Two-Class Priority Queueing System with Restricted Number of Priority Customers and Catastrophes

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Abstract - *In this paper, we analyze a two-class single-server preemptive priority queueing system with restricted number of priority customers and catastrophes. The arrivals per class follow the Poisson process with exponentially distributed service times. Customers are served on a first-come, first-served basis within their queue. Explicit expressions for the mean queue length and the joint distribution are derived in the steady-state case for the number of high and low priority customers in the system. The analysis is based on the generating function technique. Also, we study the impact of catastrophe on the system and other performance measures.*

Key Words: **Queueing, Queue length, Preemptive resume, Generating function, Catastrophe**

1. INTRODUCTION

This Priority queues occur in many aspects of daily life, especially in cases where preferential treatment is given to certain types of individuals, for example. Telecommunications field. Priority mechanisms are an invaluable scheduling method that allows messages from different categories to receive different quality of service. For this reason, the priority queue has received considerable attention in the literature. Cobham [1] was the first to consider the non-preemptive priority of the queue was considered with Poisson inputs and exponential retention time and discussed them for single and multi-channel cases. Holley [2] simplified Cobham's work. Barry [3] and Stephan [4] studied the problem of preemptive priority system problem with two priorities, Poisson inputs and exponential retention time. White and Christie [5] and Heathcote [6] studied the same problem for two channels and used the generating function method to describe the steady-state and transient queue length distributions, respectively. The main difficulty is to extend these results to multiple categories with general service time allocations in processing Laplace transformations that characterize the dynamics of the system. An extensive study of two priority plans can be cited in Miller [7] where he obtained several results in this model using the functions of generating and transformations of Laplace. A different approach to converting Laguerre to this model has also been applied, see for example Keilson and Sumita [8] and Keilson and Nunn [9]. Miller [10] considered exponential single server priority queues with two classes of customers. He Obtain repeated calculation formulas for steady distributions using Neuts [11] theory of matrix-geometric invariant probability vectors. Shack and Larson [12] are treated with a multi-server, non-preemptive, Multi-stationary queueing system with Poisson arrivals and exponential servers. They assumed a limited discipline in the priority queue and derived many performance measures. Franti [13,14] developed algorithms for a dynamic priority queue to compute performance measures such as the distribution of queue length and moments. Nishida [15] gave an approximation analysis for a heterogeneous multiprocessor system with preemptive and non-preemptive priority discipline and also presented the numerical comparison between simulation and approximation analysis. Recently, Bitran and Caldentey [16] analyzed a two-class single queueing system that features state dependent arrivals and preemptive priority service discipline. For this system, they computed the mean balance queue length and studied the effective service time and other first passage time quantities for both classes. More recently, Drekic and Woolford [17] analyzed a two class, single-server preemptive priority queueing model with low priority balking customers. They considered arrivals to each class are assumed to follow a Poisson with exponentioally distributed service times and obtained the steady state joint distribution of the number of high and low priority customers in the system. Choa and Zheng [20] considered an immigration birth and death process with total catastrophes and studied its transient as well as equilibrium behavior Kirshna Kumar et al. [23,24] obtained transient solution for analytically using continued fractions for the system size in an M/M/1 queueing system with catastrophes, server failures and nonzero repair time , the steady state probability of the system size, and obtained transient solution for the system size in the M/M/1 queueing model with the possibility of catastrophes at the service station in the direct way. Dharmaraja and Rakesh

Kumar [25] used the generation function technique to derive the model's transient solution directly. Explicit time dependent probabilities are obtained for system size . Other contributions are due to Economou and Fakinos [26,27] that extend some previous results on birth-death processes with catastrophes to the more general cases of continuous-time Markov chains with catastrophes resulting from a point process, and of the non-homogeneous Poisson process with total or binomial catastrophes. Following this introduction, the present is organized as follows. In section 2, we describe the model along with the assumption and several notation used throughout the paper. Moreover, we obtain the generating function for queue size of each priority classes. Using the obtained generating function and partial fractions technique, we derive explicit expressions for the steady-state probabilities. Finding explicit expressions for $P_{i,.}$ and $P_{.,j}$ the marginal distributions of the high and low priority classes, respectively is presented in Section 3. In Sections 4 include a simple way to obtain and compute the mean queue length of the high priority classes. In Section 5, several special cases obtained to support our work and conclusion. Also, some numerical results conclude the paper in Section 6.

2. Model Description and Analysis

We consider a single server queueing system serving two types of customers; class-1 and class-2 each having its own respective line and the arrival process for both types is state independent. A higher priority is assigned to class-1. Suppose that the rule of service within each class is FCFS and that the priority system is preemptive resumed. i.e. during the service of a low priority customer, if high priority customers joins the system, then the low priority customer's service will stop and will resume working when there are no high-priority customers in the system. Let also by the number of the customers of class *i* (*i* = 1, 2).

We denote the number of customers in the first class is restricted to a finite number L including the one being served, if any, and the number of the second class is infinite. Suppose also λ_1, λ_2 denote the arrival rates for the two classes and let μ_1, μ_2 denote the service rates for the two classes. We define the traffic intensities by $\rho_1 = \frac{\lambda_1}{\mu_1}, \rho_2 = \frac{\lambda_2}{\mu_2}$ and the steady state probability that the system is in state (i, j), where i is the number of the high priority customers and j is the number of low priority customers in the system. The catastrophes also γ Whenever a catastrophe occurs at the system, all the available customers there are destroyed immediately, the server gets inactivated momentarily and the server is ready for service when a new arrival occurs. Clearly, the governing difference equations of the system under consideration are given by

$$(\lambda_1 + \lambda_2 + \gamma)P_{0,0} = \mu_1 P_{1,0} + \mu_2 P_{0,1} + \gamma \tag{1}$$

$$(\lambda_1 + \lambda_2 + \mu_2 + \gamma)P_{0,j} = \lambda_2 P_{0,j-1} + \mu_1 P_{1,j} + \mu_2 P_{0,j+1}, \qquad j \geq 1 \tag{2}$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \gamma)P_{i,0} = \lambda_1 P_{i-1,0} + \mu_1 P_{i+1,0}, \qquad 1 \leq i \leq L-1 \tag{3}$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \gamma)P_{i,j} = \lambda_1 P_{i-1,j} + \lambda_2 P_{i,j-1} + \mu_1 P_{i+1,j}, \qquad 1 \leq i \leq L-1, j \geq 1 \tag{4}$$

$$(\lambda_2 + \mu_1 + \gamma)P_{L,0} = \lambda_1 P_{L-1,0} \tag{5}$$

$$(\lambda_2 + \mu_1 + \gamma)P_{L,j} = \lambda_1 P_{L-1,j} + \lambda_2 P_{L,j-1}, \qquad j \geq 1. \tag{6}$$

Then the above system (1) – (6) can be written as

$$\left[\lambda_1+\gamma+(\mu_2-\lambda_2 s)(1-s^{-1})\right]G_0(s)-\mu_1 G_1(s)=\gamma+\mu_2(1-s^{-1})P_{0,0} \quad (7)$$

$$-\lambda_1 G_{i-1}(s)+\left[\lambda_1+\mu_1+\gamma-\lambda_2 (s-1)\right]G_i(s)-\mu_1 G_{i+1}(s)=0, \quad 1\le i\le L-1 \quad (8)$$

$$\lambda_1 G_{L-1}(s)-(\mu_1+\gamma-\lambda_2(s-1))G_L(s)=0, \quad (9)$$

where $G_i(s)$ is the generating function for the steady state probabilities $P_{i,j}$. Now, these equations can be written as

$$A(s)G(s)=B(s), \quad (10)$$

where $G_i(s)=\left[G_0(s) \quad G_1(s) \quad G_2(s) \quad ..... \quad G_L(s)\right]^t$ is the generating function state probability vector for $P_{i,j}$, $B(s)=\left[\gamma+\mu_2(1-s^{-1})P_{0,0} \quad 0 \quad 0 \quad .... \quad 0\right]^t$ and $A(s)=\left[a_{i,j}\right]$ is $(L+1)\times(L+1)$ order tridiagonal matrix and its coefficients are given by

$$a_{i,j}=\begin{cases}\lambda_1+\gamma+(\mu_2-\lambda_2 s)(1-s^{-1}), & i=j=0\\ -\mu_1, & j=1, \quad i=0,1,2,...,L\\ \lambda_1(1-\delta_{i,L})+\mu_1+\gamma+\lambda_2(s-1), & j=i, \quad i=1,2,....,L\\ -\lambda_1, & j=i-1, \quad i=1,2,....,L\\ 0, & otherwise.\end{cases}$$

We find explicit expression for $G_i(s)$ by the following theorem.

Theorem 1. For any non-negative integer i, $0\le i\le L$, we have

$$G_i(s)=\frac{\lambda_1^i(\gamma+\mu_2(1-s^{-1})P_{0,0})D_{L-i}(s)}{(\gamma-\lambda_2(s-1))\prod_{k=0}^{L}(\lambda_1+\lambda_2+\mu_1+\gamma-\alpha_{L,k}\sqrt{\lambda_1\mu_1}-\lambda_2 s)}, \quad (11)$$

where

$$D_m(s)=\beta D_{m-1}(s)-\lambda_1\mu_1 D_{m-2}(s) \quad 2\le m\le L, \quad (12)$$

with

$D_1(s)=\gamma+\mu_1-\lambda_2(s-1),$
$D_2(s)=\beta\left[\gamma+\mu_1-\lambda_2(s-1)\right]-\lambda_1\mu_1,$
$\beta=\lambda_1+\gamma+\mu_1-\lambda_2(s-1),$
$P_{0,0}=\dfrac{(a^{L+1}-b^{L+1})-\lambda_1(a^L-b^L)}{(a^{L+1}-b^{L+1})}-\rho_2+\dfrac{\gamma}{\mu_2}E(Y),$
$a=\dfrac{(\lambda_1+\gamma+\mu_1)+\sqrt{(\lambda_1+\gamma+\mu_1)^2-4\lambda_1\mu_1}}{2},$
$b=\dfrac{(\lambda_1+\gamma+\mu_1)-\sqrt{(\lambda_1+\gamma+\mu_1)^2-4\lambda_1\mu_1}}{2},$
$E(y)=\sum_{i=0}^{L}G_i^{'}(1),$
$\rho_1=\lambda_1/\mu_1,\rho_2=\lambda_2/\mu_2$ and $\alpha_{L,k}, k=1,2,3,\ldots,L$ are the root of Chebychev's polynomials of second kind.

Proof. The proof closely parallels the elementary construction of continuous-time Markovian queue, see for instance Tarabia [19]. We will start by solving above system (10) using Crammer's rule, we obtain

$$G_i(s)=\frac{|A(s)|_i}{|A(s)|},\quad i=0,1,2,3,\ldots,L, \tag{13}$$

where $|A(s)|_i, i=0,1,2,\ldots,L$ is determinant of the matrix $A(s)$ with replacing its ith column by the elements of the vector $B(s)$. Expanding the determinant $|A(s)|_i$, we easily obtain

$$|A(s)|_i=\lambda_1^i(\gamma+\mu_2(1-s^{-1})P_{0,0})\ D_{L-i}(s),\quad i=0,1,2,\ldots,L, \tag{14}$$

where $D_{L-i}(s)$ represents the determinant of the bottom right square matrix obtained by eliminating the first row and column of the matrix $A(s)$ and it can be satisfied the following recurrence relations

$$D_m(s)=\beta\,D_{m-1}(s)-\lambda_1\,\mu_1\,D_{m-2}(s)\quad 2\le m\le L, \tag{15}$$

where

$D_1(s)=\gamma+\mu_1-\lambda_2(s-1), D_2(s)=\beta\left[\gamma+\mu_1-\lambda_2(s-1)\right]-\lambda_1\mu_1,$ and $\beta=\lambda_1+\gamma+\mu_1-\lambda_2(s-1).$

On other hand, we do some operations on the matrix $A(s)$ and it can be seen that satisfies the following recurrence relations

$$|A(s)|=(\beta-\lambda_1-\mu_1)\left[\left[\beta+\mu_2(1-s^{-1})\right]C_{L-1}(s)-\lambda_1\mu_1C_{L-2}(s)\right], \tag{16}$$

$$C_n(s)=\beta\,C_{n-1}(s)-\lambda_1\mu_1\,C_{n-2}(s),\quad 2\le n\le L, \tag{17}$$

where $C_1=\beta$ and $D_2=\beta^2-\lambda_1\mu_1.$

So

$$G_i(s)=\frac{\lambda_1^i\left(\gamma+\mu_2(1-s^{-1})P_{0,0}\right)D_{L-i}(s)}{(\beta-\lambda_1-\mu_1)\left[\left[\beta+\mu_2(1-s^{-1})\right]C_{L-1}(s)-\lambda_1\mu_1C_{L-2}(s)\right]},\qquad i=0,1,2,\ldots,L \tag{18}$$

Now, to compute $P_{0,0}$ we add all the Equations (7) - (9) for $j=0,1,\ldots.$ then we take limited at $s\to1$, we get

$$\mu_2P_{0,0}=\lim_{s\to1}\left[\mu_2G_0(s)+\mu_2(s-1)G_0^{'}(s)+\gamma\left(s\sum_{i=0}^{L}G_i^{'}(s)+\sum_{i=0}^{L}G_i(s)\right)\right.$$
$$\left.-\mu_2(s-1)\sum_{i=0}^{L}G_i(s)-\lambda_2s\left((s-1)\sum_{i=0}^{L}G_i^{'}(s)+\sum_{i=0}^{L}G_i(s)\right)-\gamma\right]$$

$$\mu_2P_{0,0}=\left[\mu_2G_0(1)+\gamma\left(\sum_{i=0}^{L}G_i^{'}(1)+\sum_{i=0}^{L}G_i(1)\right)-\lambda_2\sum_{i=0}^{L}G_i(s)-\gamma\right]. \tag{19}$$

We use the facts $\sum_{i=0}^{L}G_i(1)=1$ and $\sum_{i=0}^{L}G_i^{'}(1)=E(y)$, we obtain

$$G_0(1)=P_{0,0}+\rho_2-\frac{\gamma}{\mu_2}E(y), \tag{20}$$

Using Equation (18) with i=0,s=1, we get

$$G_0(1)=\frac{D_L(1)}{\left[\beta C_{L-1}(1)-\lambda_1\mu_1C_{L-2}(1)\right]}, \tag{21}$$

where

$$D_L(1)=(a-b)^{-1}[(a^{L+1}-b^{L+1})-\lambda_1(a^L-b^L)], \tag{22}$$

$$C_{L-1}(1)=(a-b)^{-1}(a^L-b^L), \tag{23}$$

$$C_{L-2}(1)=(a-b)^{-1}(a^{L-1}-b^{L-1}). \tag{24}$$

Substituting from Equations (22)-(24) into Equation (21), the marginal probability $P_{0,.}$ can be written as

$$G_0(1)=P_{0,.}=\frac{(a^{L+1}-b^{L+1})-\lambda_1(a^L-b^L)}{(a^{L+1}-b^{L+1})}, \tag{25}$$

On the other hand, Equations (20) and (25) together with some little simplifications imply that

$$P_{0,0}+\rho_2-\frac{\gamma}{\mu_2}E(y)=\frac{(a^{L+1}-b^{L+1})-\lambda_1(a^L-b^L)}{(a^{L+1}-b^{L+1})}.$$

Then

$$P_{0,0}=\frac{(a^{L+1}-b^{L+1})-\lambda_1(a^L-b^L)}{(a^{L+1}-b^{L+1})}-\rho_2+\frac{\gamma}{\mu_2}E(y). \tag{26}$$

Proposition 1. For any i, $0\le i\le L$ we have

$$P_{i,j}=\frac{A_0}{(\gamma+\lambda_2)}\left(\frac{\lambda_2}{(\gamma+\lambda_2)}\right)^j+\lambda_2^j\sum_{k=1}^{L}\frac{A_k}{\beta_k^{j+1}} \tag{27}$$

Proof. The proof this proposition is directly from the using of partial fractions and using the definition of Chebychev's polynomial of second kind (see Abramowitz and Stegun [28]) $|A|$ can be written as

$$|A|=\left(\gamma-\lambda_2(s-1)\right)\prod_{k=0}^{L}(\lambda_1+\lambda_2+\mu_1+\gamma-\alpha_{L,k}\sqrt{\lambda_1\mu_1}-\lambda_2 s), \tag{28}$$

where $\beta_k=\lambda_1+\lambda_2+\mu_1+\gamma-\alpha_{L,k}\sqrt{\lambda_1\mu_1}$, $k=1,2,\ldots,L$. Hence

$$G_i(s)=\frac{A_0}{(\gamma-\lambda_2(s-1))}+\sum_{k=1}^{L}\frac{A_k}{\left(\lambda_1+\lambda_2+\mu_1+\gamma-2\alpha_{L,k}\sqrt{\lambda_1\mu_1}-\lambda_2 s\right)}$$

$$G_i(s)=\frac{A_0}{(\gamma+\lambda_2)\left(1-\frac{\lambda_2 s}{\gamma+\lambda_2}\right)}+\sum_{k=1}^{L}\frac{A_k}{\beta_k\left(1-\frac{\lambda_2 s}{\beta_k}\right)},\qquad i=0,1,2,\ldots,L. \tag{28}$$

Extending in power of s and comparing the coefficient of in the both sides drives to $P_{i,j}$.

$$P_{i,j}=\frac{A_0}{(\gamma+\lambda_2)}\left(\frac{\lambda_2}{\gamma+\lambda_2}\right)^j+\lambda_2^j\sum_{k=1}^{L}\frac{A_k}{\beta_k^{j+1}},$$

with

$$A_0=\frac{\gamma\,\mu_1\rho_1^i(1-\rho_1)(\gamma+\lambda_2+\mu_2P_{0,0})}{\mu_1(\gamma+\lambda_2)(1-\rho_1^{L+1})+\gamma\,\mu_2(1-\rho_1^L)},$$

$$A_k=\frac{\lambda_1^i(\gamma+\mu_2(1-s_k^{-1})P_{0,0})D_{L-i}(s_k)}{(\gamma-\lambda_2(s_k-1))\prod_{\substack{k=0\\k\ne i}}^{L}(\beta_k-\beta_i)},\qquad s_k=\frac{\beta_k}{\lambda_2}.$$

3. The Marginal Distributions of the High and Low Priority Classes

Finding explicit expressions for $P_{i,.}$ and $P_{.,j}$ the marginal distributions of the high and low priority classes, respectively is obtained in the following propositions

Proposition 2. The marginal distributions of the high priority is given by

$$P_{i,.} = \frac{\lambda_1^i \left[(a^{L-i+1} - b^{L-i+1}) - \lambda_1 (a^{L-i} - b^{L-i}) \right]}{(a^{L+1} - b^{L+1})}, \qquad i = 0,1,2,\dots,L \tag{29}$$

Proof. As is clearly defined, $G_i(1)$ represents the marginal distribution of high priority, thus one can put into Equation(18), we may get

$$G_i(1) = \frac{\lambda_1^i D_{L-i}(1)}{\left[\beta C_{L-1}(1) - \lambda_1 \mu_1 C_{L-2}(1)\right]}, \qquad i = 0,1,2,\dots,L,$$

where

$$D_{L-i}(1) = (a-b)^{-1}[(a^{L-i+1} - b^{L-i+1}) - \lambda_1 (a^{L-i} - b^{L-i})], \tag{30}$$

now using Equations (23),(24) and (30), we obtain

$$P_{i,.} = G_i(1) = \frac{\lambda_1^i \left[(a^{L-i+1} - b^{L-i+1}) - \lambda_1 (a^{L-i} - b^{L-i}) \right]}{\left[(a+b)(a^L - b^L) - a\,b\,(a^{L-1} - b^{L-1}) \right]},$$

where $\beta = a + b$ and $\lambda_1 \mu_1 = a\,b$, then

$$P_{i,.} = \frac{\lambda_1^i \left[(a^{L-i+1} - b^{L-i+1}) - \lambda_1 (a^{L-i} - b^{L-i}) \right]}{(a^{L+1} - b^{L+1})}, \qquad i = 0,1,2,\dots,L$$

Proposition 3. The marginal distributions of the low priority is given by

$$P_{.,j} = \frac{\mu_2}{(\lambda_2 + \gamma)} P_{0,j+1} - \frac{\mu_2\, \gamma}{(\lambda_2 + \gamma)^2} \sum_{k=1}^{j} \left(\frac{\lambda_2}{\lambda_2 + \gamma} \right)^{j-k} P_{0,k} + \frac{\gamma}{(\lambda_2 + \gamma)} \left(\frac{\lambda_2}{\lambda_2 + \gamma} \right)^{j}, \qquad j \ge 0 \tag{31}$$

Proof. By adding Equations. (1), (3) and (5) for i, $0 \le i \le L$, we get

$$(\lambda_2 + \gamma) P_{.,0} + \lambda_1 \sum_{i=0}^{L-1} P_{i,0} + \mu_1 \sum_{i=0}^{L} P_{i,0} = \mu_1 \sum_{i=0}^{L} P_{i,0} + \lambda_1 \sum_{i=0}^{L-1} P_{i,0} + \mu_2 P_{0,1} + \gamma,$$

which leads to

$$P_{.,0} = \frac{\mu_2}{(\lambda_2 + \gamma)} + \frac{\gamma}{(\lambda_2 + \gamma)}.$$

In general and using similar approach by adding Equations. (2), (4) and (6) for i, $0 \le i \le L$, we obtain

$$(\lambda_2 + \gamma) P_{.,j} + \lambda_1 \sum_{i=0}^{L-1} P_{i,j} + \mu_1 \sum_{i=0}^{L} P_{i,j} + \mu_2 P_{0,j} = \mu_1 \sum_{i=0}^{L} P_{i,j} + \lambda_1 \sum_{i=0}^{L-1} P_{i,j} + \lambda_2 P_{.,j-1} + \mu_2 P_{0,j+1},$$

which simply leads to

$$(\lambda_2+\gamma)P_{.,j}+\mu_2 P_{0,j}=\lambda_2 P_{.,j-1}+\mu_2 P_{0,j+1}.$$

By induction method, the marginal probabilities $P_{.,j}$ satisfy the following recursion relation

$$P_{.,j}=\frac{\mu_2}{(\lambda_2+\gamma)}P_{0,j+1}-\frac{\mu_2\,\gamma}{(\lambda_2+\gamma)^2}\sum_{k=1}^{j}\left(\frac{\lambda_2}{\lambda_2+\gamma}\right)^{j-k}P_{0,k}+\frac{\gamma}{(\lambda_2+\gamma)}\left(\frac{\lambda_2}{\lambda_2+\gamma}\right)^{j},\qquad j\geq 0$$

4. The Mean Queue Length of the High Priority Class

In this section, our main objective is to get the mean queue length of high priority class. To obtain it, we have supposed that N_1 be the queue length of the first class and define

$$E(N_1)=\sum_{i=0}^{L}\sum_{j=0}^{\infty} i\;P_{i,j}=\sum_{i=0}^{L} i\;P_{i,.}$$

By substituting from Proposition 2 into the above equation, we have

$$E(N_1)=\sum_{i=0}^{L} i\;\frac{\lambda_1^i\left[(a^{L-i+1}-b^{L-i+1})-\lambda_1(a^{L-i}-b^{L-i})\right]}{(a^{L+1}-b^{L+1})}$$

$$=\frac{1}{(a^{L+1}-b^{L+1})}\left[a^{L+1}\sum_{i=0}^{L} i\left(\frac{\lambda_1}{a}\right)^i-b^{L+1}\sum_{i=0}^{L} i\left(\frac{\lambda_1}{b}\right)^i-\lambda_1 a^L\sum_{i=0}^{L} i\left(\frac{\lambda_1}{a}\right)^i+\lambda_1 b^L\sum_{i=0}^{L} i\left(\frac{\lambda_1}{b}\right)^i\right]$$

$$=\frac{1}{(a^{L+1}-b^{L+1})}\left[\lambda_1 a^L\sum_{i=0}^{L}\frac{d}{d(\lambda_1/a)}\left(\frac{\lambda_1}{a}\right)^i-\lambda_1 b^L\sum_{i=0}^{L}\frac{d}{d(\lambda_1/b)}\left(\frac{\lambda_1}{b}\right)^i\right.$$

$$\left.-\lambda_1^2 a^{L-1}\sum_{i=0}^{L}\frac{d}{d(\lambda_1/a)}\left(\frac{\lambda_1}{a}\right)^i+\lambda_1^2 b^{L-1}\sum_{i=0}^{L}\frac{d}{d(\lambda_1/b)}\left(\frac{\lambda_1}{b}\right)^i\right]$$

$$E(N_1)=\frac{\lambda_1}{(a^{L+1}-b^{L+1})}\left[\frac{b\sum_{k=0}^{L}a^k\lambda_1^{L-k}-a\,\lambda_1\sum_{k=0}^{L-1}a^k\lambda_1^{L-(k+1)}-b^{L+1}}{(b-\lambda_1)}\right]\qquad(32)$$

5. Special Cases

a) If we take $a=\lambda_1$ and $b=\mu_1$, we obtain

$$E(N_1)=\frac{\rho_1\left[1-(L+1)\rho_1^L+L\,\rho_1^{L+1}\right]}{(1-\rho_1)(1-\rho_1^{L+1})}$$

which agress with the know result given by Tarabia [20] and Sharma and Gupta [21] when $t\to\infty$. Moreover, it shows that high priority queue is functioning independently as an $M/M/1/L$ queue.

b) When there is no catastrophe, i.e., $(\gamma = 0)$ and we put $\rho_2 = 0$, we get all of result the well known $M/M/1/L$ model such as

$$E(N_1) = \frac{\rho_1^i (1-\rho_1)}{(1-\rho_1^{L+1})}, \quad i = 0,1,2,\ldots,L, \rho_1 < 1.$$

6. Numerical Results

To conclude, we shall now make use of the results outlined in the foregoing to indicate by means of some examples the goodness of our continuous approximation to the queueing system with catastrophes. For different values of parameters $\lambda_1, \mu_1, \lambda_2, \mu_2$ and γ, we study the impact of catastrophe γ on the steady state probability of the system, when $\lambda_1 = 0.12, \mu_1 = 0.2, \lambda_2 = 0.2, \mu_2 = 0.57$ and $L = 25$ on $P_{0,0}$ of the given system and the results are shown in Table1 .

Table -1: Effect of γ on the steady state probability of the system $P_{0,0}$ when $\lambda_1 = 0.12, \mu_1 = 0.2, \lambda_2 = 0.2, \mu_2 = 0.57$ and $L = 25$

γ	0.02	0.2	0.5	0.6	0.85	0.98
$P_{0,0}$	0.152632	0.428241	0.584867	0.620027	0.693817	0.727395

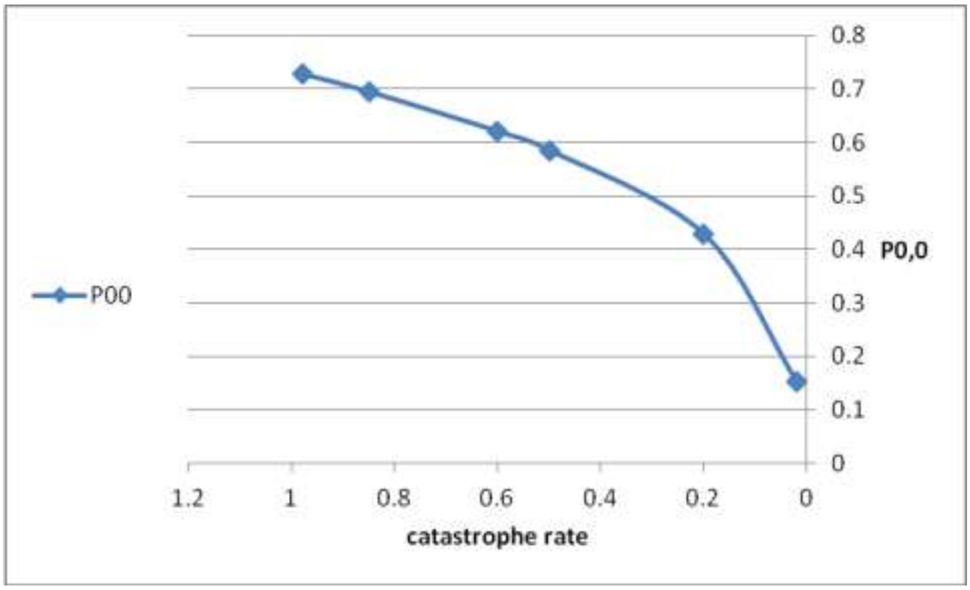

Chart -1: Effect of γ on the steady state probability of the system $P_{0,0}$

From Table (1) and Chart (1) all show that the $P_{0,0}$ increase as the catastrophe rate increases.

7. CONCLUSIONS

We remark that the explicit expressions obtained for the mean queue length and the steady state joint distribution of the number of high priority customers in the system are simple and with catastrophe . Moreover, on the whole, the analysis of the first class behaves like M/M/1/L Markovian queue.

REFERENCES

[1] A. Cobham, "Priority assignment in waiting line problem", J. Oper. Res. Soc. Am., vol. 2, 1954, pp. 70–76.
[2] J.L. Holly, '' Waiting line subject to priorities", J. Oper. Res. Soc. Am., vol. 2, 1954, pp. 341–343.

[3] J.Y. Barry, ''A priority queueing problem'', J. Oper. Res., vol. 4, 1954, pp. 385–386.

[4] F.F. Stephan, ''Two queues under preemptive priority with Poisson arrival and service rates '', J. Oper. Res., vol. 6, 1958, pp. 399–418.

[5] H. White, L. Christie, ''Queueing with preemptive priorities or with breakdown'', J. Oper. Res., vol. 6 1958, pp. 79–96.

[6] C. Heathcote, ''The time-dependent problem for a queue with preemptive priorities, '' J. Oper. Res., vol. 7, 1959, pp. 670–680.

[7] R.G. Miller, ''Priority queues'', J. Ann. Math. Stat., vol. 31, 1960, pp. 86–103.

[8] J. Keilson, U. Sumita, ''Evaluation of the total time in system in a preempt/resume priority queue via modified Lindley process'', J. Adv. Appl. Prob., vol. 15, 1983, pp. 840–856.

[9] J. Keilson, W. Nunn, '' Laguerre transformation as a tool for the numerical solution of integral equations of convolutions type'', J. Appl. Math. Comput., vol. 5, 1979, pp. 313–359.

[10] D.R. Miller, "Computation of steady-state probabilities for M/M/1 priority queues", J. Oper. Res., vol. 29, issue. 5, 1981, pp. 945–958.

[11] M.F. Neuts, ''Matrix-geometric solutions in stochastic models. Baltimore'', The Johns Hopkins University Press, 1981.

[12] C. Schaak, , R. C. Larson, ''An N-server cut off priority queue'', J. Oper. Res., vol. 34, 1986, pp. 257–266.

[13] S.S. Franti, ''Algorithms for a dynamic priority queue with two types of customers'', Drexel University, Philadelphia, 1985.

[14] S.S. Franti, ''Analysis of a dynamic priority queue'', J. Commun. Stat. Stoch. Model, vol. 6, issue. 3, 1990, pp. 415–444.

[15] T. Nishida, ''Approximate analysis for heterogeneous multiserver systems with priority jobs'', J. Perform. Eval., vol. 15, 1992, pp.77–88.

[16] G. Bitran, R. Caldentey, ''Two-class priority system with state dependent arrivals'', J. Queueing Syst., vol. 40, issue. 4, 2000, pp. 355–382.

[17] S. Drekic, D.G. Woolford, ''A preemptive priority queue with balking'', Eur. J. Oper. Res., vol. 164, 2005, pp. 387–401.

[18] A.M.K. Tarabia, ''Transient analysis of a non-empty M/M/1/N queue an alternative approach'', J. Oper. Res. Soc. India, vol. 38, issue. 4, 2001, pp. 431–440.

[19] A.M.K. Tarabia, ''Analysis of M/M/1/N queueing system with two priority classes'', J. Oper. Res. Soc. India, vol. 44, issue. 4, 2007, pp. 370–389.

[20] A.M.K. Tarabia, ''Two-class priority queueing system with restricted number of priority customers'', Int. J. Electron. Commun. (AEU), vol. 61, 2007, pp. 534–539.

[21] O.P. Sharma, U.C. Gupta, ''Transient behaviour of an M/M/1/N queue'', Stoch. Process. Appl., vol. 13, 1982, pp. 327–331.

[22] A. Chao, Y. Zheng, "Transient analysis of immigration-birth-death process with total catastrophes", Prob. Eng. Inf. Sci., vol. 17, 2003, pp. 83–106.

[23] B. Krishna Kumar, A. Krishnamoorthy, S.Pavai Madheswari, S. Sadiq Basha, "Transient analysis of a single server queue with catastrophes, failures and repairs", Springer Sci. , Bus. Med., vol. 56, 2007, pp. 133–141.

[24] B. Krishna Kumar, D. Arivudainambi, "Transient solution of an M/M/1 with catastrophes", Oper. Res. Soc., India, vol. 52, issue. 4, 2015, pp. 810–826.

[25] Dharmaraja, Rakesh Kumar, "Transient solution of an Markoving queuing model heterogenous servers and catastrophes", comp. and math. with appl., vol. 40, 2000, pp. 1233–1240.

[26] A. Economou, D. Fakinos, "A continuous-time Markov chain under the influence of a regulating point process and applications in stochastic models with catastrophes", Eur. J. Oper. Res. vol. 149, 2003, pp. 625-640.

[27] A. Economou, D. Fakinos, "Alternative approaches for the transient analysis of Markov chains with catastrophes", J. Stat. Theor. Pract. vol. 2, 2008, pp. 183-197.

[28] M. Abramowitz, I.A. Stegun. Handbook of mathematical functions. New York: Dover, 1970.