International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 06 Issue: 03 | Mar 2019 www.irjet.net p-ISSN: 2395-0072

Soft Pre Generalized b-Closed Sets in a Soft Topological Space

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Abstract:- *This paper introduces soft pre generalized b-closed sets in a soft topological space. In a soft topological space, a soft set F_A is said to soft pre generalized b- closed if bcl (F_A) $\tilde{\subseteq}$ F_0 whenever $F_A \tilde{\subseteq} F_0$ and F_0 is a soft P- open set in X. A detail study carried out on properties of soft Pre generalized b- closed sets.*

Keywords: **Soft Pgb- closed set, Soft Pgb- open set, Soft Pgb-closure, soft Pgb b-interior**.

1. INTRODUCTION

Soft set theory was first introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on.

In 2010 Muhammad shabir, Munazza Naz [2] used soft sets to define a topology namely Soft topology. In general topology the concept of generalized closed set was introduced by Levine [3]. This notation has been studied extensively in recent years by many topologies.

The investigation of generalized closed sets has led to several new and interesting concepts, e.g. new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology.

Soft generalized closed set was introduced by K.Kannan [4] in 2012. Soft pre generalized closed set was introduced by J.Subhashinin [5] in 2014. In this paper we introduce soft pre generalized b-closed set in a soft topological space and studied some of its properties.

2. Preliminaries

2.1. Definition [6] A soft set F_A on the universe X is defined by the set of ordered pairs $F_A=\{(e,f_A(e)): e\in E, f_A(e)\in P(X)\}$, where E is the parameters, $A\subseteq E$, P(X) is the power set of X, and f_A: A→P(X) such that $f_A(e)=\Phi$ if $e\notin A$. Here f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary, some of them may be empty, some may have non-empty intersection. Note that the set of all soft set over X is denoted by S(X).

2.2. Definition [6] A soft set F_A over X is called a null soft set, denoted by F_ϕ, if $e\in A$, $F(e)=\phi$.

2.3. Definition [6] A soft set F_A over X is called a absolute soft set, denoted by $\tilde{A}$, if $e\in A$, F(e)=X.

If A=E then the A-universal soft set is called a universal soft set denoted by $\tilde{X}$.

2.4. Definition [6] Let $F_E\in S(X)$. A soft topology on F_E denoted by $\tilde{\tau}$ is a collection of soft subsets of F_E having the following properties:

(i) F_ϕ, $F_E\in\tilde{\tau}$
(ii) $\{F_{E_i}\tilde{\subseteq} F_E : i\in I\subseteq N\}\tilde{\subseteq}\tilde{\tau} \Rightarrow \cup_{i\in I} F_{Ei}\in\tilde{\tau}$
(iii) $\{F_{E_i}\tilde{\subseteq} F_E : 1\le i\le n, n\in N\}\tilde{\subseteq}\tilde{\tau} \Rightarrow \cap^n F_{Ei}\in\tilde{\tau}$.
The pair (F_E, $\tilde{\tau}$) is called a soft topological space.

2.5. Definition [6] Let (F_E, $\tilde{\tau}$) be a soft topological space. Then every element of $\tilde{\tau}$ is called a soft open set. Clearly F_ϕ and F_E are soft open sets. F_C is said to be soft closed if the soft set $F_C^{\tilde{c}}$ is soft open in F_E.

2.6. Definition [5] Let (F_E, $\tilde{\tau}$) be a soft topological space over X, a soft set F_A is said to be soft pre-open (soft P-open) if $F_A\tilde{\subseteq}int(cl(F_A))$. F_A is said to be soft pre-closed (soft P-closed) if the soft set $F_A^{\tilde{c}}$ is soft P- open in F_E.

2.7. Proposition [5]

i) Every soft open set is a soft pre open set.
ii) Every soft closed set is a soft pre closed set.

2.8. Definition [7] Let (F_E, $\tilde{\tau}$) be a soft topological space over X, a soft set F_A is said to be soft b-open if $F_A \tilde{\subseteq} int(cl(F_A)) \tilde{\cup} cl(int(F_A))$. A soft set F_A is said to be soft b-closed if $int(cl(F_A)) \tilde{\cap} cl(int(F_A)) \tilde{\subseteq} F_A$

2.9. Theorem[7]

In a soft topological space X and let F_A and F_B are soft sets over X.Then

i) $bcl(\phi)=\phi$
ii) $bint(\phi)=\phi$
iii) $(bcl(bcl\ F_A))=bcl\ F_A$
iv) $bcl(F_A\tilde{\cup}F_B)= bcl(F_A)\ \tilde{\cup}\ bcl(F_B)$
v) $bcl(F_A\tilde{\cap}F_B)\ \tilde{\subseteq}\ bcl(F_A)\ \tilde{\cap}\ bcl(F_B)$

Proof: Refer theorem 7[7]

2.10. Definition [8] Let (F_E, $\tilde{\tau}$) be a soft topological space over X . A soft set F_A is called a soft generalized closed (soft g-closed set) in U if cl(F_A) $\tilde{\subseteq}$ F_0 whenever F_A $\tilde{\subseteq}$ F_0 and F_0 is soft open in X.

2.11. Definition [5] Let (F_E, $\tilde{\tau}$) be a soft topological space over X. A soft set F_A is called a soft pre- generalized closed set (soft pg-closed set) in X if pcl(F_A) $\tilde{\subseteq}$ F_0 whenever F_A $\tilde{\subseteq}$ F_0 and F_0 is soft pre open in X.

3. Soft Pre Generalized b-closed set

In this section we introduce soft pre generalized b-closed sets in a soft topological space and study some of their properties.

3.1. Definition Let (F_E, $\tilde{\tau}$) be a soft topological space over X. A soft set F_A is called a soft pre generalized b-closed set (briefly soft pgb-closed set) in X if bcl(F_A) $\tilde{\subseteq}$ F_0 whenever F_A $\tilde{\subseteq}$ F_0 and F_0 is soft pre open in X.

3.2. Definition Let (F_E, $\tilde{\tau}$) be a soft topological space and let F_A be soft subset of F_E. Then the soft pre generalized b-closure (briefly soft pgb-closure) of F_A denoted by pgbcl(F_A) is defined as the intersection of all soft pre generalized b-closed supersets of F_A.

3.3. Example Let (F_E,$\tilde{\tau}$) be a soft topological space over X, where X={h_1,h_2,h_3} ,E={e_1,e_2} , $\tilde{\tau}$ ={ $\tilde{X}$, F_ϕ, $F_{E_1}, F_{E_2}, F_{E_3}, F_{E_4}, F_{E_5}, F_{E_6}, F_{E_7}, F_{E_8}$}and the following soft sets over X can be defined as follows:

$\tilde{X}$= {(e_1, {h_1, h_2, h_3}), (e_2, {h_1, h_2, h_3})}, F_ϕ= {(e_1, ϕ), (e_2, ϕ)}

F_{E_1}= {(e_1, {h_3}), (e_2, ϕ)}, F_{E_2}= {(e_1, {h_3}), (e_2, {h_1})}

F_{E_3}= {(e_1, {h_3}), (e_2, {h_1, h_3})}, F_{E_4}= {(e_1, ϕ), (e_2, {h_3})}

F_{E_5}= {(e_1, {h_3}), (e_2, {h_3})}, F_{E_6}= {(e_1, {h_2}), (e_2, {h_3})}

F_{E_7}= {(e_1, {h_2, h_3}), (e_2, {h_1, h_3})}, F_{E_8}= {(e_1, {h_2, h_3}), (e_2, {h_3})}

The soft set A_E= {(e_1, {h_1, h_2}), (e_2, $\tilde{X}$)} is a soft pgb-closed set.

3.4. Theorem. Every soft closed set is soft pgb-closed set.

Proof: Let F_A be any soft closed set in X such that $F_A$$\tilde{\subseteq}$ F_0, where F_0 is soft P-open. Since bcl(F_A) $\tilde{\subseteq}$ cl(F_A)=F_A, bcl(F_A) $\tilde{\subseteq}$ F_0. Hence F_A is soft pgb-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

3.5. Example

In example 3.3, consider the soft subset, B_E={(e_1,ϕ),(e_2,{h_2,h_3})}. The soft set B_E is a soft pgb-closed set but not a soft closed set.

3.6. Theorem. Every soft b-closed set is soft pre generalized b-closed set.

Proof: Let F_A be any soft b-closed set such that F_0 is any soft pre-open set containing F_A. Since F_A is soft b-closed, bcl(F_A)=$F_A$$\tilde{\subseteq}$ F_0, bcl(F_A) $\tilde{\subseteq}$ F_0.Hence F_A is soft pgb-closed set.

The converse of the above theorem need not true be as seen from the following example.

3.7. Example

In example 3.5, the soft subset B_E is soft pgb-closed but not a soft b-closed.

3.8. Theorem. Every soft generalized closed set is a soft pgb-closed set.

Proof: Let F_A be any soft g-closed set in a soft topological space X and F_0 is any soft open set containing F_A. Since every soft open set is a soft pre-open set, bcl(F_A) $\tilde{\subseteq}$ cl(F_A) $\tilde{\subseteq}$ F_0 .Therefore bcl(F_A) $\tilde{\subseteq}$ F_0 . Hence F_A is soft pgb-closed set.

The converse of the above theorem need not true be as seen from the following example.

3.9. Example

In example 3.3, consider the soft subset C_E= {(e_1, {h_2}), (e_2, {h_1, h_3})}. The set C_E is a soft pgb-closed set but not a soft generalized closed set.

3.10. Theorem. Every soft pg-closed set is a soft pgb-closed set

Proof: Let F_A be any soft pg closed set such that F_0 is soft pre open set containing F_A, pcl(F_A) $\tilde{\subseteq}$ F_0. Also, bcl(F_A) $\tilde{\subseteq}$ pcl(F_A) $\tilde{\subseteq}$ F_0, bcl(F_A) $\tilde{\subseteq}$ F_0. Hence F_A is soft pgb-closed set.

The converse of the above theorem need not be true as seen from the following example.

3.11. Example

In example 3.3, Consider the soft subset D_E = {(e_1,{h_2,h_3}),(e_2,{h_1})}. The set D_E is a soft pgb-closed set but not a soft pg-closed set.

3.12. Theorem. If F_A and F_B are soft pgb-closed sets in X then $F_A \tilde{\cup} F_B$ is a soft pgb-closed set in X.

Proof: Let F_A and F_B be soft pgb-closed sets in X and F_0 be any soft pre open set containing F_A and F_B. Therefore $bcl(F_A) \tilde{\subseteq} F_0$; $bcl(F_B) \tilde{\subseteq} F_0$. Since $F_A \tilde{\subseteq} F_0$, $F_B \tilde{\subseteq} F_0$ then $F_A \tilde{\cup} F_B \tilde{\subseteq} F_0$. Hence $bcl(F_A \tilde{\cup} F_B)=bcl(F_A) \tilde{\cup} bcl(F_B) \tilde{\subseteq} F_0$. Therefore $F_A \tilde{\cup} F_B$ is a soft pgb-closed set in X.

3.13. Theorem. A set F_A is a soft pgb-closed set iff $bcl(F_A)-F_A$ contains no non-empty soft pre closed set.

Proof: Assume that F_C is a soft pre closed set in X such that $F_C \tilde{\subseteq} bcl(F_A)-F_A$. Then $F_A \tilde{\subseteq} F_C^{\tilde{c}}$. Since F_A is soft pgb-closed set and $F_C^{\tilde{c}}$ is soft pre open then $bcl(F_A) \tilde{\subseteq} F_C^{\tilde{c}}$. ie) $F_C \tilde{\subseteq} bcl(F_A^{\tilde{c}})$, $F_C=F_\phi$. Hence $bcl(F_A)-F_A$ contains null soft pre closed set.

Conversely, Assume that $bcl(F_A)-F_A$ contains no non-empty soft pre closed set. Let $F_A \tilde{\subseteq} F_0$, F_0 is soft pre open. Suppose that $bcl(F_A)$ is not contained in F_0, $bcl(F_A) \tilde{\cap} F_0^{\tilde{c}}$ is a non –empty soft pre closed set of $bcl(F_A)-F_A$,which is contradiction. Therefore $bcl(F_A) \tilde{\subseteq} F_0$. Hence F_A is soft pgb-closed set.

3.14. Theorem. The intersection of any two soft pgb-closed sets in X is also a soft pgb-closed set in X.

Proof: Let F_A and F_B be any two soft pgb-closed sets. $F_A \tilde{\subseteq} F_0$, F_0 is any soft pre open and $F_B \tilde{\subseteq} F_0$, F_0 is soft pre open. Then $bcl(F_A) \tilde{\subseteq} F_0$, $bcl(F_B) \tilde{\subseteq} F_0$, therefore $bcl(F_A \tilde{\cap} F_B) \tilde{\subseteq} bcl(F_A) \tilde{\cap} bcl(F_B) \tilde{\subseteq} F_0$, F_0 is soft pre open in X. Hence $F_A \tilde{\cap} F_B$ is a soft pgb-closed set.

3.15. Theorem. If F_A is a soft pgb-closed set in X and $F_A \tilde{\subseteq} F_B \tilde{\subseteq} bcl(F_A)$, Then F_B is a soft pgb-closed set in X.

Proof: Since $F_B \tilde{\subseteq} bcl(F_A)$, we have $bcl(F_B) \tilde{\subseteq} bcl(F_A)$. Clearly $bcl(F_B)- F_B \tilde{\subseteq} bcl(F_A) - F_A$ .By theorem 3.13, $bcl(F_A)-F_A$ contains no non-empty soft pre closed set.Hence $bcl(F_B)-F_B$ contains no non-empty soft pre closed set. Therefore F_B is soft pgb-closed set in X.

4. Soft Pre Generalized b-open set

In this section we introduce soft pre generalized b-open sets in a soft topological space and study some of their properties.

4.1. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space over X. A soft set F_A is called a soft pre generalized b-open set (pgb-open set) if the complement $F_A^{\tilde{C}}$ is a soft pgb-closed set in X. Equivalently, F_A is called a soft pgb-open set if $F_C \tilde{\subseteq} bint(F_A)$ whenever $F_C \tilde{\subseteq} F_A$ and F_C is soft pre closed set in X.

4.2. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and let F_A be a soft subset of F_E. Then the soft pre generalized b-interior (soft pgb-interior) of F_A denoted by $pgbint(F_A)$ is defined as the soft union of all soft pre generalized open subsets of F_A.

4.3. Example

In example 3.3, consider the soft subset F_{E_1}= {(e1, {h3}), (e2, ϕ)}. The soft set F_{E_1} is a soft pgb-open set.

4.4. Theorem. Every soft b-open set is a soft pre generalized b-open set.

Proof: Let F_A be any soft b-open set such that F_C is any soft pre closed set contained in F_A. Since F_A is soft b-open, $bint(F_A)=F_A \tilde{\supseteq} F_C$. Therefore $bint(F_A) \tilde{\supseteq} F_C$. Hence F_A is a soft pgb-open set.

The converse of the above theorem need not true be as seen from the following example.

4.5. Example. In example 3.3, consider the soft subset H_E= {(e1, $\tilde{X}$), (e2, {h1})} .The soft set H_E is a soft pgb-open set but not a soft b-open set.

4.6. Theorem. The intersection of two soft pgb-open sets in X is also a soft pgb-open set in X.

Proof: Let F_A and F_B be soft pgb-open sets in X. Then $F_A^{\tilde{C}}$ and $F_B^{\tilde{C}}$ are soft pgb-closed sets. By theorem3.12, $F_A^{\tilde{C}} \tilde{\cup} F_B^{\tilde{C}}$ is also a soft pgb-closed set in X. (ie) $F_A^{\tilde{C}} \tilde{\cup} F_B^{\tilde{C}} = (F_A \tilde{\cap} F_B)^{\tilde{c}}$ is a soft pgb-closed set in X. Therefore $F_A \tilde{\cap} F_B$ is a soft pgb-open set in X.

4.7. Theorem. If $bint(F_A) \tilde{\subseteq} F_B \tilde{\subseteq} F_A$ and F_A is soft pgb-open in X, then F_B is soft pgb-open in X.

Proof: Suppose that $bint(F_A) \tilde{\subseteq} F_B \tilde{\subseteq} F_A$ and F_A is soft pgb-open in X. $F_A^{\tilde{c}} \tilde{\subseteq} F_B^{\tilde{c}} \tilde{\subseteq} bcl(F_A^{\tilde{c}})$.Since $F_A^{\tilde{c}}$ is soft pgb-closed in X, by theorem 3.15 $F_B^{\tilde{c}}$ is soft pgb-closed in X. Hence F_B is soft pgb-open in X.

Conclusion

Soft set theory was introduced by D.Molodtsov[1] in 1999.[2] Muhammad shabir and Munazza Naz introduced the concept of soft topological space. In 2014,J.Subhashini and C.Sekar[5] introduced the concept of soft pre generalized closed sets. In this paper, we introduce soft pre generalized b- closed sets based on the soft pre open sets. Also we introduce soft pre generalized b- open sets and study some theorems.

References

[1]D.Molodtsov, Soft Set Theory-First Results, Computers and Mathematics with applications, Vol. 37 (1999) 19-31

[2]Muhammad Shabir and Munazza Naz, On Soft topological spaces, Computers and Mathematics with Applications, Vol.63 (2011) 1786-1799.

[3]N.Levine, Generalised closed sets in topology, Rend.Circ.Mat.Palermo, Vol. 19 (2) (1970) 89-96.

[4] K.Kannan, Soft Generalized closed sets in Soft topological spaces, Journal of Theoretical and Applied Information Technology, Vol.37 (1) (2012) 1-20.

[5]J.Subhashini, Soft Pre Generalized closed sets in a soft topological space, International Journal of Engineering Trends and Technology, Vol.12, Number 7-Jun 2014.

[6] Naim Cagman, Serkan Karatas and Serdar Enginoglu, Soft Topology, Computers and Mathematics with Applications, Vol.62 (2011) 351-358.

[7] Metin Akdag and Alkan Ozkan, Soft b-open sets and soft b-continuous functions , Original Research , Math Sci (2014) 8:124

[8] Saziye Yuksel, Naime Tozlu, Zehra Guzel Ergul, On soft generalized closed sets in soft Topological spaces, Journal of Theoretical and Applied Information Technology, Vol.55(2) (2013) 273-279.