

STRUCTURAL ANALYSIS OF VISCOELASTIC MATERIALS: REVIEW PAPER

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ABSTRACT: This paper is aimed at for the presentation of the research carried out on the analysis of the viscoelastic material since 1964 to till date. Viscoelastic material has both viscous and elastic behaviour together, due to this for static loading a structural configurations embedded or with such material will exhibit time dependent static response. To capture this phenomenon rectangular, trapezoidal integration are used. Other than this time dependent behaviour of material, these materials are nearly incompressible. Hence instead of displacement formulation mixed up formulation is considered for the appropriate behaviour of such structure.

1. INTRODUCTION

Viscoelastic materials are used in structural elements of many engineering structures such as asphalt pavements, Solid fuels in rockets etc. The difference between viscoelastic media and elastic media lies essentially in the relation between stress and strain. Elastic analysis is based upon the constant of proportionality such as young's modulus whereas viscoelastic relation must allow for time or strain rate effects. For the analysis of viscoelastic materials the straight forward application of displacement method of finite element method fails and it requires special formulations.

2. PAST LITERATURES

This paper gives an overview of different studies carried out in analysis of viscoelastic materials.

Leonard R. Herrmann (1896) presented a variational theorem by expressing elastic field equations in terms of the displacements and a function of the mean pressure. This takes the advantages of theorem of minimum potential energy and solves the difficulty in analysing incompressible materials. It uses Ritz technique in conjunction with the variational principle. This paper involves comparison of results obtained by using theorem of minimum potential energy and new variational theorem. In this paper several numerical examples were tested to validate this theorem.

E H Lee (1955) in this paper stress and strain analysis of linear viscoelastic under quasi static loading is studied. Here Laplace transform is used for time variable to get solution in terms of associated elastic problem.

E H Lee (1956) here stress analysis of viscoelastic materials were carried out. Mainly three aspects were considered in this paper. Measurement of material properties, using this equivalent viscoelastic model was determined and use of this in theoretical analysis of stress distributions.

L W Morland et al (1960) this paper presents stress analysis of linear viscoelastic materials under non-homogeneous temperature conditions. Analysis is done for thermorheologically simple materials which is having constant change of temperature, relaxation modulus and creep with respect to logarithm of time.

R. A. Schapery (1961) here two approximate methods of Laplace transform inversions for the linear viscoelastic analysis is described. Of this one uses mathematical property of Laplace transform and other one is based on results got from irreversible thermodynamics and variational principles. Two problems are solved using this method and found to have accuracy. Addition to this stress analysis two techniques for computing operational moduli and compliance from the experimental stress strain data also discussed here.

J L White (1968) in his paper presented a two-dimensional, plane strain, finite element formulation for a thermoviscoelastic problem. This solution not depends on equivalent elastic solutions or transform methods. Time constant bulk modulus is assumed which generates simple time varying stiffness matrices and reduces the regeneration of matrices at each step.

Theodore H H Piang et al (1969) this paper reviews various approaches in finite element analysis of solid continua. In addition to reviewing the two hybrid models and mixed model by Reissner's variational principle are presented. He suggested mixed model by Reissner as the best tool for analysis of plates and shells. Also concluded that finite element method with assumed stresses are more accurate than those with assumed displacements.

D J Naylor (1974) in this paper a method for analysing porous media which may be linear or nonlinear by splitting the stiffness into effective and pore fluid components. Its applications in to saturated soils are also given. In addition to this some problems

are analysed for assess the stress accuracy at very low temperatures by displacement method. Parabolic isoparametric elements are used.

Issac Fried (1974) introduced mesh refinement by balancing residual discretization energy to solve incompressibility. This paper shows this methods practical feasibility. Also application of this method for the finite element modelling of thin plates and shells is described.

S. Yadagiri and C Pappi Reddy (1985) developed an efficient special purpose code for visco-elastic analysis of nearly incompressible solids. These were developed using isoparametric elements with selective integration procedure, which is a third order Gauss rule for deviatoric response and second order Gauss rule for volumetric response. The volumetric locking is solved by splitting the constitutive relations into shear modulus and bulk modulus. Three problems were considered in this study and they were subjected to pressure, gravity and thermal loadings respectively. Elements considered are 8 noded plane strain, 8-noded axisymmetric and 20 noded three-dimensionalelement. Also, procedure is provided to obtain the Visco-elastic solution.

Carleton J. Moore (1988) examined stress relaxation test data and a new math model is proposed. The SRM propellant, insulation, inhibitor, liners, and seals have been generally characterized as being made of viscoelastic materials. Although the viscoelastic classification has been generally accepted, close examination of these materials reveal that they are either more complex and nonlinear than classic viscoelastic models or the actual mechanisms should be redefined in a different mathematical form.

M R Lajczok (1994) the objective of the study is to determine the effective propellant modulus which will predict the deformation of the propellant during rise portion of ignition event. Firstly a time dependent effective time dependent modulus was found by assuming constant strain rate then another time dependent modulus was calculated by assuming constant stress rate. To find out which one of this modulus represent the propellant behaviour finite element analysis was done. Its result shown that displacement is represented more accurately by constant stress rate effective modulus.

Tzer-ming chen (1995) in this paper linear viscoelastic beams are solved by hybrid Laplace transform for quasi static and dynamic responses. Timoshenko beam theory and conventional beam theory were used for solving problem. In hybrid method the Laplace transform with respect to time is applied and finite element model is developed by applying Hamilton's variational principle without any integral transformation. Also, quasi static and dynamic response for Maxwell fluids and three parameter solid types are given. From the study it is found that using this method a wide variety of dimensional, composite structures with irregular geometry and time dependent loading condition in linear range can be solved.

Shiang-Woei Chyuan (2002) in his paper presented nonlinear thermoviscoelastic analysis of solid propellant. Normally linear analysis is done since nonlinear effect is normally done for some critical designs. To incorporate material and geometrical irregularities a step by step finite element model, time temperature shift principle, reduced integration and thermorheologically simple material assumption were used. From the results of the study it can be concluded that the nonlinear effects are important for structural integrity analysis under high temperature. Also, material nonlinearity is found to be more predominant than geometrical irregularity.

K Renganathan et al (2006) For the design of solid propellant rocket motor one condition which is to be satisfied is the structural integrity under inertia loading. In this study a simple methodology is presented to do the viscoelastic finite element analysis for slump estimations in rocket grain under vertical storage. An eight-node quadrilateral axisymmetric Herrmann element is used for the analysis in MARC software. Actual measured slump displacement is found to be in good agreement with that obtained from finite element analysis results.

K. Kanakaraju et. al. (2007) developed three efficient hybrid-stress-displacement finite elements based on Spilker's concept for mechanical, thermal and body force loads suitable for modelling propellant grains. This involves independent interpolations of intra-element equilibrating stress and compatible (boundary or intra-element) displacements The efficiency of the developed elements is demonstrated through several test problems and comparing the existing analytical and other finite element solutions based on displacement formulation

Jaeseung Kim (2011) in the design of road way pavements linear elastic analysis was used from older days. The primary assumption in the design is layered system consist of materials which are linearly elastic even though the asphalt used is a well-known viscoelastic material. So, this study is intended to solve this limitation and need to derive a viscoelastic solution taking in to account time and rate dependent nature of this material in multilayered system. Here for the system subjected to static load was derived using principle of elastic viscoelastic correspondence principle and numerical inversion of Laplace

transforms. Then this solution was extended to pavement subjected to moving loads by Boltzmann's superposition principle. Solution is also compared with finite element analysis software.

R. Marimuthu & B. Nageswara Rao (2013) done a study to assess structural integrity of solid propellant grains under the specified gravity, thermal and pressure loading conditions. For this purpose finite elements developed are twenty node brick element (BH20), eight node quadrilateral plane strain element (PH8) and eight node axi-symmetric solid of revolution element (AH8) using Herrmann formulations. The adequacy of elements is examined considering structural elements having high modulus to low modulus. The developed elements are tested in the software package Finite Element Analysis of Structures (FEAST). Several Solid Rocket Motor (SRM) configurations are analysed to assess the structural integrity under different loading conditions. Results are found to be in good agreement with those obtained from MARC software.

Nithesh Kumar et al (2015) done a finite element analytical study based on Herrmann formulation to overcome the limitation of computing the distribution of stress and strain on the solid propellants which are viscoelastic in nature. They were analysed for stress and strain distribution of head and mid segments of solid propellant rocket motor under thermal loading. The casing material is compressible in nature which is idealized using the standard eight-node isoparametric quadrilateral axisymmetric element having two degrees of freedom. The propellant grain which is nearly incompressible in nature is idealized using the Herrmann element having three degrees of freedom. Results obtained from the study were compared to that obtained using MARC, commercial Finite Element Analysis software and they are in good agreement.

Merin Kurian et al (2016) in this study elastic, viscoelastic and dynamic analysis of a typical cylindrical port is carried out for storage and pressure loading. In both elastic and viscoelastic analysis, a close relation is observed among the measured deformations. Dynamic analysis shows lesser deformation and so not considered for the design.

Kyeong-Soo Yun (2016) here the damage due to cyclic loading and dewetting is modelled on an isotropic nonlinear viscoelastic model. Dewetting damage function is represented as the ratio of volumetric strains before and after dewetting. Cycling loading damage function is expressed as a function of octahedral shear strain. The effects of strain rate, temperature and loading conditions are considered. Computational results are compared with experimental results.

3. CONCLUSION

There are shortcomings in the assumed displacement model such as those involving nearly incompressible materials. For nearly incompressible materials such as solid propellants having Poisson's ratio $\nu \approx 0.5$ it is not possible to determine the complete state of stress from the strain alone. Modern analytical techniques used in treating incompressibility effects in finite element codes are based on the Hellinger-Reissner and Hu-Washizu variational principles.

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