Parabolic Loading in Fixed Deep Beam using 5th Order Shear Deformation Theory

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Abstract - *In this paper, a variationally consistent 5th order refined shear deformation theory for deep fixed-fixed beams is developed. The governing differential equations and boundary conditions are obtained using principle of virtual work. The general solution technique is developed to solve the governing differential equations. The theory is applied to static flexural analysis of fixed-fixed beams of homogeneous and isotropic material with uniform solid rectangular cross-section. The general solutions for field variables w (x) and $\phi(x)$ are obtained for beams under consideration using appropriate boundary conditions. The general expressions for displacements and stresses are presented.*

***Key Words*: Fixed, Deep beam, 5th order**

1. INTRODUCTION

Beams and plates are widely used in civil and mechanical industries. As the thickness is much smaller than the length, it can be converted from 3D to 1D also it is conceivable to work-out the variation of the stress in the thickness coordinate. Bernoulli-Euler established the most commonly used classical or elementary theory of beam. Galileo in 1638 have made first attempt till 1856 mentioned by Saint Venant Barre de is also presented by Love . The classical theory of beam bending (ETB) is founded on the hypothesis that the plane sections remain plane and normal to the axis after bending, implying that the transverse shear strain is zero. Due to negligence of the transverse shear deformation, it is acceptable for the analysis of thin beams. Due to underestimation of deflections in case of thick beams where shear deformation effects are more pronounced.

1.1 Theoretical Formulation

The beam is made up of isotropic material and occupies in $0-x-y-z$ Cartesian coordinate system the region: $0 \le x \le L$; $-\frac{b}{2} \le y \le \frac{b}{2}$; $-\frac{h}{2} \le z \le \frac{h}{2}$

where,

x, y, z = Cartesian coordinates,

L = Length of beam in *x* direction

b = breadth of beam in *y* direction, and

h = thickness of the beam in the *z*-direction. The beam is up of homogeneous, linearly elastic isotropic material.

1.2 Equilibrium Equations

Using the expressions for strains and stresses (2) through (4) and the principle of virtual work, following equilibrium equations can be obtained. The expression obtained by using principle of virtual work as follows:

$$\int_{x=0}^{x=L}\int_{y=-b/2}^{y=b/2}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x\delta\varepsilon_x+\tau_{zx}\delta\gamma_{zx}\right)dx\,dy\,dz-\int_{x=0}^{x=L}q(x)\delta w\,dx=0 \quad (5)$$

Integrating Eqn. (5) successively, we obtain the coupled equilibrium equations of the beam.

$$EI\frac{d^4w}{dx^4}-A_0EI\frac{d^3\phi}{dx^3}=q(x), \qquad A_0EI\frac{d^3w}{dx^3}-B_0EI\frac{d^2\phi}{dx^2}+C_0GA\phi=0$$

where the constants, $A_0=\frac{12}{7}$, $B_0=2.96$, $C_0=2.4635$

A fixed beam with parabolic load $q(x)=q_0\frac{x^2}{L^2}$

A fixed-fixed beam has its origin at left hand side support and is fixed at *x* = 0 and *L*. The beam is subjected to parabolic load, $q(x)=q_0\frac{x^2}{L^2}$ on surface z = +*h*/2 acting in the downward *z* direction with maximum intensity of load q_0. parabolic load is considered as shown in Fig. 1. The material assumed for beam are as modulus of elasticity (*E*) = 2.1 x10^5 MPa, Poisson's ratio (*μ*) = 0.3 and density (ρ)= 7800 kg/m³.

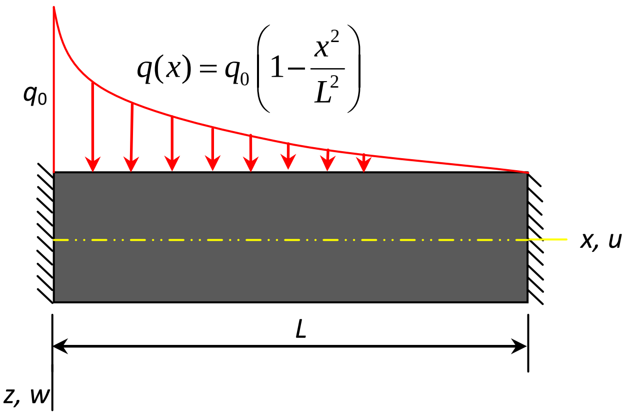

Fig. 1: Fixed beam with parabolic load

Boundary conditions for the beam used as

$$\frac{dw}{dx} = \phi = w = 0 \text{ at } x = 0, L$$

2. Numerical Results

The results for flexural and transverse shear stresses are mentioned in Table 1 in the following non-dimensional form.

$$\bar{u} = \frac{Ebu}{q_0 h}, \quad \bar{w} = \frac{10Ebh^3 w}{q_0 L^4}, \quad \bar{\sigma}_x = \frac{b\sigma_x}{q_0}, \quad \bar{\tau}_{zx} = \frac{b\tau_{zx}}{q_0}$$

The transverse shear stresses ($\bar{\tau}_{zx}$) are obtained directly by constitutive relation and, alternatively, by integration of equilibrium equation of two dimensional elasticity and are denoted by ($\bar{\tau}_{zx}^{CR}$) and ($\bar{\tau}_{zx}^{EE}$) respectively. The transverse shear stress satisfies the stress free boundary conditions on the top $\left(z = -\frac{h}{2}\right)$ and bottom $\left(z = +\frac{h}{2}\right)$ surfaces of the beam when these stresses are obtained by both the above mentioned approaches.

Non-Dimensional Axial Displacement ($\bar{u}$) at (x = 0.25L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = 0.25L, z =0.0) Axial Stress ($\bar{\sigma}_x$) at (x = 0.0, z = h/2) Maximum Transverse Shears Stresses $\bar{\tau}_{zx}^{CR}$ at (x=0.01L, z =0.0) and $\bar{\tau}_{zx}^{EE}$ at (x=0.01L, z =0.0) of the Fixed Beam Subjected to Parabolic Load for Aspect Ratio 4

V order	0.2890	15.5504	1.4037	-1.832
HPSD	0.289	16.309	1.458	-3.017
HSDT	0.2965	14.6098	1.2744	1.9020
FSDT	0.1943	6.4000	0.1905	2.5400
ETB	0.1340	6.4000	—	2.5400

Table -1: Variation of axial displacement

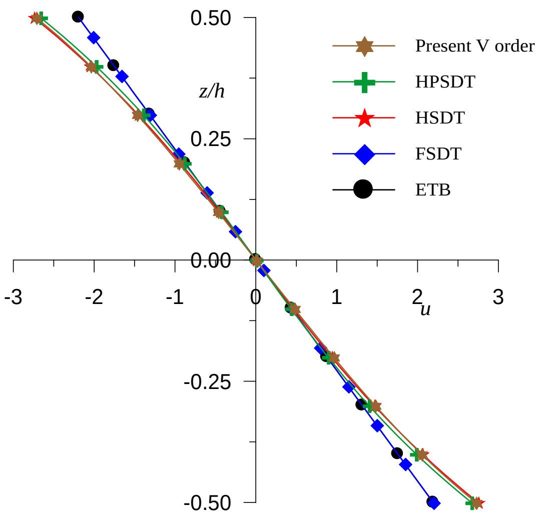

Fig. 2: Variation of axial displacement ($\bar{u}$) through the thickness of fixed-fixed beam at (x = 0.25L, z) when subjected to parabolic load for aspect ratio 4.

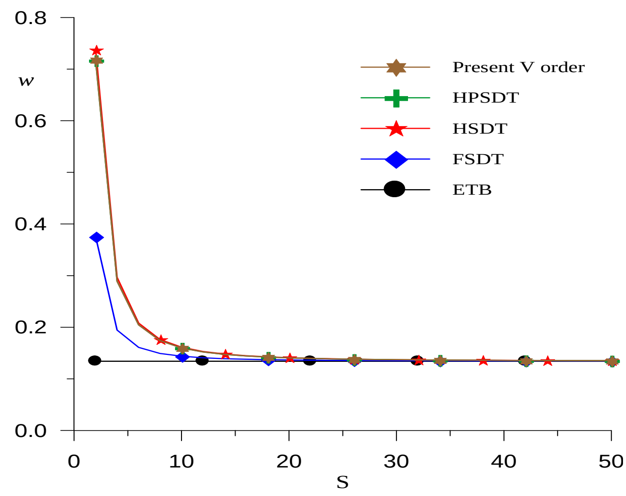

Fig. 3: Variation of maximum transverse displacement ($\bar{w}$) of fixed-fixed beam at (x=0.25L, z = 0) when subjected to parabolic load with aspect ratio S.

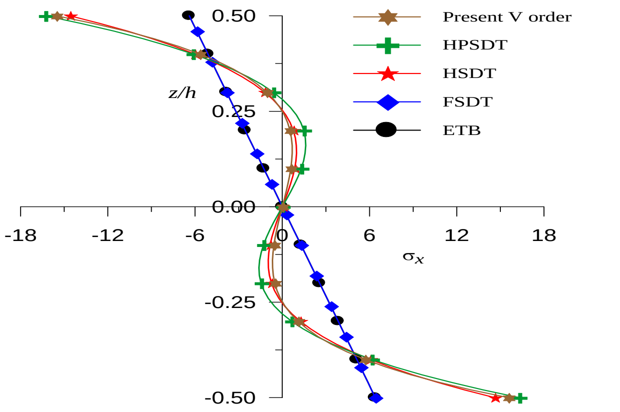

Fig. 4: Variation of axial stress ($\bar{\sigma}_x$) through the thickness of fixed-fixed beam at (x = 0, z) when subjected to parabolic load for aspect ratio 4.

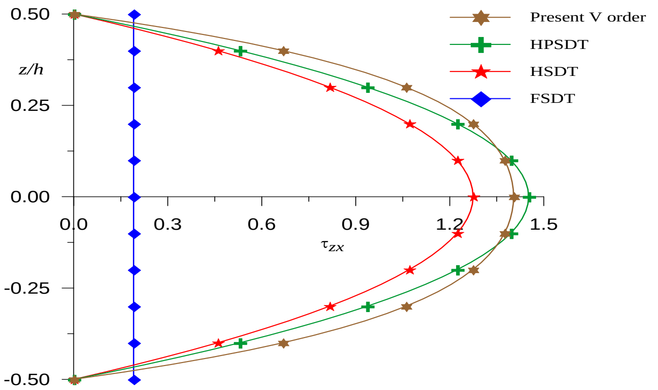

Fig. 5: Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to parabolic load and obtain using constitutive relation for aspect ratio 4.

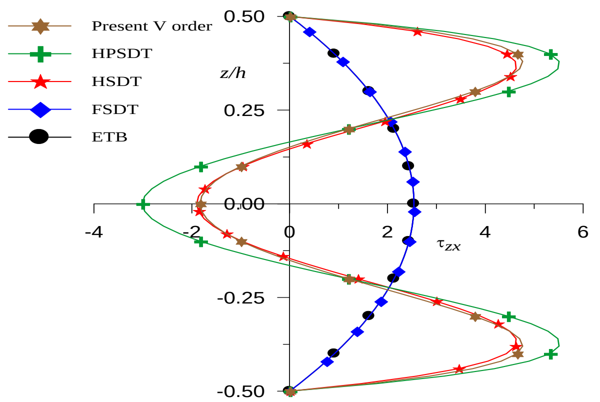

Fig. 6: Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to parabolic load and obtain using equilibrium equation for aspect ratio 4.

3. CONCLUSIONS

1. The flexural stresses and their distributions through the thickness of beam given by proposed theory are in excellent agreement with those of other refined shear deformation theories.
2. The shear stresses and their distributions over the thickness of beam from constitutive and equilibrium equations are matching with that of other refined shear refined theories.
3. In general, use of proposed theory gives precise results by the numerical considered.
4. This validates the usefulness of the 5th order shear deformation theory.

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