Analysis of Random Error of Portable-Coordinate Measuring Arm

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Abstract - *To improve the measurement and calibration accuracy of PorTable-Coordinate Measuring Arm(PCMA), it is very important to accurately identify the system and random errors of the measuring machine. From the kinematic error model, system errors can be identified during calibration. But identifying random errors remains a difficult problem. First, we use the Solidworks software to analyze the structure of each joint. Second, Then, CETOL 6σ tolerance analysis software is used to calculate the random error of the probe generated by the clearance of bearings in each joint. Finally, the random error due to the systematic uncertainty of the rotary encoder is calculated. The experimental results show that the total value of the random error duo to the bearing clearance, the systematic uncertainty of the rotary encoder and the thermal expansion of the mechanical structure according to the temperature variation does not exceed 0.1112mm. This value is theoretically the limit of the single point repeatability accuracy that can be achieved by this measuring machine. The use of the CETOL 6σ not only allows the design to be carried out scientifically, but also reduces production costs.*

Key Words: PCMA, Random error, Measurement accuracy, Calibration; 3D tolerance analysis

1. INTRODUCTION

An articulated arm coordinate measuring machine (AACMM) is a multi-degree-of-freedom non-orthogonal coordinate measuring system that replaces an angle scale with the length scale. It connects several links and one probe in series through a rotary joint. AACMM has the advantages of small size, lightweight, good flexibility, large measuring range, flexibility, low cost, and the ability to move the measuring machine to the workspace (e.g. [1-4]). The measurement accuracy of the PCMA depends on the interaction of many factors, such as the dimensional error of the components in the mechanism of PCMA, the assembly error, the gap in the motion pair, the skew of the axis of the moving part, the elastic deformation caused by the self-weight or thermal deformation caused by temperature change, and the initial error such as friction and wear, and also, in actual operation, the load changes, acceleration or deceleration and other factors may cause the geometric deviation and motion deviation on PCMA, thus affecting on the measurement accuracy of the measuring machine.

Based on statistical evaluation techniques, the measurement error of the measuring arm can be divided into systematic error and random error (e.g. [5,6]). Systematic error is the error that occurs during the manufacture and assembly of the measuring arm. Random error refers to the measurement error caused by unpredicTable-factors. Because it cannot obtain its certain mathematical model, it can only be estimated by statistical processing of many experimental measurement data.

The main error sources of PCMA can be divided into the following (e.g. [7-12]).

(1) The dynamic error caused by the joint axis sway is mainly caused by the radial runout of the bearing and the axial displacement during the rotation.
(2) The angle measurement random error, which is caused by the limited the measurement accuracy of the angle encoder.
(3) The thermal deformation error caused by temperature changes.
(4) The probe detection error, the probe sphericity error and the detection error caused by excessive or too small contact force during measurement (e.g. [13]).

The greatest impact on random errors is the systematic uncertainty of the rotary encoder, thermal deformation caused by temperature changes, and bearing clearance. Here, the random error caused by the bearing and the encoder is an adjusTable-amount, which can be reduced by selecting a bearing and an encoder with higher precision. The experiment was carried out in a sTable-environment with a temperature change of ±2°C. Standard procedures are also strictly followed during the measurement process.

In this paper, we will only analyze the influence of the random error caused by the bearing runout using CETOL 6σ tolerance analysis software, the limit of random error at the probe center due to systematic uncertainty of the encoder and the thermal expansion in the maximum range of the operating temperature variation. These errors are affecting repeatability of the measuring machine, and are also the key to improving the measurement accuracy of the measuring machine (e.g. [14]).

2. Structure and tolerance analysis of PCMA

2.1 Structure of PCMA

The manufactured structure of the 5-DOF PCMA is shown in Fig- 1. The machine has a five-axis revolute joints. Fig- 2 shows the reference frames of the 5 links and end-effector, and some of DH parameters. The internal structures of the first joint to the third joint are identical.

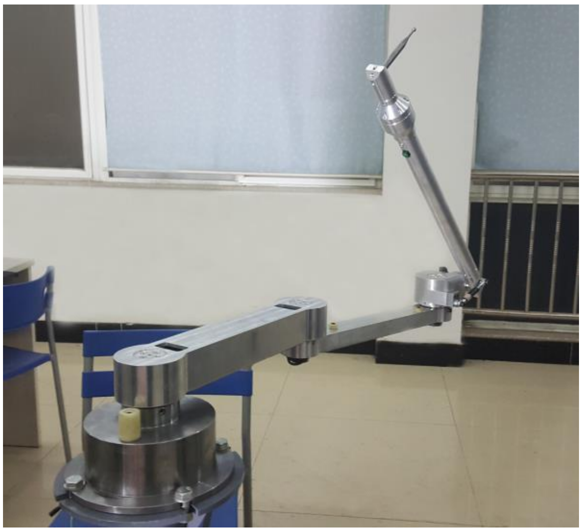

Fig- 1: Photograph of the 5DOF PCMA

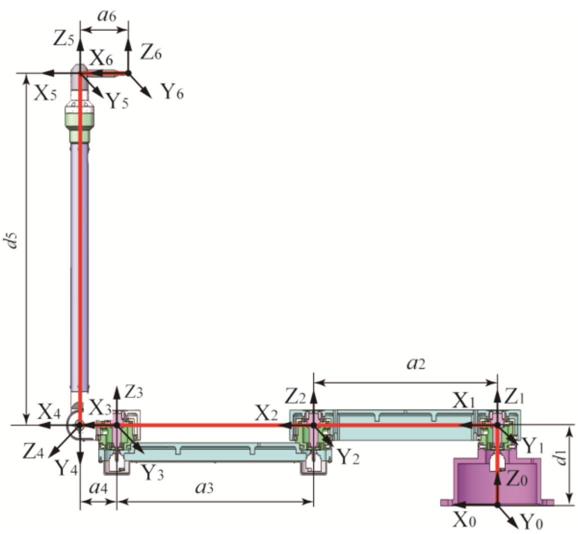

Fig-2: Reference frames

2.2 Tolerance analysis of joint 1~3

2.2.1 Internal structure

The internal structure of the first joint is shown in Fig-3. The cross arrow in Fig-3 shows the effect of the bearing clearance. Due to the existence of the bearing clearance, there is a random error in the joint. This error can be obtained by performing a tolerance analysis at the joint using the CETOL 6σ.

CETOL 6σ provides a complete solution for the analysis and management of part and assembly deviations. CETOL 6σ uses a more general form of the equation for performing statistical performance analysis called the Method of System Moments (MSM). The method of system moments allows the use of any combination of distribution types to represent the feature manufacturing processes. And the MSM is able to predict the first four moments of the assembly functional requirement distribution, not just the standard deviation(e.g. [15]).

Performance analysis is used to determine if the design satisfies the assembly functional requirements.

The most common form of statistical performance analysis is the 1D RSS (Root Sum of Squares) equation. This equation assumes that all processes have the same distribution type:

$$\sigma_{U} = \left(\sum_{i=1}^{n} \sigma_i^2 \right)^{\frac{1}{2}} \tag{1}$$

where σ_i : Standard deviation of the dimension

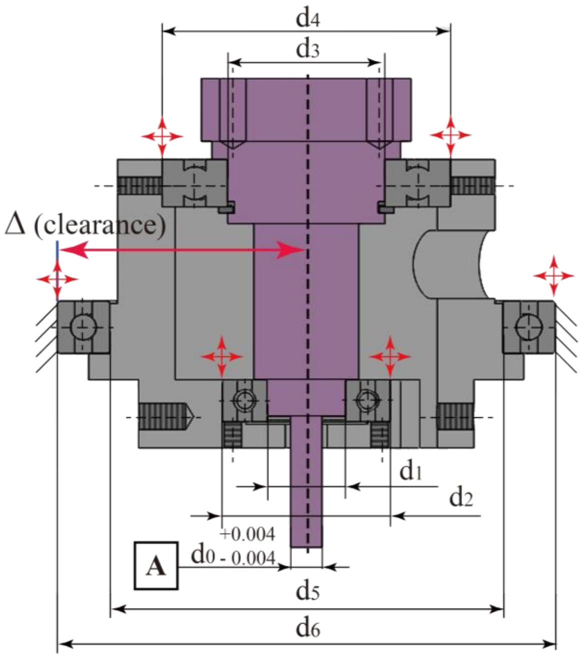

Fig-3: Internal structure of the first joint

In a derivative-based analysis, CETOL uses sensitivities to predict the measurement variation caused by dimensional variability. Sensitivities are the partial derivatives of a measurement with respect to each of the variables in the model. Sensitivity values are calculated based on part geometry and assembly constraints (joints) CETOL calculates sensitivities numerically using the Central Difference formula:

$$\frac{\partial F}{\partial h} \approx \frac{F(h+\Delta h)-F(h-\Delta h)}{2\Delta h} \quad (2)$$

where F is the measurement of interest, and h is a model variable.

The value obtained from the structure of the first joint assembled with the three bearings is the clearance Δ from the central axis of the joint to the outer radius of the outer bearing. This is the total value of random error due to the gap between the bearings in the first joint.

Precision bearings are classified according to ISO grading standards: P0, P6, P5, P4, P2. The grades are sequentially increased, of which P0 is ordinary precision, and other grades are precision grades. The bearing we use is the deep groove ball of the high precision bearing P4 series. The inner ring and the outer ring and the shaft achieved an interference fit, the interference value is 0~+4μm, the runout of the bearing seat hole shoulder is less than 4μm; the inner end of the front cover of the main shaft has a runout of 4μm or less. Bearing's gap was already fixed at 4μm.

2.2.2 First joint assembly constraint DOF status

The assembly constraint state of the first joint using CETOL6σ is shown in Fig-4. The CETOL 6σ model logic network diagram shows the degree of freedom for each component and assembly constraint.

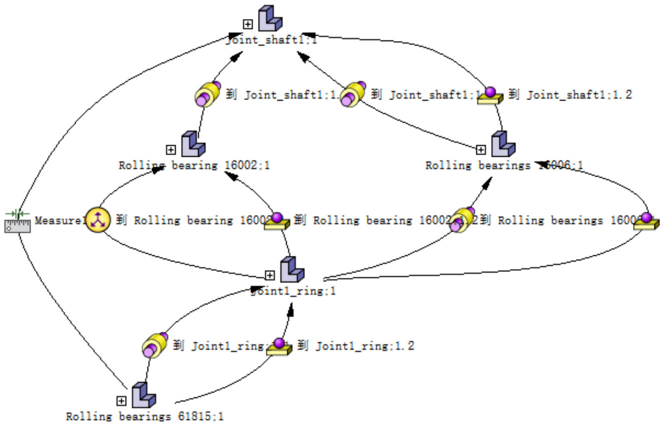

Fig-4: Assembly constrains DOF state of the first joint

2.2.3 Measurement size analysis of the first joint

Fig-5 shows the CETOL6 sigma analyzer interface. The tolerances of the given bearings are fixed and a reasonable tolerance analysis is performed while changing the other dimensional tolerances.

Fig-6-Fig-8 show the sensitivity, statistical contribution, and worst case contribution of each variables in the measurement model of the first joint. Sensitivity can be used to identify the critical dimensions. For a given measurement, the most sensitive variable value is the most important variable. Conversely, sensitivity can also be used to identify non-critical dimensions, and variables with low sensitivity are non-critical variables. These variables are a possible source of cost savings. If all of the key measurements of the assembly have low sensitivity values for a particular variable, then the assembly will be able to use low-cost process manufacturing.

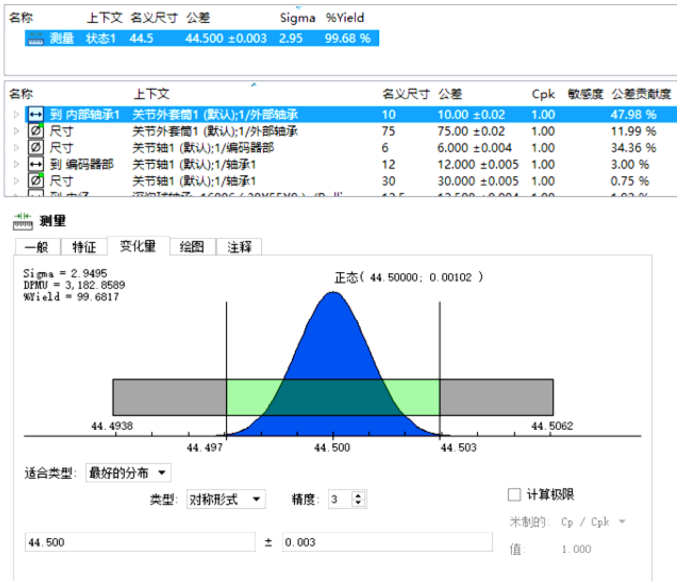

Fig-5: Measurement results of the first joint

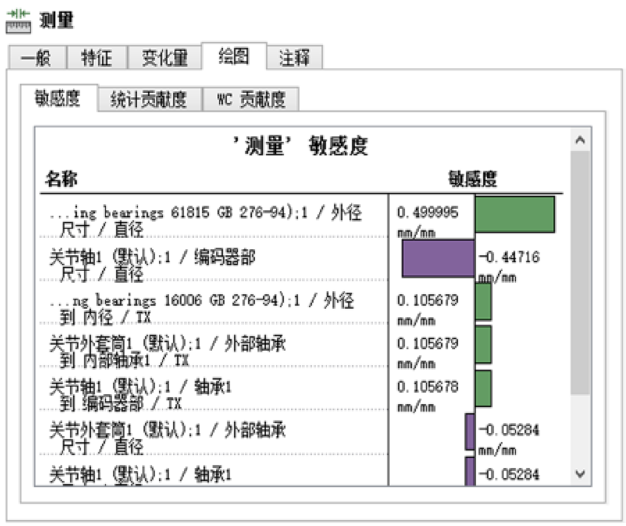

Fig-6: Sensitivity of the first joint

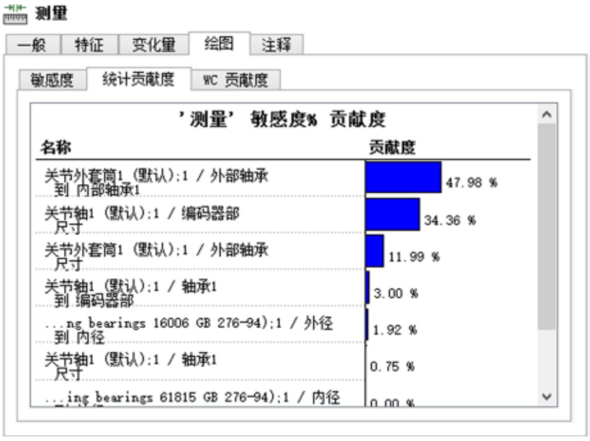

Fig-7: Statistical contribution of the first joint

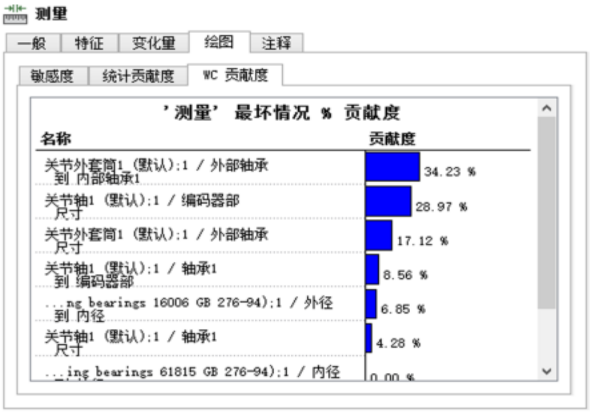

Fig-9: Worst case contribution of the first joint

Fig-6-Fig-8 show the sensitivity, statistical contribution, and worst case contribution of each variables in the measurement model of the first joint. Sensitivity can be used to identify the critical dimensions. For a given measurement, the most sensitive variable value is the most important variable. Conversely, sensitivity can also be used to identify non-critical dimensions, and variables with low sensitivity are non-critical variables. These variables are a possible source of cost savings. If all of the key measurements of the assembly have low sensitivity values for a particular variable, then the assembly will be able to use low-cost process manufacturing.

Table-1: XML Data structure of the first joint(unit:mm)

Name	content	Nominal size	Tolerance	Sigma	%Yield
Measurement	State 1	44.5	44.50 ±0.003	2.95	99.68%
Tolerance tree data:(unit: mm)					
Name	**content**	**Nominal size**	**Tolerance**	**Sensitivity**	
To internal bearing 1	Outer joint sleeve1 to outer bearing	10	10.00 ±0.02	0.105679 mm/mm	
size	Outer joint sleeve1 to outer bearing	75	75.00 ±0.02	-0.0528398 mm/mm	
size	Joint axis1 to Encoder section	6	6.00 ±0.004	-0.44716 mm/mm	
To encoder section	Joint axis1 to bearing1	12	12.00 ±0.005	0.105678 mm/mm	
size	Joint axis1 to bearing1	30	30.00 ±0.005	-0.0528396 mm/mm	
To the inside diameter	Deep groove ball bearings 16006 to 1/OD	12.5	12.50 ±0.004	0.105679 mm/mm	

As shown in Figure 6, the most sensitive is the outer diameter tolerance d6 of the outer bearing. Therefore, one of the most important ways to improve the accuracy of such PCMA is to select high-precision bearings that satisfy the accuracy requirements of the measuring machine.

Fig-7 and Fig-8 show the statistical contribution and worst case contribution. The next one is the connection dimension d0 between the joint axis and the rotary encoder, which is fixed in design and difficult to change. In addition to bearing tolerances and d0 dimensions, other dimensions can reduce manufacturing costs by reasonably controlling tolerances. The random error due to the gap tolerance of the bearings in the first joint satisfies 0.003mm with a probability of 99.68%. The tolerance analysis structure of the first joint is shown in Table-1.

2.3 Tolerance analysis of the 4th joint

The internal structure of the fourth joint is shown in Fig- 9. The analysis method of the fourth joint is the same as that of the first joint.

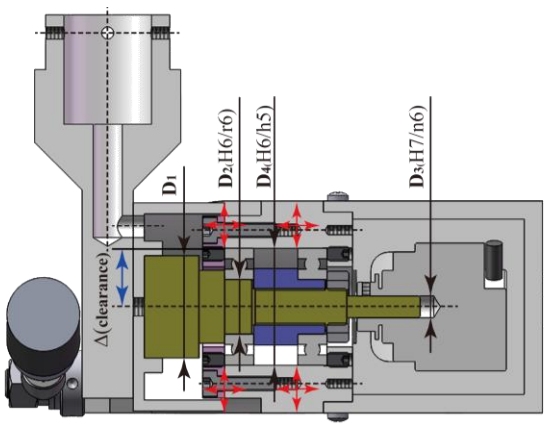

Fig-9: Internal structure of the fourth joint

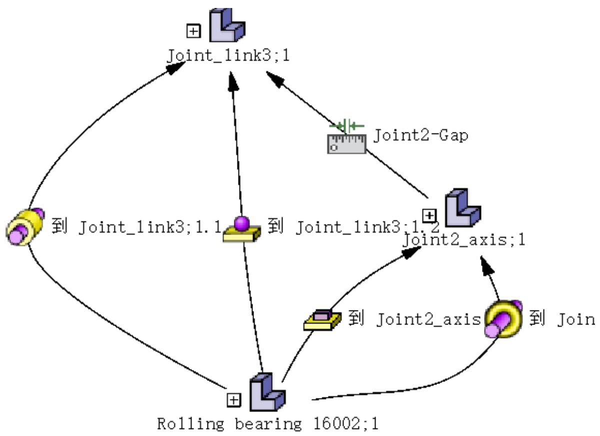

Fig--10: Assembly constrains of the fourth joint DOF state

Fig-10 shows the assembly constraint state of the fourth joint using CETOL6σ. The dimension to be measured is the clearance error (Δ) of the swing joint connection radius D1/2 and the center axis of the 4th joint.

The random error due to the gap tolerance of the bearings in the fourth joint satisfies 0.004mm with a probability of 99.64%.

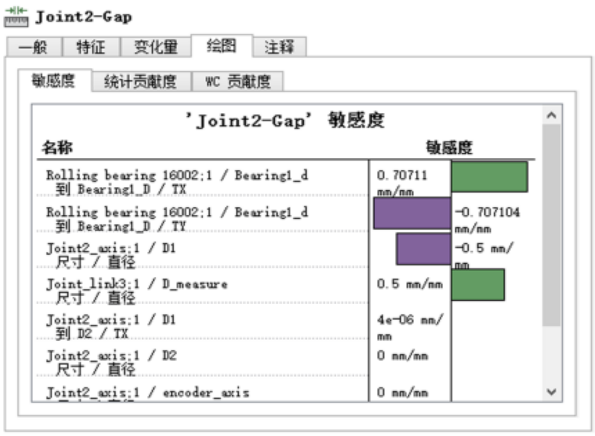

Fig-11: Sensitivity of the 4th joint

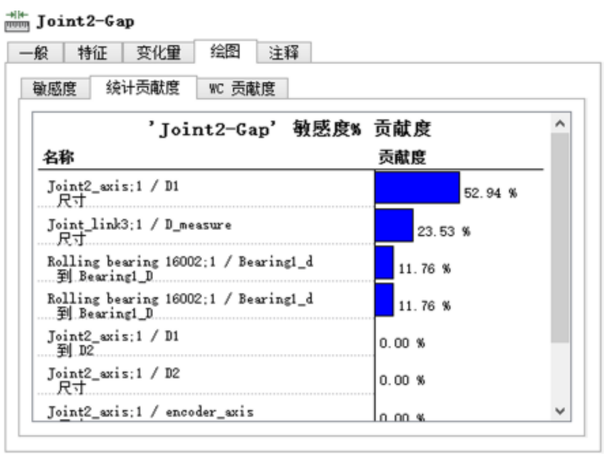

Fig-12: Statistical contribution of the 4th joint

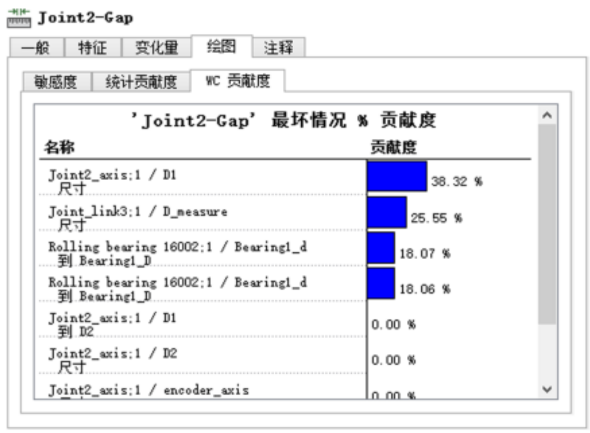

Fig-13: Worst case contribution of the 4th joint

2.4 Tolerance analysis of the fifth joint

The internal structure of the probe joint is shown in Fig-14. The analysis method of the five joint is the same as that of the first joint. Using CETOL6σ, the assembly constraint state of the fifth joint is as shown in Fig-15. The measured dimension is the gap clearance from the center axis of the probe to the centerline of the joint.

Table-2: XML Data structure of the 4th joint(unit:mm)

Name	content	Nominal size	Tolerance	Sigma	%Yield
Joint4-Gap	Gap_measure	32	0.000 ±0.004	2.91	99.64%
Tolerance tree data:(unit：mm)					
Name	**content**	**Nominal size**	**Tolerance**	**Sensitivity**	
to Bearing1_D	Rolling bearing 16002;1/Bearing1_d	0	0.000 ±0.002	-0.707104 mm/mm	
To Bearing1_D	Rolling bearing 16002;1/Bearing1_d	8.5	8.500 ±0.002	0.70711 mm/mm	
size	Joint_link3;1/Dmeasure	32	32.000 ±0.004	0.5 mm/mm	
size	Joint4_axis;1/D1	28	28.000 ±0.006	-0.5 mm/mm	

Table-3: XML Data structure of the 5th joint(unit:mm)

Name	content	Nominal size	Tolerance	Sigma	%Yield
measure	Joint5	11	11.000 ±0.003	3.18	99.85%
Tolerance tree data:(unit：mm)					
Name	**content**	**Nominal size**	**Tolerance**	**Sensitivity**	
TY	Roller bearing_1 16001;1/To Ring1;1.1	0	±0.0015	1 mm/mm	
To bearing1_D1	Roller bearing_1 16001;1/bearing1_d1	8	8.000 ±0.002	0.51171 mm/mm	

size	probe_shaft;1/encoder_D1	6	6.000 ±0.004	-0.5 mm/mm
size	Ring1;1/ring_d1	28	28.000 ±0.003	0.5 mm/mm

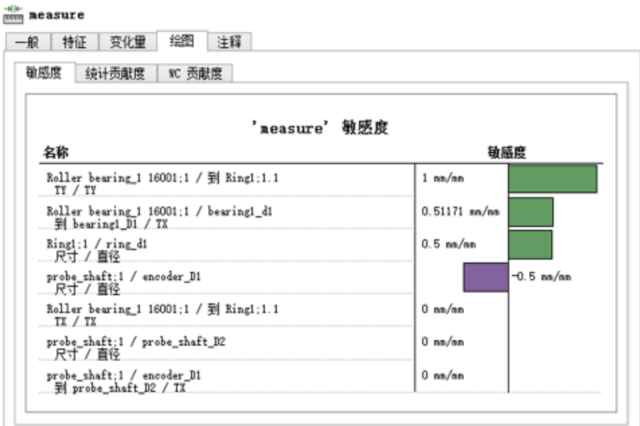

Fig-16: Sensitivity of the 5th joint

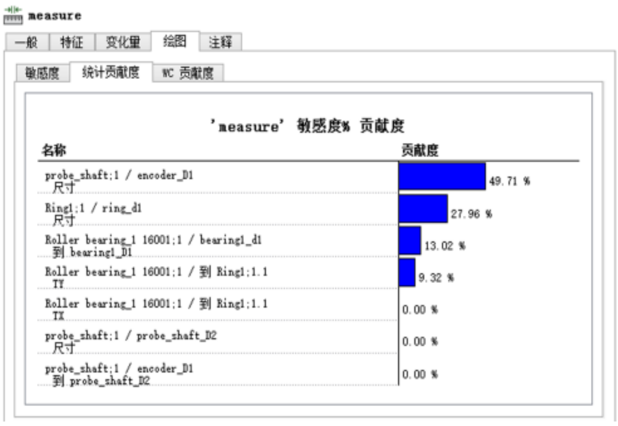

Fig-17: Statistical contribution of the 5th joint

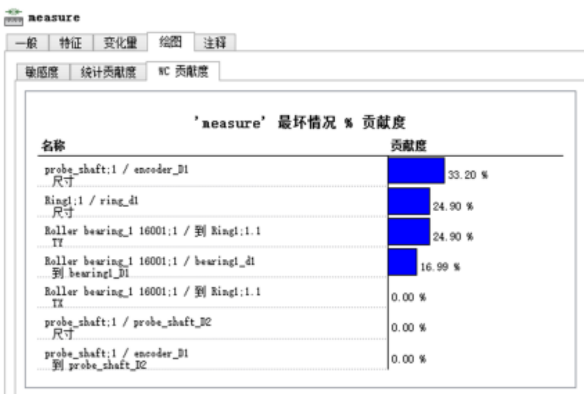

Fig-18: Worst case contribution of the 5th joint

Fig-16-Fig-18 show the sensitivity, statistical contribution, and worst case contribution of each variable in the fifth joint measurement model calculated here.

As shown in Fig-16, the most sensitive is the outer diameter tolerance D3 of the outer bearing. This reduction in clearance tolerance through this dimension requires an interference fit.

Fig-17 and Fig-18 show the statistical contribution and worst case contribution.

The dimensional tolerance of the 5th joint can guarantee a dimensional accuracy of 0.003 mm with a probability of 99.85%. That is, the random error in the 5th joint is 0.003 mm. The tolerance analysis structure of the 5th joint is shown in Table-3.

2.5 Calculation of random errors caused by the clearance of bearings

In this paper, the MDH structural parameters of the PCMA are shown in Table-4.

For each pair of joints, the random errors caused by the clearance bearings are calculated by statistical method as follows (Table-5).

$$\pm T_{bearing}^{total} = \sqrt{\sum_{i=1}^{n} {}^{bearing}T_i^2} \text{ (mm)} \tag{3}$$

where ${}^{bearing}T_i$ is the gap tolerance of the ith joint, $T_{bearing}^{total}$ is the total tolerance

Table-4: Nominal MDH parameters

№	θ_i [°]	d_i [mm]	α_i [°]	a_i [mm]	β_i[°]	Range of θ_i
1	θ_1	156.5	0	0	0	[-145, 145]
2	θ_2	-	0	420	0	[-158, 158]
3	θ_3	-	0	450	0	[-30, 145]
4	θ_4	-65.5	-90	85	-	[-45, 180]
5	θ_5	682	90	-118.4	-	[0, 360]

Table-5: Gap tolerance in joint pairs, (unit:mm)

№	${}^{bearing}T_i$	d_i	a_i	${}^{bearing}T_i$	%Yield
1	${}^{bearing}T_1$	156.5	0	±0.003	99.68
2	${}^{bearing}T_2$	-	420	±0.003	99.68
3	${}^{bearing}T_3$	-	450	±0.003	99.68
4	${}^{bearing}T_4$	-65.5	85	±0.004	99.64
5	${}^{bearing}T_5$	682	-118.4	±0.003	99.85
$T_{bearing}^{total}$		0.0072[mm]			

2.6 Effects of random errors caused by encoder systematic uncertainty

In this paper, we used the rotary encoder ERN1070 produced by the German company HEIDENHAIN, which has a cycle of 3600 signal periods per revolution and 40 times subdivision.

The maximum directional deviation at 20°C ambient temperature and slow speed(scanning frequency between 1kHz and 2kHz) lies within ±18″)[(e.g. [16]).

For each joint, the angular errors are converted to the distance errors due to the systematic uncertainty of the encoder:

$$\pm {}^{link}T_i = L_i \cdot {}^{encod}T_i \tag{4}$$

where ${}^{encod}T$ is the error of the ith encoder, L_i is the length of the ith joint arm, (mm)

$$L_i = [d_1, a_2, a_3, (\overline{d_4 a_4}), (\overline{d_5 a_5})]$$

According to the statistical method, the total error of the center of the probe can be obtained as follow:

$$\pm T_{link}^{total} = \sqrt{\sum_{i=1}^{n} (L_i \cdot {}^{encod}T_i)^2} = 0.0814 \text{ (mm)} \tag{5}$$

Table-6: Length tolerance according to the systematic uncertainty of the encoder, (unit:mm)

№	d_i	a_i	${}^{encod}T_i$	${}^{link}T_i$
1	156.5	0	±18	-
2	-	420	±18	±0.0366
3	-	450	±18	±0.0393
4	-65.5	85	±18	±0.0094
5	682	-118.4	±18	±0.0604
T_{link}^{total}		0.0814 [mm]		

2.6 Calculation of total random errors

According to the statistical method, the total of random errors of PCMA is calculated by the following formula:

$$\Delta_{rand}^{total} = \sqrt{(\Delta_{link}^{total})^2 + (\Delta_{bearing}^{total})^2 + (\Delta_{temp}^{L})^2 + (\Delta_{other})^2} \tag{6}$$

where

Δ_{link}^{total} is a random error caused by the systematic uncertainty of the encoder.

$\Delta_{bearing}^{total}$ is the random error caused by the bearing clearances in each joint

Δ_{temp}^{L} is the thermal expansion error of the measuring machine due to temperature changes

Δ_{other} is other error (this error may be caused by thermal deformation, operator error, plastic deformation caused by measuring operating force, and deformation due to its own weight, etc. These are unpredicTable-metrics that can be reduced when strictly following standard operation.)

In a simple first-order model, the change in the length of an object caused by temperature changes is represented by the following equation [(e.g. [17,18]):

$$\Delta_{temp}^{L} = \alpha \cdot L \cdot \Delta T \tag{7}$$

where

L : nominal length of PCMA (L = 1650mm)

T : ambient temperature during testing or measurement

α: coefficient of material thermal expansion,

α = 23×10-6/°C (aluminum alloy)

∆L: change in length of arm length

∆T: temperature change (T - 20), °C

$$\Delta_{temp}^{L} = 23 \times 10^{-6} \times 1650 \times 2 = 0.0759 \text{(mm)}$$

Δ_{other} is ignored (as described in Section 1)

The total random error of PCMA is calculated by equation (6).

$$\Delta_{rand}^{total} \approx \sqrt{(\Delta_{link}^{total})^2 + (\Delta_{bearing}^{total})^2 + (\Delta_{temp}^{L})^2}$$
$$= \sqrt{0.0814^2 + 0.0072^2 + 0.0759^2}$$
$$= 0.1112 \text{mm}$$

We can be seen from equation (6) that the accuracy of PCMA depends on the accuracy of the bearing, the system accuracy of the rotary encoder, and the influence of temperature. In particular, it can be seen that the systematic uncertainty of the encoder has the greatest influence on the random error of the measuring machine, and the influence of random errors due to bearing accuracy and temperature cannot be ignored. This is because of the characteristics of PCMA with a large measuring space, and the amplification error caused by these factors is reflected at the center of the probe. This random error has a large impact on the repeatability of the measuring machine, so it is not possible to find a more accurate and optimal solution in the calibration. Moreover, random errors due to temperature changes are very significant in the case of measuring machine in the work-site. The analysis results show that to improve the measurement accuracy of PCMA, the mechanical structure must be compact and the rotary encoder must be reasonably determined to satisfy the precision requirements of the measuring machine. At the same time, a temperature compensation system must be provided to ensure higher precision.

3. CONCLUSION

This paper has presents a method for analyzing the random error of PCMA and given a method for evaluating it. CETOL 6σ was used to calculate the random error due to the gap of the bearings in each pair of joints. And also the total value of random errors is obtained by calculating of the thermal expansion variation of the measuring arm due to the temperature variation and the systematic uncertainty of the encoder. The limit of the random error of PCMA proposed in this paper is ±0.1112 mm, which represents the theoretically achievable repeatability of PCMA. The random error estimation method proposed in this paper will help theoretically evaluate the accuracy limit that the machine can reach at the design stage.

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