5th Order Shear Deformation Theory for Fixed Deep Beam

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Abstract - *The research work presents the flexural response for fixed deep beam using 5th order shear deformation theory subjected to varying load. The theory is built upon the classical beam theory including fifth order function used in thickness coordinate for consideration of shear deformation effects. The theory satisfied the condition at the top and bottom plane of the beam as zero shear stresses. The distribution of shear stress through the thickness is genuine and avoiding shear correction factor. Principle of virtual work is used to determine the equilibrium equations. To prove the usefulness of the theory, the stress analysis is carried out for deep beam, subjected to varying load. Findings are compared with the theories of other researchers.*

Key Words: **Fixed, Deep beam, 5th order**

1. INTRODUCTION

Beams and plates are widely used in civil and mechanical industries. As the thickness is much smaller than the length, it can be converted from 3D to 1D also it is conceivable to work-out the variation of the stress in the thickness coordinate. Bernoulli-Euler [1,2,3] established the most commonly used classical or elementary theory of beam. Galileo in 1638 have made first attempt till 1856 mentioned by Saint Venant Barre de [4] is also presented by Love [5]. The classical theory of beam bending (ETB) is founded on the hypothesis that the plane sections remain plane and normal to the axis after bending, implying that the transverse shear strain is zero. Due to negligence of the transverse shear deformation, it is acceptable for the analysis of thin beams. Due to underestimation of deflections in case of thick beams where shear deformation effects are more pronounced.
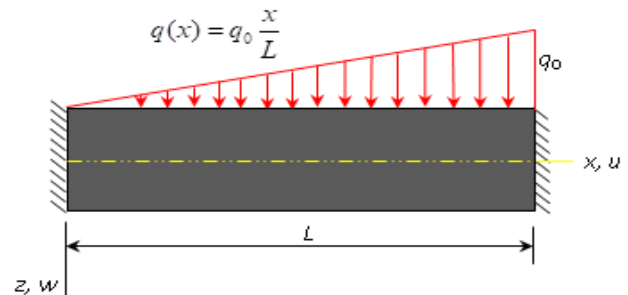
1.1 Theoretical Formulation

The beam is made up of isotropic material and occupies in $0 - x - y - z$ Cartesian coordinate system the region: $0 \leq x \leq L$; $-b/2 \leq y \leq b/2$; $-h/2 \leq z \leq h/2$

where,

x, y, z = Cartesian coordinates,

L = Length of beam in x direction

b = breadth of beam in y direction, and

h = thickness of the beam in the z-direction. The beam is up of homogeneous, linearly elastic isotropic material.

1.2 Equilibrium Equations

Using the expressions for strains and stresses (2) through (4) and the principle of virtual work, following equilibrium equations can be obtained. The expression obtained by using principle of virtual work as follows:

$$\int_{x=0}^{x=L} \int_{y=-b/2}^{y=b/2} \int_{z=-h/2}^{z=+h/2} \left(\sigma_x \delta \varepsilon_x + \tau_{zx} \delta \gamma_{zx} \right) dx\,dy\,dz - \int_{x=0}^{x=L} q(x)\delta w\,dx = 0$$

(5)

Integrating Eqn. (5) successively, we obtain the coupled equilibrium equations of the beam.

$$EI \frac{d^4 w}{dx^4} - A_0 EI \frac{d^3 \phi}{dx^3} = q(x), \qquad A_0 EI \frac{d^3 w}{dx^3} - B_0\,EI \frac{d^2 \phi}{dx^2} + C_0 GA\phi = 0$$

where the constants, $A_0 = \dfrac{12}{7}, \quad B_0 = 2.96, \quad C_0 = 2.4635$

2. Numerical Results

The results for flexural and transverse shear stresses are mentioned in Table 1 in the following non-dimensional form.

$$\bar{u} = \frac{Ebu}{q_0 h}, \quad \bar{w} = \frac{10Ebh^3 w}{q_0 L^4}, \quad \bar{\sigma}_x = \frac{b\sigma_x}{q_0}, \quad \bar{\tau}_{zx} = \frac{b\tau_{zx}}{q_0}$$

The transverse shear stresses ($\bar{\tau}_{zx}$) are obtained directly by constitutive relation and, alternatively, by integration of equilibrium equation of two dimensional elasticity and are denoted by ($\bar{\tau}_{zx}^{CR}$) and ($\bar{\tau}_{zx}^{EE}$) respectively. The transverse shear stress satisfies the stress free boundary conditions on the top $\left(z = -\dfrac{h}{2} \right)$ and bottom $\left(z = +\dfrac{h}{2} \right)$ surfaces of the

beam when these stresses are obtained by both the above mentioned approaches.

V order	24.3175	0.1834	47.9820	4.2592
HPSDT	24.3626	0.1834	49.5291	4.4978
HSDT	24.3646	0.1834	46.1200	4.2329
FSDT	24.0234	0.1757	30.0000	0.6165
ETB	24.0234	0.1563	30.0000	—

**Table -1:** Variation of axial displacement
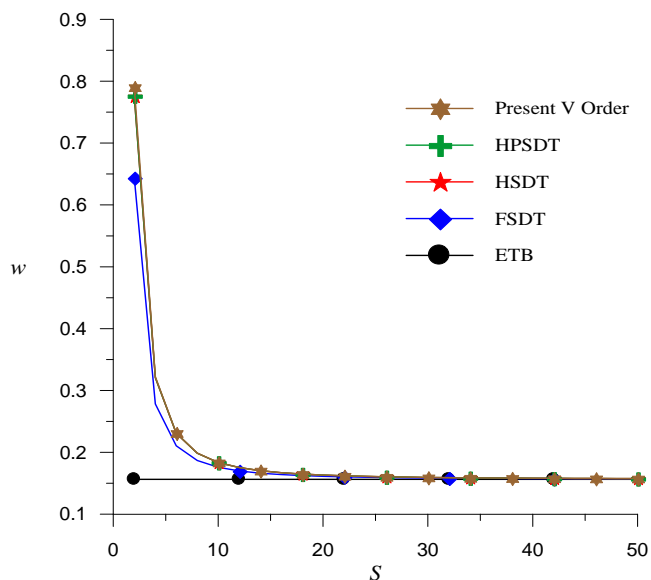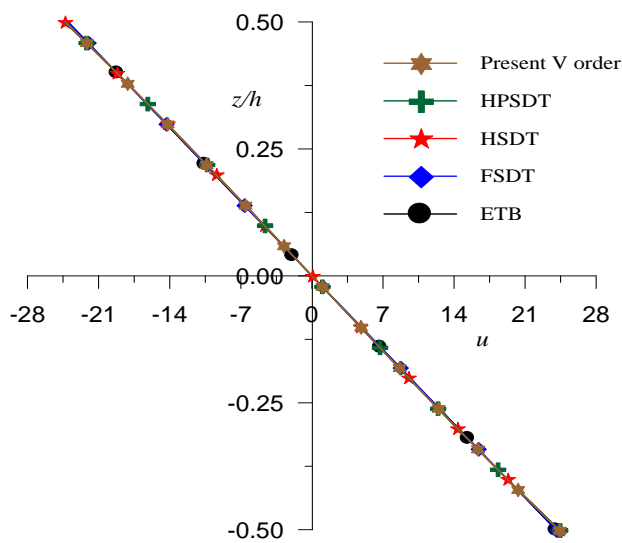
Variation of maximum transverse displacement ($\overline{w}$) of fixed-fixed beam at ($x, z = 0$) when subjected to varying load with aspect ratio S.

Below Graph Shows Variation of axial stress ($\overline{\sigma}_x$) through the thickness of fixed-fixed beam at ($x= 0, z$) when subjected to varying load for aspect ratio 10.
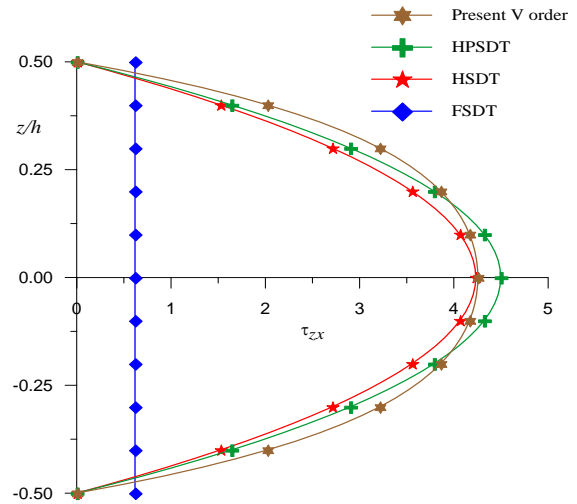
3. RESULTS

Variation of transverse shear stress ($\overline{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to varying load and obtain using equilibrium equation for aspect ratio 10.
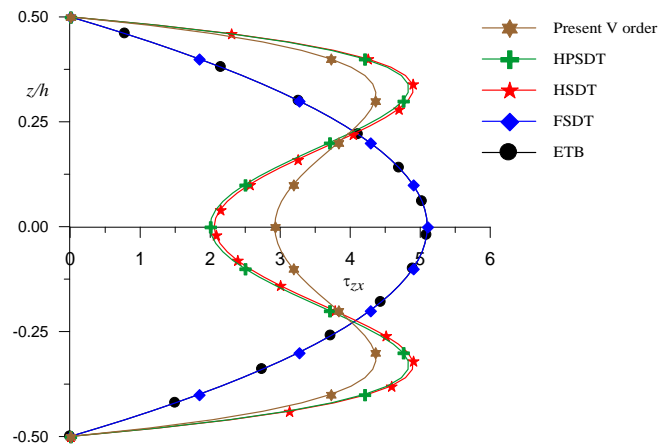
4. CONCLUSIONS

1. The flexural stresses and their distributions through the thickness of beam given by proposed theory are in excellent agreement with those of other refined shear deformation theories.
2. The shear stresses and their distributions over the thickness of beam from constitutive and equilibrium equations are matching with that of other refined shear refined theories.
3. In general, use of proposed theory gives precise results by the numerical considered.

4. This validates the usefulness of the 5th order shear deformation theory.

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