $E(N)$, $P_{0,1}$ and $P_{1,1}$ decreases.

7. CONCLUSIONS

In the previous analysis, a two queues in parallel with jockeying, and restricted capabilities and Catastrophes is considered to obtain the steady state probabilities of the system size are also studied. Finally, some important performance measures have been obtained from the steady state probabilities, Through the tables and diagrams we note the effect of Catastrophes on this system.

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