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Integrated Optimization of Multi-Period Supply Chains and Commonality of Variant Designs at Modular-Level across Product Family

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Abstract - *In an increased competition, product family design has proven to be an effective method for manufacturing products satisfying diverse customers' demand while keeping design and production cost- and time-effective. Recognizing the need for modularity and commonality in platform development, Nowadays, a comprehensive view of modularity and commonality is required to encompassing design, production, logistics, and organizational aspects and concurrently optimizes both product design and supply chain. In this paper, commonality decisions are taken concurrently with multi-period supply chain optimal operation conditions. Considering a family of products including a variant of certain module in its architectural design one of which is a high-end that can replace other variant modules with higher cost. The study also considers the possibility of using two levels of technology platforms, material discount rate, and inventory of modules.*

An integer linear programming mathematical model is developed for multi-period supply chain and solved at different values of SC parameters by LINGO software minimizing total supply chain cost. The results determine the level of commonality and selected technology/s along with any possible common modules inventory at each period. The results showed that the commonality decision may consider full or partial or no commonality based on the interaction of the different parameters controlling the operation of the supply chain. According to the level of commonality, the level of technology is determined. At low cost ratio of high-end module to variant modules, commonality is dominated by inventory in decision, while at high ratio; it is denominated by quantity discount rate.

***Key Words*: Product Modularity, Product commonality, Commonality decisions, Supply Chain optimization, Integrated commonality and supply chain optimization**

1. INTRODUCTION

Nowadays, consumers' demands are frequently changing seeking wider varieties of products at lower prices. The manufacturers faced the challenge of satisfying wide range of customers' needs at lowest possible cost. Modular design and commonality across product families is a smart approach for the stated challenge, Thevenot *et al.*[1]. Modularity has been defined by Holtta and weck [2] as "*using the same module in multiple products enabling a large variety of products while using more common component types than if the different products did not share common modules*". Ulrich and Tung [3] lists the associated modular products benefits including: "1) Component economies of scale due to the use of components across product families; 2) increased product variety from a smaller set of components, (3) Decreased order lead-time due to fewer components."

On the other hand, commonality is the use of the same version of component across multiple products, Labro [4]. Ashayeri and Selen [5] define commonality as *"the number of parts/components that are used by more than one end product and is determined for all product families".*

The decision of having common modules or standardized components is usually made by designers at the product design phase. The commonality decision may also have a great impact on the supply chain management decisions in regard of purchasing of material(s), and/or items to selecting production technologies utilized and keeping inventories. Bremner [6] states “Utilizing commonality across product families may lead to other benefits, where considerable savings from the use of product architecture commonalities across platforms can be achieved”. While Wazed *el al* [7]sates that “benefits of applying commonality include simplified planning and scheduling, lower setup and holding costs, reduction of vendor lead time uncertainty, order quantity economies, lower safety stocks, processing time and productivity”.

Many researchers investigated the relation between product commonality and modularity decisions and supply chain performance. One of the early trials was done by Guo and Gershenson [8] who investigated the relationship between product modularity and product cost. They redesigned four products and generated their recycling modularity, manufacturing modularity and assembly modularity as well as its recycling cost, manufacturing cost, and assembly cost. They used Coulter et al.[9] and Zhang and Gershenson [10,11] modularity measure as well as regression analysis to find the relationships between product modularity and product cost. Results revealed that best relationships are from the recycling viewpoint.

Chiu and Okudan [12] investigated the influence of different modularity levels of product architectural designs on supply chain lead time and costs. They generated multiple architectural designs options and evaluated modularity of each design through calculating Design for Assembly (DfA) index. For each design option the supply chain decisions are optimized using developed MIP minimizing lead time or Cost. Results revealed that modularity improves the supply chain lead time and yielded higher costs.

Dong and Chen [13] firstly presented an integrated modeling framework for multistage supply chains, to capture the interdependencies between model components. Then they used state and resource-based simulation concepts to model the supply chain network configurations. They used a novel analytical measure of component commonality that consists of a component-level commonality index and a product-level commonality index to investigate the impacts of component commonality on integrated supply chain network performance effectively. The results of analysis-of-variance and Tukey’s tests reveal that there is a significant difference in performance measures, such as delivery time and order fill rates, when comparing an integrated supply chain with higher component commonality to an integrated supply chain with lower component commonality.

Nepal *et al.* [14] used multi-objective optimization to study the sensitivity or impact of product development decision on supply chain decisions. They proposed a three-steps method to configure a supply chain for modular design. These three tasks include: 1) selection of modular design, 2) evaluation of potential suppliers, and 3) optimal configuration of supply chain. After testing their model on an automotive climate control system, it was found that selecting the optimal modular architecture will lead to an annual saving in total supply chain cost if compared to integral climate control architecture because modular architecture allows the company to outsource part of their production to module suppliers with lower production costs and offers lower inventory costs.

The problem of taking the commonality decision integrally with supply chain decision attracted researchers in the last decade. Khalaf el.al [15] presented a novel model that efficiently selected modules for products avoiding function redundancy in a constrained delivery time and constrained distant location facilities. Tabu Search Algorithm using interesting neighborhoods was employed for solving their cost optimization model. They generated many instances to verify suitable tabu size that improve the initial solution. It was proved that the tabu algorithm improves quickly the initial solution at suitable computational time of thirty minutes. Khalaf el.al [16] in subsequent work proposed different levels of standardization with functional redundancy and determined which modules will constitute the products and their manufacturing location (from a set of possible manufacturing facilities), that minimize production cost and transportations cost between the manufacturing facilities. The results showed that standardization leads to greater benefits than function redundancy; as standardization is more profitable and total standardization leads generally to offer very few different products except when variable costs are greater than fixed costs.

Baud-Lavigne el al [17] , tackled problem similar to Khalaf et al. [16] yet introduced different technologies may be used in each facility. The optimization model selects modules manufacturing locations and the technology used, yet they suggest four standardization scenarios and optimized the supply chain manufacturing cost and transportation cost between the production facilities. Based on the optimization results they introduced a graphic representation illustrate when one of the four standardization levels superior over the others. In their later work [18], instead of optimizing the impact of standardization scenarios on SC, they developed (MILP) model that jointly construct products' bill of materials and determine production facility and distribution center locations as well as suppliers' selections. For larger instances, two heuristics to solve the MILP are designed. In their work, they concluded that their model allows solving the real-life problem in a considerably accepted time. They did not discuss how the problem parameters affected the joint decision of product and supply chain.

In a different consideration of module selection problem, Wei el al [19] proposed also a multi-objective genetic optimization algorithm and fuzzy-based select mechanism to reasonably select module instances from the module sets concerning three main parameters which are product performance, cost, and task time. After they compared their algorithm results' with Zitzler el al [20] established strength evolutionary algorithm (SPEA2) and Horn el al [21] established niched pareto genetic algorithm (NPGA) results, it was proved that their algorithm outperforms the others in terms of product configuration optimization problem. They used an example of air compressor multi-objective configuration optimization to demonstrate the feasibility and validity of the proposed method.

Chung el al.[22] proposed a methodology with main objective not to maximize modularity level, but to adopt life cycle costing and life cycle assessment to identify the most beneficial modular structure. In the case study presented, processing facilities are modeled as a closed-loop supply chain, and their influence on life cycle metrics is evaluated. Using the proposed methodology, a designer can identify not only the most beneficial modular structure during configuration design, but also an optimal supply chain network structure. In their subsequent work [23] , they attempted to fill the gab related to modeling supply chain effects in the product design stage. Their work evaluated life cycle performance of a modular structure (LCC &LCEC) at various supply chain states through the employment of supply chain evaluation model and statistical techniques. Results revealed that there is significant effect of various supply chain conditions on product life cycle performance. It was shown that optimization is effective in reducing the surge of life cycle cost and life cycle environmental impact caused by various supply chain states.

From the previous literature, it can be concluded that if modularity is considered in supply chain environment, it may have negative effect on supply chain cost, however, it may improve SC lead time and fill rate. It has proven that there is a relationship between product modularity and product cost especially when comparing an integrated supply chain with higher component commonality to an integrated supply chain with lower component commonality. Modularity can benefit from outsourcing in reducing manufacturing cost and inventory cost. Higher standardization leads generally to offer very few different products except when variable costs are greater than fixed costs.

In general that the previous researches have talked the problem of integrated SC and commonality problem considering the following assumptions: full modular commonality across a product family, availability of single technology platform of specific technology; variant of modules are made common at the design stage; single period SC; no inventory is allowed within the facility and purchasing is made without any concern to ordered quantiles.

In the present work, the above-mentioned parameters are to be studied under different modularity and supply chain conditions. A mathematical model is developed to enable the decision maker to make necessary compromises between design aspects and supply chain performance aspects. The mathematical model optimizes both product commonality decisions and supply chain decisions in one step model.

The effect of supply chain operational parameters such as material discounts and availability of inventory on commonality decision are investigated. Product design decisions include mainly the determination the variant modules that will be replaced by high-end modules, are for considered commonality which minimizes the total supply chain cost.

Percent Commonality Index (%C) is used to measure commonality achieved after adjusting it to proposed model assumption. According to Siddique, et al [24] , the %C is based on three main viewpoints: (1) component, (2) component-component connections, and (3) assembly. Each of these viewpoints results in a percentage of commonality, which can then be combined to determine an overall measurement of commonality for a platform by using appropriate weights for each item that determined by designer [24] .The adjustment made for it is that we gave the component the full weight while zero weight to the component-component connections and assembly. The rest of this paper is organized as follows; the nomenclature is in section 2, section 3 demonstrates the problem definition and the developed mathematical model is presented in section 4, the numerical example is in section 5 and the results and discussion is in and section 6, section 7 is dedicated to the conclusion.

2. Nomenclature

2.1 Sets

$\boldsymbol{I}$	Set of products composing the product family
$\boldsymbol{\mathcal{J}}$	Set of all modules that can be used in all products I. $\boldsymbol{\mathcal{J}} = \boldsymbol{U}_{\mathbf{i}} \cup \boldsymbol{B}_{\mathbf{i}},\ \forall\, \boldsymbol{i} \in \mathbf{I}$
$\boldsymbol{B}_{\mathbf{i}}$	Set of variant modules that participating in product $i \in$ I.
$\boldsymbol{U}$	Set of unique modules that must exist in product $i \in$ I
$\boldsymbol{h}_{\boldsymbol{i},\boldsymbol{l}}$	Set for each variant module in product 'i' which contains this variant module 'b' all high-end modules that can replace it of the same function.
$\boldsymbol{E}_{\boldsymbol{i}}$	Union of every set $h_{i,b}$ where $h_{i,b} \neq \phi$.
$\boldsymbol{C}$	Set composed of high-end modules that may be decided to be used in more than one product where module $c \in B$
$\boldsymbol{G}$	A set equal $\boldsymbol{\mathcal{J}} - \boldsymbol{C}$
$\boldsymbol{\mathcal{M}}$	Set of material types that can be purchased by the module manufacturer
$\boldsymbol{\mathcal{T}}$	Set of planning periods $= \{1,2, \dots T\}$

2.2 Input Parameters

n	Total number of products in a product family
$D_{i,t}$	Demand of product i in period $t \in \boldsymbol{\mathcal{T}}$
$\mathbf{R}_{i,j}$	An array in which $r_{i,j} = 1$ if module $j \in \boldsymbol{\mathcal{J}}$ can used in product i
NM_i	The number of modules in product i
$\pi_{j,m}$	Percentage of material $m \in \boldsymbol{\mathcal{M}}$ constituting module $j \in \boldsymbol{\mathcal{J}}$
g_j	Total weight of module $j \in \boldsymbol{\mathcal{J}}$
K_m	Threshold quantity of material $m \in \boldsymbol{\mathcal{M}}$ after which certain discount is awarded from supplier
$\phi_{i,1}$	Opening inventory of product $i \in \boldsymbol{I}$ in period $t = 1$
$\phi_{j,1}$	Opening inventory of module $j \in \boldsymbol{\mathcal{J}}$ in period $t = 1$
$\phi_{m,1}$	Opening inventory of module $m \in \boldsymbol{\mathcal{M}}$ in period $t = 1$

2.3 Cost Parameters

c_g^T	Tooling cost to produce module $g \in G$ type in L.E
FC_c^{HT}	Fixed Tooling Cost for automated module manufacturing, $c \in \boldsymbol{C}$ in L.E
FC_c^{LT}	Fixed Tooling Cost for manual module manufacturing, $c \in \boldsymbol{C}$ in L.E
$FC_{i,j}^A$	Fixed tooling cost to assemble module '$j \in \boldsymbol{\mathcal{J}}$' with product '$i \in \boldsymbol{I}$' in L.E
vc_{HT}	Variable Processing cost per module using automated machine L.E / module
vc_{LT}	Variable Processing cost per module using manual machine L.E / module
$vc_{i,j}^A$	Assembly Cost of one unit of module j on its Product i L.E / module
c_j^I	Inventory holding cost of module j L.E / module
c_i^I	Inventory holding cost of Product i L.E / product
c_m^I	Inventory holding cost of material $m \in \mathcal{M}$ L.E / material unit
c_m^M	Unit Material purchasing cost of one unit of material $m \in \mathcal{M}$ L.E / material unit
α_m	Purchase discount rate in– percent if the ordered amount of material $m \in \mathcal{M}$ exceeds K_m

2.4 Other Parameters

$I_{i,t}$	Ending Inventory of Product i in period $t \in \mathcal{T}$
$I_{j,t}$	Ending Inventory of module j in period $t \in \mathcal{T}$
$I_{m,t}$	Ending Inventory of material m in period $t \in \mathcal{T}$

2.5 Decision variables

$X_{i,j,t}$	Binary variable, equals 1 if module j is selected for product i in period $t \in \mathcal{T}$; 0 otherwise.
$E_{i,j}$	Binary variable, equals 1 if module j is selected for product i in any period $t \in \mathcal{T}$; 0 otherwise
$Y_{j,t}$	Binary variable, equals 1 if module j is produced in period $t \in \mathcal{T}$; 0 otherwise
B_g	Binary variable, equals 1 if module g $\in \boldsymbol{G}$ is produced in any period; 0 otherwise
$U_{m,t}$	Binary variable equals 1 if $A_{m,t}$ >= K_m, 0 otherwise
$Z_{m,t}$	Binary variable equals 1 if $A_{m,t}$ <= K_m, 0 otherwise
$N_{i,t}$	Quantity of product i assembled each period $t \in \mathcal{T}$
$N_{j,t}$	Quantity of module j needed by the assembly plant in period $t \in \mathcal{T}$
$O_{j,t}$	Quantity produced from module j in period $t \in \mathcal{T}$
$W_{m,t}$	Quantity of material $m \in \mathcal{M}$ needed for manufacturing in period $t \in \mathcal{T}$
$A_{m,t}$	Quantity of material $m \in \mathcal{M}$ purchased in period $t \in \mathcal{T}$
BH_c	Binary variable, equals 1 if automated technology is used in producing high-end module in any planning period ‘t’, 0 otherwise
BL_c	Binary variable, equals 1 if manual technology is used in producing high-end module in any planning period ‘t’, 0 otherwise

3. Problem Description and Assumptions

3.1 Product Family Architecture and features

A product family is assumed to be composed of ‘n’ products; each product $i \in \mathrm{I}$ consists of a number of modules NM_i, where any module $j \in \mathcal{J}$ and ‘$\mathcal{J}$’ is the set of all modules. The modules composing the products are categorized into two main sets; unique modules set(s) ‘U_i’ and variant modules set(s) ‘B_i’. A unique module $u \in U_i$, has unique function that cannot be performed by any other module within the product family, i.e. is not common with any other module. Product family my include sets of modules each of which performing specific function, however, the members of the set may differ marginally in specification and design. It is assumed that any set of variant modules B_i, $i \in \mathrm{I}$ can be replaced by a high-end module having higher features (materials and components). For every variant module ‘b’ belong to product ‘i’ the high-end variant set ‘h_{ib}’ is defined containing the variant module(s) belong to product ‘i’ and any high-end module that can replace it. If the high-end module is selected to replace any set of variant modules, it is then considered as common module $c \in C$. Any module $j \in \mathcal{J}$ may be composed from one or more material (component) type $m \in \mathcal{M}$. The percentage of material ‘m’ by weight in module ‘j’ is defined by ‘$\pi_{j,m}$’, while g_j is the total weight of module $j \in \mathcal{J}$. Therefore, $\sum_{m\in \boldsymbol{M}} \pi_{j,m} = g_j$ $\forall j \in \boldsymbol{J}$. The material weight will reflect the module cost; therefore, a high end module has higher material weight and consequently higher cost compared to other variant modules within the set of modules. From the design point of view, the problem is to decide which high-end module will replace a specific variant module in which product. This decision will be taken in the scope of optimizing the performance of the supply chain based on assumed parameters.

3.2 Supply Chain features and parameters

A four echelons supply chain is considered, consisting of material/component supplier, Facility for modules manufacturing plant, product assembly plant and customers as shown in Figure 1. The module manufacturer can keep inventory from materials, components and of manufactured modules, while the product assembly plant cannot keep inventory as assembled products are shipped directly to the DC where they can be stored or shipped directly to customers. The role for each echelon of the proposed supply chain is as follows:

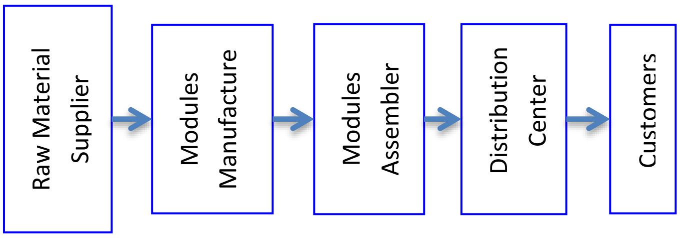

Figure 1. The Considered Supply chain

The materials are purchased from single supplier which specifies for each material ‘m’ its prices as well as quantity discounts rate ‘α_m’, if any, which is given if the purchased quantity at any period ‘$A_{m,t}$’ exceeds the required minimum purchased quantity to grant discount (threshold quantity) ‘K_m’ determined by the supplier. The supplier is assumed capable to deliver any required quantity $A_{m,t}$ from all materials. If at any planning period the quantity of the purchased material $A_{m,t}$ exceeds needed amount during that period $W_{m,t}$, the unused material quantities are kept as inventory at the manufacturing facility upcoming periods. The product modules are manufactured and then delivered to product assembly plant.

Two technology platforms exist at the manufacturing facility; the first is manual and the second is automated. The manual method requires low tooling (fixed) cost, high processing cost and can be used to manufacture any type of modules. The automated method requires high tooling cost, low variable cost and can be used to manufacture high-end modules only. The tooling cost in both methods is incurred only once throughout the planning horizon ‘T’. There is unlimited capacity available from either method.

Product assembly plant assembles various products of the product family to fulfill a deterministic demand $D_{i,t}$ from product i in period $t \in \mathcal{T}$. All assembled products are shipped directly to a distribution center (DC) to be distributed to customer(s). No transportation costs are considered between echelons. The manufactured modules or the assembled products in any period may be more than the demand in the same period in which case the residual will be kept as inventory to meet future demand. No shortages are allowed and the opening and ending inventories at all stages are set to be zero. Since the assembled quantity of a product i in period t may exceed its forecasted demand in that period.

The supply chain planning decisions includes acceptance of the discount on purchasing material and technology selection based on the decision of manufacturing the high-end module. The decisions also include the quantities of purchased materials, produced modules, assembled products and the inventory. These decisions are affected by the number of high-end modules produced. As the problem is product(s) architecture, both decisions are taken integrally to minimize the overall supply chain cost.

4. Problem Mathematical Modelling

4.1 Objective Function

The objective of this model is to minimize the total supply chain cost. The developed cost model considered mainly the supply chain operational costs, which includes: material cost, module manufacturing cost, product assembly cost and inventory costs for materials, modules and products. The cost modeling of each of these costs is given below.

4.2 Material cost

$$\sum_{t\in \boldsymbol{T}}\sum_{m\in \boldsymbol{M}} c_m^M * A_{m,t} - \alpha_m * c_m^M * A_{m,t} * U_{m,t} \tag{1}$$

Equation 1 gives the total materials purchasing cost for all periods minus the discount ‘α_m’ given if the purchased quantity ‘$A_{m,t}$’ that exceeds certain amount ‘$K_{m,t}$’, which is determined by the binary variable ‘$U_{m,t}$’ as it equals 1 if $A_{m,t} > K_{m,t}$ and zero otherwise.

4.2 Modules manufacture Cost

$$\sum_{c\in \boldsymbol{c}}(FC_c^{HT}\, BH_c + FC_c^{LT}\, BL_c) + \sum_{g\in \boldsymbol{G}} c_g^T * B_g + \left(\sum_{t\in \boldsymbol{T}}\sum_{g\in \boldsymbol{G}} O_{g,t} * c_g^P\right) + \left(\sum_{t\in \boldsymbol{T}}\sum_{c\in \boldsymbol{c}} BH_c * O_{c,t} * vc_c^{HT}\right) + \left(\sum_{t\in \boldsymbol{T}}\sum_{c\in \boldsymbol{c}} BL_c * O_{c,t} * vc_c^{LT}\right) + \tag{2}$$

The module manufacturing cost is divided into fixed and variable cost, the fixed cost represents the tooling cost for the manufacturing methodology selected, manual ‘FC_c^{LT}’ or automated ‘FC_c^{HT}’ for high-end modules c $\in C$. While for unique and variant modules that subset from the G set, will not be replaced as they will be manufactured manually. The variable cost for all modules is divided to variable processing cost of high-end modules using the selected machines and variable processing cost of other modules g ∈ G using manual machines.

4.4. Product assembly cost

$$\sum_{j\in\boldsymbol{J}} FC_{i,j}^{A} * E_{i,j} + \left(\sum_{t\in\boldsymbol{T}}\sum_{j\in\boldsymbol{J}}\sum_{i\in\boldsymbol{I}} X_{i,j,t} * N_{i,t} * vc_{i,j}^{A}\right) \quad (3)$$

The products assembly cost is divided into fixed cost incurred for every module to be assembled in the product and a variable assembly cost related to cost of assembling each module j composing product i.

4.5. Inventory costs Modelling:

$$\sum_{\boldsymbol{t}=2}^{T} \sum\nolimits_{i\in\boldsymbol{I}} \frac{1}{2} c_i^I \left[I_{i,t} + I_{i,t-1} \right] \quad (4)$$

$$\sum_{\boldsymbol{t}=2}^{T} \sum\nolimits_{j\in\boldsymbol{J}} \frac{1}{2} c_j^I \left[I_{j,t} + I_{j,t-1} \right] \quad (5)$$

$$\sum_{\boldsymbol{t}=2}^{T} \sum\nolimits_{m\in\boldsymbol{M}} \frac{1}{2} c_m^I \left[I_{m,t} + I_{m,t-1} \right] \quad (6)$$

Materials, modules and products inventory costs are given in equations (4), (5) and (6) respectively. The inventory cost is calculated based on the average inventory kept in each period i.e. half the difference between opening inventory and closing one in each period multiplied by the holding cost per unit.

Total supply chain cost given in equation (7) is optimized by selecting which high-end modules will be common. The purchased materials, manufactured module, product assembled quantities and inventory levels at each supply chain stage are to be determined.

$$\begin{aligned} MIN\ TC = \sum_{t\in\boldsymbol{T}}\sum_{m\in\boldsymbol{M}} & c_m^M * A_{m,t} - \alpha_m * c_m^M \\ & * A_{m,t} * U_{m,t} + \sum_{c\in\boldsymbol{c}} FC_c^{HT} \\ & + FC_c^{LT}\ BL_c + \sum_{g\in\boldsymbol{G}} c_g^T * B_g \\ & + \Big(\sum_{t\in\boldsymbol{T}}\sum_{g\in\boldsymbol{G}} O_{g,t} * c_g^P \Big) \\ & + \left(\sum_{t\in\boldsymbol{T}}\sum_{c\in\boldsymbol{c}} BH_c * O_{c,t} * vc_c^{HT}\right) \\ & + \left(\sum_{t\in\boldsymbol{T}}\sum_{c\in\boldsymbol{c}} BL_c * O_{c,t} * vc_c^{LT}\right) \\ & + \sum_{j\in\boldsymbol{J}} FC_{i,j}^{A} * E_{i,j} + \sum_{j\in\boldsymbol{J}} FC_{i,j}^{A} * E_{i,j} \qquad (7) \\ & + \left(\sum_{t\in\boldsymbol{T}}\sum_{j\in\boldsymbol{J}}\sum_{i\in\boldsymbol{I}} X_{i,j,t} * N_{i,t} * vc_{i,j}^{A}\right) \\ & + \left(\sum_{\boldsymbol{t}=2}^{T} \sum\nolimits_{i\in\boldsymbol{I}} \frac{1}{2} c_i^I \left[I_{i,t} + I_{i,t-1} \right]\right) \\ & + \left(\sum_{\boldsymbol{t}=2}^{T} \sum\nolimits_{j\in\boldsymbol{J}} \frac{1}{2} c_j^I \left[I_{j,t} + I_{j,t-1} \right]\right) \\ & + \left(\sum_{\boldsymbol{t}=2}^{T} \sum\nolimits_{m\in\boldsymbol{M}} \frac{1}{2} c_m^I \left[I_{m,t} \right.\right. \\ & \left.\left. + I_{m,t-1} \right]\right) \end{aligned}$$

4.6. Model Constraints

$$\sum\nolimits_{j\in J} X_{i,j,t} * R_{i,j} = NM_i \qquad \forall\, i \in \boldsymbol{I}, t \in \boldsymbol{T} \qquad (8)$$

$$\sum\nolimits_{\boldsymbol{h}_{\mathbf{i,b}}\in\boldsymbol{E}_{\mathbf{i,h}}} \sum\nolimits_{j\in\boldsymbol{h}_{\mathbf{i,b}}} X_{i,j,t} = NM_i - \sum\nolimits_{j\in\boldsymbol{U}_{\mathbf{i}}} X_{i,j,t} \qquad \forall\, i \in \boldsymbol{I}, t \in \boldsymbol{T} \qquad (9)$$

$$\sum\nolimits_{j\in\boldsymbol{h}_{\mathbf{i,b}}} X_{i,j,t} = 1 \qquad \forall\, \boldsymbol{h}_{\mathbf{i,b}} \in \boldsymbol{E}_{\mathbf{i,h}}, i \in \boldsymbol{I}, t \in \boldsymbol{T} \qquad (10)$$

$$X_{i,j,t} \le \sum\nolimits_{\boldsymbol{t}=1}^{\mathcal{T}} Y_{j,\boldsymbol{t}} \qquad \forall\, i \in \boldsymbol{I},\ j \in \boldsymbol{J}, t \in \boldsymbol{T} \qquad (11)$$

Equation (8) **e**nsures that the number of modules selected to be used in product 'i' will not exceed the maximum number of modules compose 'i'. While equation (9) ensures that the total number of variant modules $b \in B_{\mathrm{i}}$ that may be selected for product $i \in \boldsymbol{I}$ in period $t \in T$ is exactly equal to the difference between the total number of modules that composing product $i \in \boldsymbol{I}$ and the summation of the unique modules $j \in \boldsymbol{U}_{\mathbf{i}}$ that must be selected for that product $i \in \boldsymbol{I}$ in period $t \in \boldsymbol{\mathcal{T}}$. Equation (10) ensures that only one module $b \in \boldsymbol{h}_{\mathbf{i,b}}$ is selected from each set $\boldsymbol{h}_{\mathbf{i,b}}$that include variant module(s) and high-end module(s) that could replace it/them for product $i \in \boldsymbol{I}$ in period $\in \boldsymbol{\mathcal{T}}$. Equation 8, 9,10 ensures that module selection is fixed for each product in each planning period for the total produced number beyond each product. Equation (11) is the constraint that ensures that module $j \in \boldsymbol{J}$ is selected for product $i \in \boldsymbol{I}$ in period $t \in \boldsymbol{\mathcal{T}}$ only if manufactured in that period or any previous planning periods.

$$I_{i,t} = N_{i,t} - D_{i,t} + \phi_{i,1} \qquad \forall\, i \in \boldsymbol{I}\,, t = 1 \tag{12}$$

$$I_{i,t} = N_{i,t} - D_{i,t} + I_{i,t-1} \qquad \forall\, i \in \boldsymbol{I}\,,\ t \geq \mathbf{2} \tag{13}$$

$$\textstyle\sum_{t \in \boldsymbol{T}} N_{i,t} = \sum_{t \in \boldsymbol{T}} D_{i,t} \qquad \forall\, i \in \boldsymbol{I} \tag{14}$$

Since the assembled quantity of a product i in period t may exceed its forecasted demand in that period. Therefore, the product manufacturer may be kept as inventory in each period. Equation (12), represent the ending inventory of a product i in period t=1, and equation (13), represents the ending Inventory of a product i in any planning period $t \geq \mathbf{2}$. Equation (14), ensures the summation of quantities required to be produced from product $i \in \boldsymbol{I}$ in all planning periods $t \in \boldsymbol{\mathcal{T}}$ exactly equal to summation of demands of same products in all planning periods $t \in \boldsymbol{\mathcal{T}}$**.**

$$N_{j,t} = \left(\textstyle\sum_{i \in \boldsymbol{I}} X_{i,j,t} \ * \ N_{i,t}\right) \qquad \forall\, j \in \boldsymbol{J}\,, t \in \boldsymbol{\mathcal{T}} \tag{15}$$

$$O_{j,t} \geq Y_{j,\boldsymbol{t}} \qquad \forall\, j \in \boldsymbol{J}\,, t \in \boldsymbol{\mathcal{T}} \tag{16}$$

$$O_{j,t} = O_{j,t} * Y_{j,\boldsymbol{t}} \qquad \forall\, j \in \boldsymbol{J}\,, t \in \boldsymbol{\mathcal{T}} \tag{17}$$

$$I_{j,t} = O_{j,t} - N_{j,t} + \phi_{j,1} \qquad \forall\, j \in \boldsymbol{J}\,, t = 1 \tag{18}$$

$$I_{j,t} = O_{j,t} - N_{j,t} + I_{j,t-1} \qquad \forall\, j \in \boldsymbol{J}, t \geq \mathbf{2} \tag{19}$$

$$\textstyle\sum_{t \in \boldsymbol{T}} O_{j,t} = \sum_{t \in \boldsymbol{T}} N_{j,t} \qquad \forall\, j \in \boldsymbol{J} \tag{20}$$

Equation (15) represents the quantity of module j needed to be assemble $N_{i,t}$ products in period $\in \boldsymbol{\mathcal{T}}$. Constraints (16) and (17) ensure that the binary variable $Y_{j,t}$ equal to one only if module $j \in J$ is produced in period $t \in \boldsymbol{\mathcal{T}}$ equal zero otherwise. Equations (18) and (19) compute the ending inventory of a module j in period t. Constraint (20) ensures the summation of quantities of modules needed to assemble products exactly equal to summation of produced quantities of these modules.

$$W_{m,t} = \textstyle\sum_{j \in \mathcal{J}} g_j\, \pi_{j,m}\, O_{j,t} \quad \forall\, m \in \boldsymbol{\mathcal{M}}, t \in \boldsymbol{\mathcal{T}} \tag{21}$$

$$I_{m,t} = A_{m,t} - W_{m,t} + \phi_{m,1} \quad \forall\, m \in \boldsymbol{\mathcal{M}}, t = 1 \tag{22}$$

$$I_{m,t} = A_{m,t} - W_{m,t} + I_{m,t-1} \ \forall\, m \in \boldsymbol{\mathcal{M}}, t \geq 2 \tag{23}$$

$$\textstyle\sum_{t \in \boldsymbol{T}} W_{m,t} = \sum_{t \in \boldsymbol{T}} A_{m,t} \qquad \forall\, m \in \boldsymbol{\mathcal{M}} \tag{24}$$

$$A_{m,t} \geq K_m U_{m,t} \qquad \forall\, m \in \boldsymbol{\mathcal{M}}, \qquad t \in \boldsymbol{\mathcal{T}} \tag{25}$$

$$K_m \geq A_{m,t} Z_{m,t} \qquad \forall\, m \in \boldsymbol{\mathcal{M}},\ t \in \boldsymbol{\mathcal{T}} \tag{26}$$

$$U_{m,t} + Z_{m,t} = 1 \qquad \forall\, m \in \boldsymbol{\mathcal{M}}, \qquad t \in \boldsymbol{\mathcal{T}} \tag{27}$$

Equation (21) represents the quantity of materials needed to produce exactly '$O_{j,t}$' from module 'j' during period 't'. Equations (22) and (23) compute the ending Inventory of a material m in any planning period. Constraint (24) ensures the summation of material quantities purchased in all planning periods exactly equal to summation of needed material quantities in all planning periods. Constraints (25), (26) and (27) ensure that if the purchased materials is higher than certain threshold 'K_m', after which discount is granted on purchased materials, then the variable $U_{m,t}$ equals one and variable $Z_{m,t}$ equals zero, otherwise their values are reversed.

$$B_g \geq Y_{g,t} \qquad \forall\, g \in \boldsymbol{G}, t \in \boldsymbol{\mathcal{T}} \tag{28}$$

$$B_g \leq \sum_{t \in \boldsymbol{T}} Y_{g,t} \qquad \forall\, g \in \boldsymbol{G} \tag{29}$$

$$E_{ij} \geq X_{i,j,t} \qquad \forall\, i \in \boldsymbol{I},\ \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{\mathcal{T}} \tag{30}$$

$$E_{ij} \leq \sum_{t \in \boldsymbol{T}} X_{i,j,t} \qquad \forall\, i \in \boldsymbol{I} \tag{31}$$

$$BH_c + BL_c = 1 \qquad \forall\, c \in \boldsymbol{C} \tag{32}$$

Table 1: Assumed parameters in the numerical example

Symbol	Value	Symbol	Value
$D_{i,t}$	1000 from each product every periods	c_g^T	1000 L.E.
c_j^I	1 L.E. / unit	c_g^P	10 L.E. / unit
c_i^I	1.5 L.E. / unit	FC_c^{HT}	10000 L.E.
c_m^I	3 L.E. / unit	vc_c^{HT}	5 L.E. / unit
c_m^M	10	FC_j^{LT}	1000 L.E.
g_j	1 Kg / Module	vc_c^{LT}	10 L.E. / unit
$\pi_{j,m}$	1	$FC_{i,j}^A$	1000 L.E. / unit
K_m	3000, 5000, 7000, 20000	vc_i^A	10 L.E.
α_m	10 %, 50 %		

Constraints (28) and (29) ensure that B_g equals 1 only if the module g is produced in any planning period $t \in \mathcal{T}$, 0 otherwise, while constraints (30) and (31) ensure that E_{ij} equals 1 if module j is selected for product i in any planning period $t \in \mathcal{T}$, 0 otherwise. Constraint (32) ensures that either high or low technology is used in producing high-end module c.

5. Numerical Example

In order to determine how the commonality decision is affected by supply chain decisions a product family consists of four products each composed from three modules as shown in figure 2. High-end Module 4 (M4) exists in product 2 and can replace the following variant modules: M1, M9 and M12 in products P1, P3 and P4. Module 4 can be manufactured using manual or automated technologies, while all other modules are manufactured using manual technology only. The values of problem parameters are given in table 1.

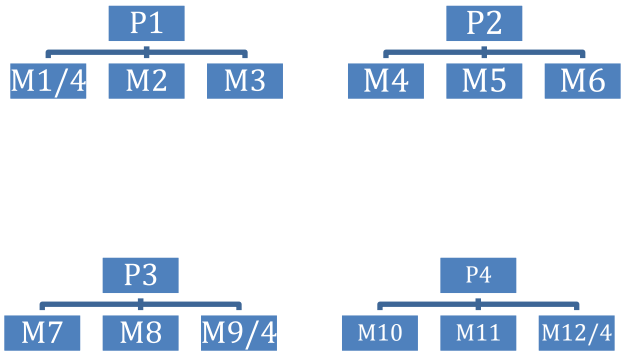

Figure 2: Structure of assumed product family

The Percent Commonality Index (%C) is used to measure commonality achieved after adjusting it to proposed model assumption.

6. Results and Discussion

The objective of the present research is to study the effect of different SC parameters on commonality decision. The parameters under consideration are related to the level of technology used and cost of increase of high end variant to average cost of variant modules.

As has been explained before, two levels of technology are available within the manufacturing facility. Low technology, manual or semi-automated, is used for the manufacturing of variant modules, while high technology, automated, can only be used to manufacture high-end module. One of the supply chain parameters under consideration is inventory of materials, modules, and products for future use during subsequent periods. Another parameter is the application of the offered quantity discount rate, offered from the supplier, in case that the total ordered quantity exceeds certain predetermined value. The variant module parameter is mainly the cost increase ratio (weight) in case of replacing a variant module with high-end module.

All analysis is based on the assumption that a high-end variant module can replace variant module of the same functional category with high-end module. The replacement of variant modules with high-end module may be fully applied, i.e. the high-end modules replace all variant of corresponding modules. The preplacement may also be partial i.e. the high-end module replaces limited number of variant modules of the same functional category. This will mean that the group of variants made common can use high technology however; the rest of variant modules will remain using low level of technology in their manufacturing. The remaining case is there will be no commonality due to increased total supply chain cost.

The commonality decision is made based on the results of optimization of the supply chain total cost after considering all parameters. The cost elements that increase the total cost are high-end module cost when it replaces variant module, inventory cost and fixed high technology tooling costs, if used. On the other hand, there are other cost elements that help decreasing the total supply chain cost which are quantity discount rate at certain threshold quantity and operating costs of using high technology in manufacturing if the high-end module quantity used for product 2 already justify high technology selection instead of low technology.

6.1 Product Family Architecture and features

Commonality decision and supply chain total cost are studied in relation with the possible increase rate in material cost of high-end module relative to average variant modules cost. Consider the case of 10% discount rate in material cost that can be awarded for order quantities exceeding 7000 units. It is evident from figure 2 that the lower the high-end material cost ratio (weight), the more the tendency towards considering full commonality and vise versa. However, when considering inventory, partial commonality may be favorable decision instead of the no commonality decision with no inventory. This is consistent with the results shown in figure 3, where the total SC cost increases with increasing material cost ratio. However, up to a ratio of 1.7 of material cost, inventory plays a considerable role in decreasing the total SC cost as it gives a chance for total material required quantity to exceed the threshold quantity for material discount. Figure 4 shows that the total SC cost, in case of inventory, is less than that without inventory. As the cost ratio exceeds 1.5, the inventory cost increases up to a ratio of 1.7 after which it decreases until it reaches the same value of no inventory at ratio of 2.0. Normally at ratio of 2.0 or above, no inventory is expected to exist because the material is considerably expensive and consequently the inventory cost is high. The drop in total SC cost before a ratio of 1.7 is mainly due to the low operating cost of automated manufacturing system used with common modules. This is also clear from figure 5, as the production cost is less for cost ratios above 1.5. As a conclusion, it can be said that inventory helps in inducing reduction in material cost and operating cost provided that the total common module quantity exceeds or equal to the threshold value. In case of low cost increase ratios, inventory becomes of limited value in commonality decision even if the quantity discount rate is limited. The quantity discount rate becomes more influencing the commonality decisions at relatively high material cost ratios, in the present case is 1.5.

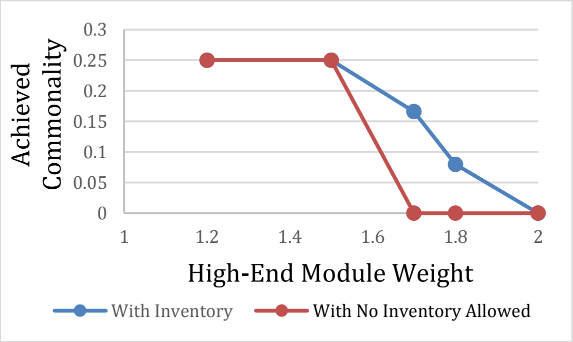

Figure 2: Commonality decsion with inventory & without inventory at Km=7000, 10% discount

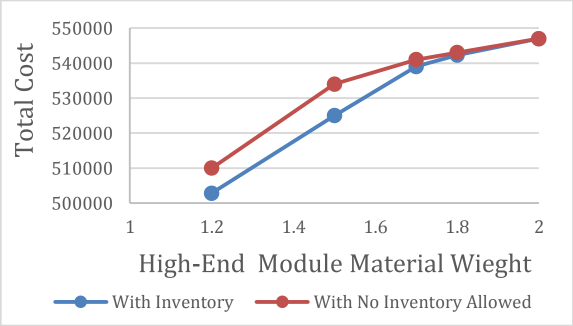

Figure 3: Total Cost with inventory & without inventory at Km=7000, 10% discount

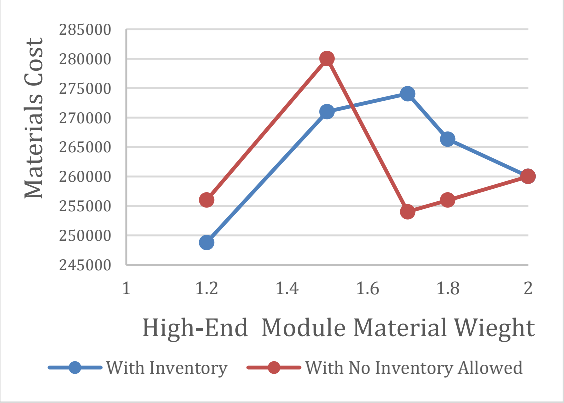

Figure 4: Materail cost with inventory & without inventory at Km=7000, 10% discount

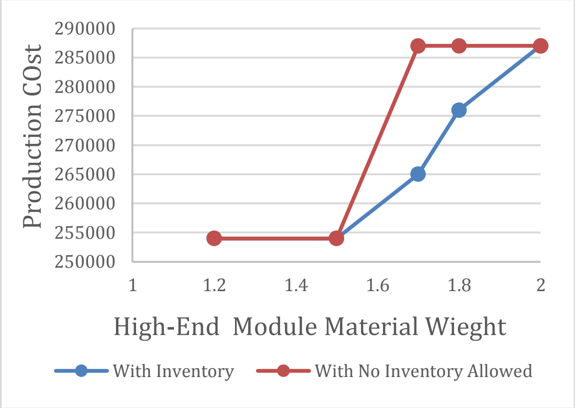

Figure 5: Production cost with inventory & without inventory at Km=7000, 10% discount

6.2 Discount Rate and commonality

In figure 6, the effect of two extreme discount rates on commonality decisions is examined in addition to the 10% discount rate which was discussed earlier. Figure 6 is exactly the same as figure 2 except it includes the results of 50% discount rate. The results of 50% discount rate show that full commonality is always considered for all material cost ratios. In other words, all parameters are dominated by the discount rate. This result is supported by the values of total cost at 10% discount rate in figure 7 where the total cost is far below other cases under consideration. Figure 8 also shows that material cost is the lowest compared to other cases. The production cost without discount is higher than 10%, while at 50%, the cost is almost constant. Figure 9 shows that in case of high discount rate, the commonality decision is to consider full commonality at all values of cost ratio and therefore the production cost is low as high technology is used. In case of no discount or limited discount, the commonality decision is affected by the cost ratio. High the cost ratios will suppress the commonality decision and hence modules are being manufactured using low technology of high cost.

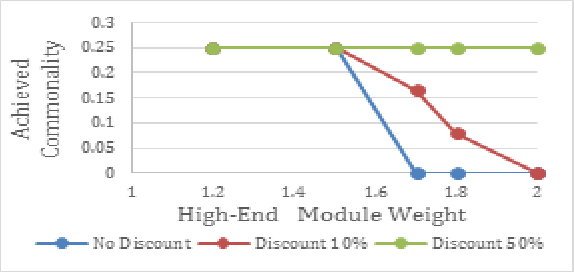

Figure 6: Commonality decsion at km = 7000 in case of No, 10%, 50 % disount respectively

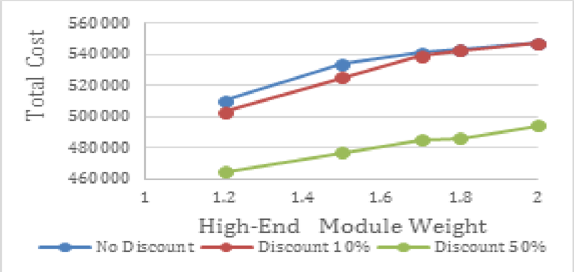

Figure 7: Total Cost at km = 7000 in case of No, 10%, 50 % discount respectively

Figure 8: Material Cost at km = 7000 in case of No, 10%, 50 % discount respectively

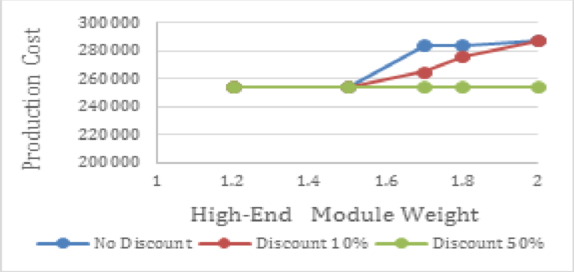

Figure 9: Production Cost at km = 7000 in case of No, 10%, 50 % discount respectively

6.3 Threshold Quantity for quantity discount

From figure 10, it is evident that when the threshold quantity is between 5000 and 7000 units, there is a possibility that the optimal total SC cost may seek partial commonality. However, higher values of threshold (20,000 units in our case) can either lead to full commonality or no commonality depending on the high-end cost ratio relative to average variant modules. The total cost shown in figure 11 is consistent with the commonality decision taken such that a considering threshold quantity of 3000 and 5000 units, the total SC cost is less than that of high threshold quantities

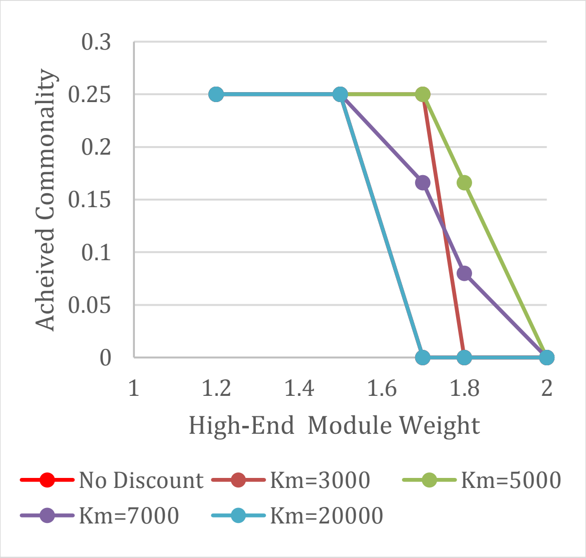

Figure 10: Commonality decsion at 10 %discount in case of discount = zero, km = 3000, 5000, 7000, 20000

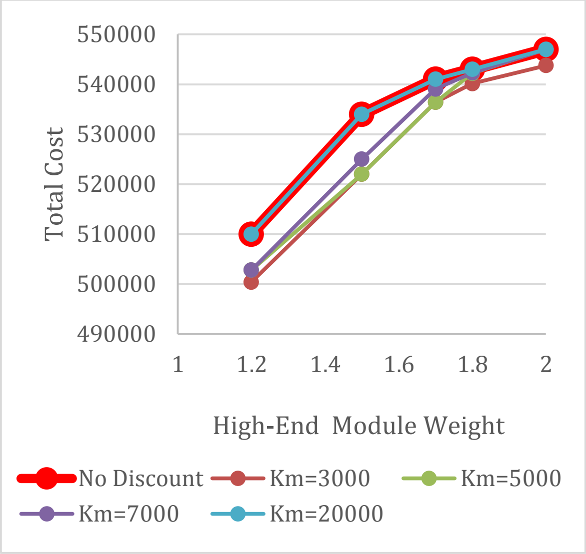

Figure 11: Total Cost at 10 % discount in case of discount = zero, km = 3000, 5000, 7000, 20000

7. Conclusions

Increasing product diversify has increased complexity on the side of manufacturing firms and their supply chains. Such complexity increases costs and lead times of customer order fulfilment process. In general, product modularity is a common solution to cope with such diversified demands while reducing costs and lead times. On the other hand, the supply chain decision maker may have some concerns for applying maximum possible commonality across product family as it might increase the SC total cost. In some instances, considering no or partial commonality may have positive economic impact on supply chain operations, performance, and total cost.

The objective of the present work is study the effect of different SC and product modules parameters on the commonality decision for optimal SC cost while considering the availability of two levels of manufacturing technologies. SC parameters include material cost of unique and variant modules, manufacturing cost, inventory holding cost, quantity discount rates and discount threshold quantities. A number of products, belonging to the family, are considered. The family products have variants of certain module one of which is high-end module that can replace other variants of same type across product family.

A mixed integer non-linear programming mathematical model is developed for multi-period supply chain and solved at different values of SC parameters by LINGO software. The results determine the level of commonality and selected technology/s along with any possible material, modules, and product inventory at each period.

It was found that the commonality decision depends mainly on the competition between the supply chain costs elements throughout production periods with consideration of the applied level of technology for each module of the products family. The total quantities required from potential common modules may decide on the possibility of using high technology with lower processing time and possible material quantity which exceeds the threshold quantity to grant quantity discounts. As the supply chain is multi-period, the system may benefit of stocking the material for next periods against predetermined cost. In conclusion, the following cases may take place;

If the total quantity of the material of any common module doesn't merit quantity discount and the increase in cost of the common module is high and the total quantity produced does not justify the use of high technology, the supply chain will consider no commonality. In the contrary, discounts can be achieved due to high quantities that exceed the threshold quantity specified by the supplier and the increase in common module cost is lower than the discount rate, the supply chain may favor maximum possible commonality of all similar modules within the products family. Moreover, in case that a number of modules can satisfy the first condition and some others satisfy the second condition, the supply chain may have partial commonality for optimization purposes. In general, the commonality decision is dominated by inventory decision at low cost ratio while it is dominated by quantity discount at low cost ratio.

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