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Finite Difference Scheme for Heat Transfer in Three Dimensional Flows through a Porous Medium with Periodic Permeability

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Abstract: - The effect of permeability variation on heat transfer and flow through a highly porous medium bounded by infinite flat porous surface is studied. The problem is three dimensional due to periodic permeability .The solution of governing equations are obtained, using finite difference method-numerical technique. The expression for the velocity, temperature, skin friction and rate of heat transfer are obtained. The skin friction and rate of heat transfer are discussed with the help of tables.

Keywords: Periodic permeability, three dimensional flow, finite difference method.

Introduction

Heat and mass transfer in porous media is discussed in a wide range of disciplines as Mechanical Engineering, Biomedical, Chemical Engineering, Food Technology, Petroleum Engineering, Groundwater Hydrology, and Industrial Engineering. The important applications include packed bed reactors, Filtering dry fuel cell, Contamination migration in groundwater, Insulation nuclear reactors and many more. Heat transfer problems have many practical applications in industrial manufacturing processes such as polymer composition. This rapidly increase in research activity has been many due to number of important applications in modern industries ranging from chemical reactors , underground spread of pollutants , heating of rooms, combustion , fires and many other heat transfer processes , both natural and artificial. Heat transfer plays a vital role in designing of car radiators, solar collection, various parts of power plant and even space craft. The basic three mechanism of heat transfer are convection, conduction and radiation.

The problems of flow through porous medium has numerous scientific and engineering applications, in view of these applications a wide research is done and still going on. Raptis (1981) ,studied the free convective flow and mass transfer through a porous medium bounded by infinite vertical surface with constant suction. Raptis (1983), investigated unsteady free convective flow through a porous medium. Adrian Bejan and R.Khair(1985), studied the heat and mass transfer by natural convection in porous medium.Bestman(1989), analysed the unsteady flow of an incompressible fluid in horizontal porous medium with suction.K.D Singh(1993),studied three dimensional viscous flow and heat transfer along a porous plate. K.D.Singh and G.N.Verma(1995), studied three dimensional oscillatory flow through a porous medium with periodic permeability. They also analysed heat transfer in three dimensional flow through a porous medium with periodic permability. N.Ahmed and D.Sarma (1997), studied three dimensional free convective flow and heat transfer through a porous medium. S.S.Das and U.K. Tripathy(2010) , analysed the effect of periodic suction on three dimensional and heat transfer past a vertical porous plate embedded in porous medium. Ujwanta , Hamza and Ibrahim (2011) ,analysed heat and mass transfer through a porous medium with variable permeability and periodic suction. The aim of this paper is to study the effect of periodic suction on three dimensional flow of a viscous incompressible fluid through a highly porous medium. The porous medium is bounded by an infinite horizontal porous surface with periodic suction. The analysis of skin friction which is very important from industry point of view is also studied.

Mathematical Formulation

For mathematical formulation of the problem, we consider the viscous incompressible fluid flow through a highly porous medium bounded by infinite porous surface. The surface lying horizontally on the x^*-z^* plane, with variable suction. The x^*-axis is taken along the infinite surface being the direction of the flow and y^*-axis is taken normal to surface directed into fluid flowing laminarly with free stream velocity U. Since surface is considered infinite in x^*-direction so all physical quantities are independent of x .

The permeability of the porous medium is assumed to be of the form

$$k^*(z^*) = \frac{K_0^{\ *}}{\left[\ 1+\varepsilon\cos\frac{\pi z^*}{L}\ \right]} \qquad (1)$$

where K^*_0 is mean permeability of the medium , L is wavelength of permeability distribution and ϵ(<<1) is amplitude of permeability variation. Denoting velocity components u* , v* ,w* in the direction x* , y* , z* respectively and temperature T*. The flow in highly porous medium is governed by following equations:

Continuity equation:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \qquad (2)$$

Momentum equation

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=\upsilon\left(\frac{\partial^2 u^*}{\partial\ y^{*2}}+\frac{\partial^2 u^*}{\partial\ z^{*2}}\right)-\frac{\upsilon}{k^*}(u^*-U) \qquad (3)$$

$$v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\upsilon\left(\frac{\partial^2 v^*}{\partial\ y^{*2}}+\frac{\partial^2 v^*}{\partial\ z^{*2}}\right)-\frac{\upsilon}{k^*}(v^*) \qquad (4)$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}+\upsilon\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial\ z^{*2}}\right)-\frac{\upsilon}{k^*}(w^*) \qquad (5)$$

Energy equation:

$$\rho c_p\left(v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}\right)=k\left(\frac{\partial\ T^*}{\partial\ y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right)+\mu\phi^* \qquad (6)$$

Where

$$\phi^*=2\left\{\left(\frac{\partial v^*}{\partial y^*}\right)^2+\left(\frac{\partial w^*}{\partial z^*}\right)^2\right\}+\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2+\left(\frac{\partial w^*}{\partial y^*}+\frac{\partial v^*}{\partial z^*}\right)^2+\left(\frac{\partial u^*}{\partial z^*}\right)^2\right\}$$

ρ is the density , μ is the dynamic viscosity, ν is the kinematic viscosity, p* is the pressure , k* is the permeability of porous medium, k is the thermal conductivity of fluid and c_p is the specific heat of fluid at constant pressure.

The initial and boundary condition of the problem are

$y^* = 0$; $u^* = 0$, $w^* = 0$, $T^* = T^*_w$, $v^* = -\alpha$

(7)

$y^* = \infty$; $u^* = U$, $w^* = 0$, $T^* = T^*_\infty$, $p^* = p^*_\infty$, $v^* = V$

The subscripts ‘ w’ and ‘∞’ denote the physical quantities at the surface and in free stream respectively.

Introducing the following non-dimensional quantities

$$y=\frac{y*}{L}, z=\frac{z*}{L}, u=\frac{u*}{U}, v=\frac{v*}{U}, w=\frac{w*}{U}, p=\frac{p*}{\rho U^2}, p_\infty=\frac{p*_\infty}{\rho U^2}, T=\left(\frac{T*-T*_\infty}{T*_w-T*_\infty}\right)$$

The other parameters introduced are defined as

$R=\frac{UL}{\upsilon}$ - Reynolds number $\alpha=\frac{V}{U}$ - Suction parameter

$\Pr=\frac{\mu c_p}{k}$ - Prandlt number

$E=\frac{U^2}{c_p(T*_w-T*_\infty)}$ - Eckert number

$K=\frac{k*U^2}{\upsilon^2}$ - permeability parameter

Equations (3) –(6) reduces to the following forms:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{8}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{R}{K_0}(u-1)[1+\varepsilon\cos\pi z] \tag{9}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{R}{K_0}v(1+\varepsilon\cos\pi z) \tag{10}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{R}{K_0}w(1+\varepsilon\cos\pi z) \tag{11}$$

$$v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{1}{R\Pr}\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{E}{R}\phi \tag{12}$$

Where

$$\phi=2\left\{\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left\{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right\}$$

The corresponding boundary condition now reduce to the form

y= 0 ; u = 0 , w = 0 , T = 1 , v = -α

y= ∞ ; u = 1 , w = 0 , T = 0 , p → p_∞ (13)

Method of solution

In order to solve these differential equations from (8) – (12) , we assume the solution using Perturbation technique. Since ϵ (<<1) the amplitude of permeability variation is very small :

$$u(y,z) = u_0(y) + \epsilon u_1(y,z) + \epsilon^2 u_2(y,z) + \ldots$$

$$v(y,z) = v_o(y) + \epsilon v_1(y,z) + \epsilon^2 v_2(y,z) + \ldots$$

$$w(y,z) = w_o(y) + \epsilon w_1(y,z) + \epsilon^2 w_2(y,z) + \ldots \quad (14)$$

$$p(y,z) = p_o(y) + \epsilon p_1(y,z) + \epsilon^2 p_2(y,z) + \ldots$$

$$T(y,z) = T_o(y) + \epsilon T_1(y,z) + \epsilon^2 T_2(y,z) + \ldots$$

When $\epsilon = 0$, the problem reduces to the two dimensional flow through a porous medium with constant suction. In this case equations (8) – (12) reduces to

$$\frac{dv_0}{dy} = 0 \quad (15)$$

$$\frac{d^2u_0}{dy^2} - v_0 R \frac{du_0}{dy} - \frac{R^2}{K} u_0 = -\frac{R^2}{K} \quad (16)$$

$$\frac{d^2T_0}{dy^2} - v_0 R \Pr \frac{dT_0}{dy} = -E \Pr u_0'^2 \quad (17)$$

The corresponding boundary condition now become

$$y = 0 \;; \quad u_0 = 0, \quad w_0 = 0, \quad T_0 = 1, \quad v_0 = -\alpha$$

$$y = \infty \;; \quad u_0 = 1, \quad w_0 = 0, \quad T_0 = 0, \quad p_0 = p_\infty \quad (18)$$

The solution for $u_0(y)$ and $T_0(y)$ using the boundary condition (19) are

$$u_0(y) = 1 - e^{-my} \quad (19)$$

$$T_0(y) = (1 + M)\, e^{-\alpha R \Pr y} - M e^{-2my} \quad (20)$$

With $v_0 = -\alpha$, $w_0 = 0$, $p_0 = p_\infty$ (21)

Where

$$m = \frac{R}{2}\left[\alpha + \sqrt{\left(\alpha^2 + \frac{4}{K}\right)}\right] \quad \text{and} \quad M = \frac{E \Pr m}{2(2m - \alpha R \Pr)}$$

When $\epsilon \neq 0$, substituting equations (14) into equations (8) – (12) and comparing the coefficients of like power of ϵ and neglecting higher power of ϵ^2 , we get the following with equations using equation (21):

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (22)$$

$$v\frac{\partial u_0}{\partial y}-\alpha\frac{\partial u_1}{\partial y}=\frac{1}{R}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)-\frac{R}{K}\left[u_1+(u_0-1)\cos\pi z\right] \tag{23}$$

$$-\alpha\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right)-\frac{R}{K}(v_1-\alpha\cos\pi z) \tag{24}$$

$$-\alpha\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-\frac{R}{K}w_1 \tag{25}$$

$$v_1\frac{\partial T_0}{\partial y}-\alpha\frac{\partial T_1}{\partial y}=-\frac{1}{R\Pr}\left(\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial z^2}\right)-\frac{2E}{R}\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} \tag{26}$$

The corresponding boundary condition now become

$y=0$; $u_1=0$, $w_1=0$, $T_1=0$, $v_1=0$

$y=\infty$; $u_1=0$, $w_1=0$, $T_1=0$, $p_1=0$ (27)

Equations (21) – (25) are linear partial differential equations which describe the three dimensional

flow. We assume the solution of equations (23)-(25) of the form

$u_1(y,z)=u_{11}(y)\cos\pi z$

$v_1(y,z)=v_{11}(y)\cos\pi z$

$w_1(y,z)=-\frac{1}{\pi}v'_{11}(y)\sin\pi z$ (28)

$p_1(y,z)=p_{11}(y)\cos\pi z$

$T_1(y,z)=T_{11}(y)\cos\pi z$

where the $v'_{11}(y)$ denotes the differentiation with respect to y. The expressions for $v_1(y,z)$ and $w_1(y,z)$ have been selected so that the equation of continuity (22) is satisfied. Substituting these expressions (28) into (24) and (25) and solving these under transformed boundary conditions (27) :

$$-\alpha v'_{11}=-p'_{11}+\frac{1}{R}\left[v''_{11}-\pi^2 v''_{11}\right]-\frac{R}{K_0}\left[v_{11}-\alpha\right] \tag{29}$$

$$\frac{\alpha}{\pi}v_{11}=-\pi p_{11}+\frac{1}{R}\left[-\frac{v'''_{11}}{\pi}-\pi v'_{11}\right]+\frac{R}{K_0\pi}v'_{11} \tag{30}$$

Eliminating the terms $p_{11'}$ and p_{11} in equations (29) and (30) , we get

$$v''''_{11}+R\alpha v'''_{11}-\left[\frac{R^2}{K_0}-2\pi^2\right]v''_{11}+\alpha R\pi^2 v'_{11}-\left[\frac{R^2\pi^2}{K_0}+\pi^4\right]v_{11}+\frac{R^2\pi^2\alpha}{K_0}=0 \tag{31}$$

Substituting the following finite difference formula

$$v'_{11}(i) = \frac{v_{11}(i+1) - v_{11}(i-1)}{2h}$$

$$v''_{11}(i) = \frac{v_{11}(i+1) - 2v_{11}(i) + v_{11}(i-1)}{h^2}$$

$$v'''_{11}(i) = \frac{v_{11}(i+2) - 2v_{11}(i+1) + 2v_{11}(i-1) - v_{11}(i-2)}{2h^3}$$

$$v''''_{11}(i) = \frac{v_{11}(i+2) - 4v_{11}(i+1) + 6v_{11}(i) - 4v_{11}(i-1) + v_{11}(i-2)}{h^4}$$

Put above results in equation (31), we get

$$A_1 v_{11}(i+2) - A_2 v_{11}(i+1) + A_3 v_{11}(i) - A_4 v_{11}(i-1) + A_5 v_{11}(i-2) + \frac{2h^4 \pi^2 R\alpha}{K_0} = 0 \qquad (32)$$

Where $A_1 = 2 + \alpha Rh$

$$A_2 = 8 + 2Rh - 2h^2 \left[\frac{R^2}{K_0} - 2\pi^2 \right] - \alpha R \pi^2 h^3$$

$$A_3 = 12 - 4h^2 \left[\frac{R^2}{K_0} - 2\pi^2 \right] - 2h^4 \left[\frac{R^2 \pi^2}{K_0} + \pi^4 \right]$$

$$A_4 = 8 - 2\alpha Rh + 2h^2 \left(\frac{R^2}{K_0} - 2\pi^2 \right) + \alpha R \pi^2 h^3$$

$A_5 = 2-\alpha Rh$

Similar finite difference formulae for equation (23) and (25)

$A_1 = u_{11}(i+1) - B_1 u_{11}(i) + A_5 u_{11}(i-1) = B(i)$ (33)

$D_1 T_{11}(i+1) - D_2 T_{11}(i) + D_3 T_{11}(i-1) = F(i)$ (34)

$D_1 = 2+RPrh$

$D_2 = 4+2\pi^2 h^2$

$D_3 = 2-\alpha RPrh$

$D(i) = 2(hRPr)^2 v_{11}(i)$ e-RPrih

$$B_1 = 4 + 2h^2\left[\pi^2 + \frac{1}{K_0}\right]$$

$$B(i) = v_{11}e^{-my} - \frac{2h^2R^2e^{-my}}{K_0}$$

$$F(i) = R\Pr v_{11}2h^2\left[(1+M)e^{-\alpha R\Pr y} - Me^{-2my}\right] - 2E\Pr u_{11}(1-e^{-my})$$

Equations (32) ,(33) and (34) have been solved by Gauss-seidal iteration method for velocity and temperature.To prove the convergence of finite difference scheme , the calculations are carried out slightly changed value of h .Smaller change is observed in the value of velocity and Temperture. Thus , it is concluded that finite difference scheme is convergent and stable.

Skin friction

Now we discuss the important characteristics of the problem skin friction component in the x*-direction in the wall non-dimensional form are given by

$$\tau_x = \frac{\tau'_x}{\rho UV} = \frac{\nu}{VL}\left(\frac{du_0}{dy}\right)_{y=0} \qquad \tau_x = \frac{1}{R}\left(\frac{du_0}{dy}\right)_{y=0} + \frac{1}{R}\varepsilon\left(\frac{du_{11}}{dy}\cos\pi z\right)_{y=0} \tag{35}$$

Rate of heat transfer

The rate of heat transfer i.e heat flux at the surface in terms of Nusselt number(Nu) is given by

$$Nu = \frac{q_w}{\rho Vc_p(T_w - T_\infty)} = \frac{k}{\rho Vc_pL}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{1}{Rpr}\left[\frac{dT}{dy} + \varepsilon\frac{dT_{11}}{dy}\cos\pi z\right]_{y=0} \tag{36}$$

Results and discussion

The velocity of the flow field found to change with variation of suction parameter α , permeability parameter K and Reynolds number R. The effect of permeability of medium on velocity of flow is shown in Figure 1.

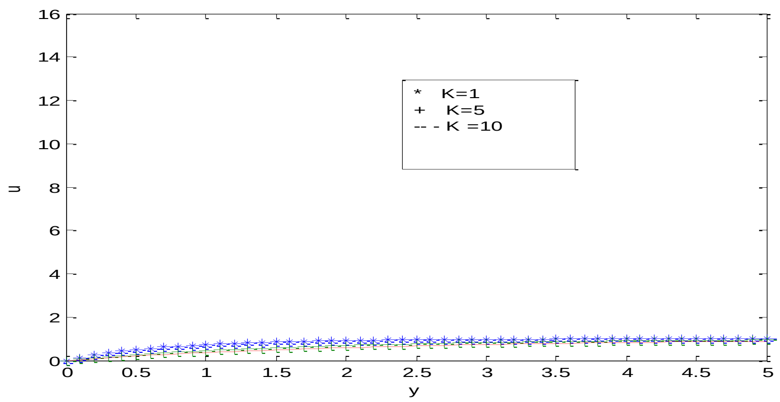

Figure1: velocity profile against y for different values of K with R = 1 , pr=0.71 ,K=1,ε = 0.2 and z=0.

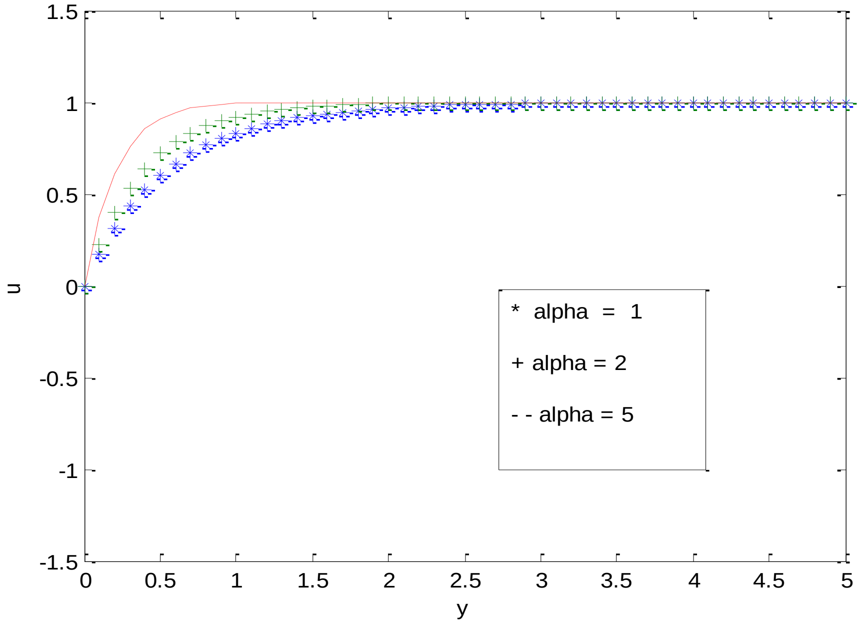

Figure2: velocity profile against y for different values of α with R = 1 , α=0.2 ,ε = 0.2 and z=0.

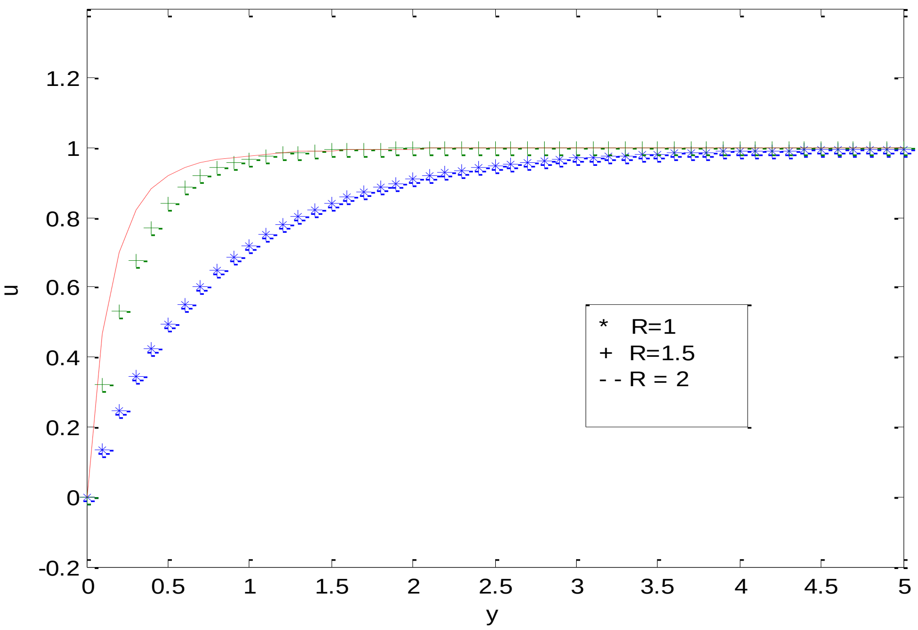

Figure3: velocity profile against y for different values of R with α=0.2 ,K=1,ε = 0.2 and z=0

It is also observed that the variation in the temperature of the flow field is due to suction parameter α and Reynolds number R. Also, increasing suction parameter (α) results a decrease in temperature.

Skin friction

The variations in the value of x component of the skin friction at the wall for different values of α and K given in Table1. It is observed that permeability parameter K decreases skin friction. The increasing the suction parameter (α) there is enhancement in the skin friction at the wall.

Table1: x component of skin friction τ_x against α for different values of K with R = 1,

α =0.2 , ϵ = 0.2

	K = 0.2	K=1	K=5
α	τ_x	τ_x	τ_x
0	2.404	1.0466	0.5602
0.2	2.506	1.1465	0.5708
0.5	2.669	1.3263	0.8623
2.0	3.637	2.4485	2.1101

Rate of heat transfer

The rate of heat transfer at the wall is calculated in terms of Nusselt number (Nu) for different values of α and K in the table2.It is observed that increasing suction parameter α heat flux at the wall grows and effect is reverse on increasing of permeability parameter K.

Table2: Rate of heat transfer(Nu) against α and different values of K with R = 1 , Pr = 0.71 , α = 0.2,

E =0.01 and ϵ = 0.2

α	K=0.2	K=1	K=5
0	0.0114	0.005	0.003
0.2	0.2003	0.1931	0.1869

Conclusion:

- The effect of permeability is medium on the velocity of the flow field .
- Increasing suction parameter results decrease in temperature.
- The permeability parameter decreases x-component and increase the magnitude of z-component of skin friction at the wall. On the other hand growing suction parameter increases skin friction.
- A growing suction parameter enhance the rate of transfer and effect is reverse on increasing skin friction.

References

[1] Singh K.D.,Sharma R. Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability. Ind.J.Pure Appl.Math.2002,33(6),941-949.

[2] Govindarajulu T., Thangaraj C.J. The effect of variable suction on free convection on a vertical plate with suction or injection,Int.J.Num.Methods Fluids.1993,17,927-941.

[3] Das S.S., Tripathy U.K. Effect of periodic suction on three dimensional flow and heat transfer past a vertical porous plate embedded in a porous medium.Int.J.Energy and Environment.2010 1(5) ,757-768.

[4] Singh K.D.,Verma G.N. Heat transfer in three dimensional flow through a porous medium with periodic permeability.ZAMM.,75(12),1995,950.

[5] Uwanta I.J,Hamza M.M. , Ibrahim M.O. Heat and mass transfer through a porous medium with variable permeability and periodic suction. Int.J. Computer applications.2011,36(2).

[6] Singh P. , Mishra J.K,Narayan K.A. Three dimensional convective flow and heat transfer in a porous medium Ind.J.Pure Appl.Math.1988,19(11),1130-1135.

[7] Ahmed S. The effect of viscous dissipative heat on three dimensional oscillatory flow with periodic suction velocity. Ind.J.Science and Tech.2010,3(3).

[8] P.Ravinder Reddy, K.Srihaari and S.Raji Reddy Combined heat and mass transfer in mhd three Dimensional porous flow with periodic permeability and heat absorption.

[9] Ravikumar V., Raju M.C., Raju G.S.S. MHD three dimensional coquette flow past a porous plate with heat transfer IOSRJM,2012, 1(3),03-09.