On the Positive Pell Equation $y^2 = 35x^2 + 29$

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Abstract – *The binary quadratic Diophantine equation represented by the positive Pellian* $y^2 = 35x^2 + 29$ *is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

Key Words: **Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions**

1. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer, has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values [1-4]. For an extensive review of various problems, one may refer [5-17]. In this communication, yet another an interesting equation given by $y^2 = 35x^2 + 29$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

2. METHOD OF ANALYSIS

The Positive pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 29 \qquad (1)$$

The smallest positive integer solutions of (1) are

$$x_0 = 1, \; y_0 = 8$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1 \qquad (2)$$

whose initial solution is

$$\tilde{x}_0 = 1, \; \tilde{y}_0 = 6$$

The general Solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n, \; \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$$

$$g_n = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1}, \; n = -1, 0, 1, \ldots$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{4}{\sqrt{35}} g_n$$
$$y_{n+1} = 4 f_n + \frac{\sqrt{35}}{2} g_n$$

The recurrence relations satisfied by the solutions x and y are given by

$$x_{n+3} - 12x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 12y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x_n and y_n satisfying (1) are given in the Table: 1 below:

Table: 1 Examples

n	x_n	y_n
0	1	8
1	14	83
2	167	988
3	1990	11773
4	23713	140288
5	282566	1671683
6	3367079	19919908

From the above table, we observe some interesting relations among the solutions which are presented below:

- Both x_n and y_n values are alternatively odd and even.

2.1. Relations among the solutions are given below:

- $x_{n+2} = y_{n+1} + 6x_{n+1}$
- $x_{n+3} = 12y_{n+1} + 71x_{n+1}$
- $y_{n+2} = 6y_{n+1} + 35x_{n+1}$
- $y_{n+3} = 71y_{n+1} + 420x_{n+1}$
- $y_{n+2} = 6x_{n+2} - x_{n+1}$
- $y_{n+3} = 71x_{n+2} - 6x_{n+1}$
- $12x_{n+2} = x_{n+3} + x_{n+1}$
- $x_{n+3} = 2y_{n+2} - x_{n+1}$
- $6y_{n+3} = 71y_{n+2} + 35x_{n+1}$
- $71x_{n+3} = 12y_{n+3} + x_{n+1}$
- $6x_{n+3} = y_{n+1} - 71x_{n+2}$
- $6y_{n+1} = 71y_{n+2} - 35x_{n+3}$
- $6x_{n+2} = x_{n+3} - y_{n+2}$

- $y_{n+1} = 71y_{n+3} - 420x_{n+3}$
- $x_{n+2} = 6x_{n+3} - y_{n+3}$
- $y_{n+2} = 6y_{n+3} - 35x_{n+3}$
- $6y_{n+2} = y_{n+1} + 35x_{n+2}$
- $y_{n+3} = y_{n+1} + 70x_{n+2}$
- $y_{n+3} = 12y_{n+2} - y_{n+1}$
- $y_{n+3} = 6y_{n+2} + 35x_{n+2}$

2.2. Each of the following expression is a nasty number:

- $\frac{6}{29}\left[58 + 16y_{2n+2} - 70x_{2n+2}\right]$
- $\frac{6}{29}\left[58 + 16x_{2n+3} - 166x_{2n+2}\right]$
- $\frac{1}{29}\left[348 + 8x_{2n+4} - 988x_{2n+2}\right]$
- $\frac{1}{29}\left[348 + 16y_{2n+3} - 980x_{2n+2}\right]$
- $\frac{6}{2059}\left[4118 + 16y_{2n+4} - 11690x_{2n+2}\right]$
- $\frac{1}{29}\left[348 + 166y_{2n+2} - 70x_{2n+3}\right]$
- $\frac{6}{2059}\left[4118 + 1976y_{2n+2} - 70x_{2n+4}\right]$
- $\frac{6}{29}\left[58 + 28y_{2n+2} - 2y_{2n+3}\right]$
- $\frac{1}{29}\left[348 + 167y_{2n+2} - y_{2n+4}\right]$
- $\frac{6}{29}\left[58 + 166x_{2n+4} - 1976x_{2n+3}\right]$
- $\frac{6}{29}\left[58 + 166y_{2n+3} - 980x_{2n+3}\right]$
- $\frac{1}{29}\left[348 + 166y_{2n+4} - 11690x_{2n+3}\right]$
- $\frac{1}{29}\left[348 + 1976y_{2n+3} - 980x_{2n+4}\right]$
- $\frac{6}{29}\left[58 + 1976y_{2n+4} - 11690x_{2n+4}\right]$
- $\frac{6}{29}\left[58 + 334y_{2n+3} - 28y_{2n+4}\right]$

2.3. Each of the following expressions is a cubical integer:

- $\frac{1}{29}\left[16y_{3n+3} - 70x_{3n+3} + 48y_{n+1} - 210x_{n+1}\right]$
- $\frac{1}{29}\left[16x_{3n+4} - 166x_{3n+3} + 48x_{n+2} - 498x_{n+1}\right]$

- $\frac{1}{87}\left[4x_{3n+5}-494x_{3n+3}+12x_{n+3}-1482x_{n+1}\right]$
- $\frac{1}{87}\left[8y_{3n+4}-490x_{3n+3}+24y_{n+2}-1470x_{n+1}\right]$
- $\frac{1}{2059}\left[16y_{3n+5}-11690x_{3n+3}+48y_{n+3}-35070x_{n+1}\right]$
- $\frac{1}{87}\left[83y_{3n+3}-35x_{3n+4}+249y_{n+1}-105x_{n+2}\right]$
- $\frac{1}{2059}\left[1976y_{3n+3}-70x_{3n+5}+5928y_{n+1}-210x_{n+3}\right]$
- $\frac{1}{29}\left[28y_{3n+3}-2y_{3n+4}+84y_{n+1}-6y_{n+2}\right]$
- $\frac{1}{174}\left[167y_{3n+3}-y_{3n+5}+501y_{n+1}-3y_{n+3}\right]$
- $\frac{1}{29}\left[166x_{3n+5}-1976x_{3n+4}+498x_{n+3}-5928x_{n+2}\right]$
- $\frac{1}{29}\left[166y_{3n+4}-980x_{3n+4}+498y_{n+2}-2940x_{n+2}\right]$
- $\frac{1}{87}\left[83y_{3n+5}-5845x_{3n+4}+249y_{n+3}-17535x_{n+2}\right]$
- $\frac{1}{87}\left[988y_{3n+4\backslash}-490x_{3n+5}+2964y_{n+2}-1470x_{n+3}\right]$
- $\frac{1}{29}\left[1976y_{3n+5\backslash}-11690x_{3n+5}+5928y_{n+3}-35070x_{n+3}\right]$
- $\frac{1}{29}\left[334y_{3n+4\backslash}-28y_{3n+5}+1002y_{n+2}-84y_{n+3}\right]$

2.4. Each of the following expressions is a biquadratic integer:

- $\frac{1}{29}\left[16y_{4n+4}-70x_{4n+4}+64y_{2n+2}-280x_{2n+2}+174\right]$
- $\frac{1}{29}\left[16x_{4n+5}-166x_{4n+4}+64x_{2n+3}664x_{2n+2}+174\right]$
- $\frac{1}{87}\left[4x_{4n+6}-494x_{4n+4}+16x_{2n+4}-1976x_{2n+2}+522\right]$
- $\frac{1}{87}\left[8y_{4n+5}-490x_{4n+4}+32y_{2n+3}-1951x_{2n+2}+522\right]$
- $\frac{1}{2059}\left[16y_{4n+6}-11690x_{4n+4}+64y_{2n+4}-46760x_{2n+2}+12354\right]$
- $\frac{1}{87}\left[83y_{4n+4}-35x_{4n+5}+332y_{2n+2}-140x_{2n+3}+522\right]$
- $\frac{1}{2059}\left[1976y_{4n+4}-70x_{4n+6}+7904y_{2n+2}-280x_{2n+4}+12354\right]$
- $\frac{1}{29}\left[28y_{4n+4}-2y_{4n+5}+112y_{2n+2}-8y_{2n+3}+174\right]$
- $\frac{1}{174}\left[167y_{4n+4}-y_{4n+6}+668y_{2n+2}-4y_{2n+4}+1044\right]$

- $\frac{1}{29}\left[166x_{4n+6}-1976x_{4n+5}+664x_{2n+4}-7904x_{2n+3}+174\right]$
- $\frac{1}{29}\left[166y_{4n+5}-980x_{4n+5}+664y_{2n+3}-3920x_{2n+3}+174\right]$
- $\frac{1}{87}\left[83y_{4n+6}-5845x_{4n+5}+332y_{2n+4}-23380x_{2n+3}+522\right]$
- $\frac{1}{87}\left[988y_{4n+5}-490x_{4n+6}+3952y_{2n+3}-1960x_{2n+4}+522\right]$
- $\frac{1}{29}\left[1976y_{4n+6}-11690x_{4n+6}+7904y_{2n+4}-46760x_{2n+4}+174\right]$
- $\frac{1}{29}\left[334y_{4n+5}-28y_{4n+6}+1336y_{2n+3}-112y_{2n+4}+174\right]$

2.5. Each of the following expression is a quintic integer:

- $\frac{1}{29}\left[16y_{5n+5}-70x_{5n+5}+80y_{3n+3}-350x_{3n+3}+160y_{n+1}-700x_{n+1}\right]$
- $\frac{1}{29}\left[16x_{5n+6}-166x_{5n+5}+80x_{3n+4}-830x_{3n+3}+160x_{n+2}-1660x_{n+1}\right]$
- $\frac{1}{87}\left[4x_{5n+7}-494x_{5n+5}+20x_{3n+5}-2470x_{3n+3}+40x_{n+3}-4940x_{n+1}\right]$
- $\frac{1}{87}\left[8y_{5n+6}-490x_{5n+5}+40y_{3n+4}-2450x_{3n+3}+80y_{n+2}-4900x_{n+1}\right]$
- $\frac{1}{2059}\left[16y_{5n+7}-11690x_{5n+5}+80y_{3n+5}-58450x_{3n+3}+160y_{n+3}-116900x_{n+1}\right]$
- $\frac{1}{87}\left[83y_{5n+5}-35x_{5n+6}+415y_{3n+3}-175x_{3n+4}+830y_{n+1}-350x_{n+2}\right]$
- $\frac{1}{2059}\left[1976y_{5n+5}-70x_{5n+7}+9880y_{3n+3}-350y_{3n+5}+19760y_{n+1}-700x_{n+3}\right]$
- $\frac{1}{29}\left[28y_{5n+5}-2y_{5n+6}+140y_{3n+3}-10y_{3n+4}+280y_{n+1}-20y_{n+2}\right]$
- $\frac{1}{174}\left[167y_{5n+5}-y_{5n+7}+835y_{3n+3}-5y_{3n+5}+1670y_{n+1}-10y_{n+3}\right]$
- $\frac{1}{29}\left[166x_{5n+7}-1976x_{5n+6}+830x_{3n+5}-9880x_{3n+4}+1660x_{n+3}-19760x_{n+2}\right]$
- $\frac{1}{29}\left[166y_{5n+6}-980x_{5n+6}+830y_{3n+4}-4900x_{3n+4}+1660y_{n+2}-9880x_{n+2}\right]$
- $\frac{1}{87}\left[83y_{5n+7}-5840x_{5n+6}+415y_{3n+5}-29220x_{3n+4}+830y_{n+3}-58450x_{n+2}\right]$
- $\frac{1}{87}\left[988y_{5n+6}-490x_{5n+7}+4940y_{3n+4}-2450x_{3n+5}+9880y_{n+2}-4900x_{n+3}\right]$
- $\frac{1}{29}\left[1976y_{5n+7}-11690x_{5n+7}+9880y_{3n+5}-58450x_{3n+5}+19760y_{n+3}-116900x_{n+3}\right]$
- $\frac{1}{29}\left[334y_{5n+6}-28y_{5n+7}+1670y_{3n+4}-140y_{3n+5}+3340y_{n+2}-280y_{n+3}\right]$

REMARKABLE OBSERVATIONS

I. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in Table:2 below:

Table: 2 Hyperbolas

S.NO	Hyperbolas	(Y,X)
1	$Y^2-35X^2=117740$	$\begin{pmatrix}16y_{n+1}-70x_{n+1}, \\ 16\sqrt{35}x_{n+1}-2\sqrt{35}y_{n+1}\end{pmatrix}$
2	$Y^2-35X^2=117740$	$\begin{pmatrix}16x_{n+2}-166x_{n+1}, \\ 28\sqrt{35}x_{n+1}-2\sqrt{35}x_{n+2}\end{pmatrix}$
3	$Y^2-35X^2=16954560$	$\begin{pmatrix}16x_{n+3}-1976x_{n+1}, \\ 334\sqrt{35}x_{n+1}-2\sqrt{35}x_{n+3}\end{pmatrix}$
4	$Y^2-35X^2=4238640$	$\begin{pmatrix}16y_{n+2}-980x_{n+1}, \\ 166\sqrt{35}x_{n+1}-2\sqrt{35}y_{n+2}\end{pmatrix}$
5	$Y^2-35X^2=593527340$	$\begin{pmatrix}16y_{n+3}-11690x_{n+1}, \\ 1976\sqrt{35}x_{n+1}-2\sqrt{35}y_{n+3}\end{pmatrix}$
6	$Y^2-35X^2=4238640$	$\begin{pmatrix}166y_{n+1}-70x_{n+2}, \\ 16\sqrt{35}x_{n+2}-28\sqrt{35}y_{n+1}\end{pmatrix}$
7	$Y^2-35X^2=593527340$	$\begin{pmatrix}1976y_{n+1}-70x_{n+3}, \\ 16\sqrt{35}x_{n+3}-334\sqrt{35}y_{n+1}\end{pmatrix}$
8	$35Y^2-X^2=4120900$	$\begin{pmatrix}28y_{n+1}-2y_{n+2}, \\ 16\sqrt{35}y_{n+2}-166\sqrt{35}y_{n+1}\end{pmatrix}$
9	$35Y^2-X^2=593409600$	$\begin{pmatrix}334y_{n+1}-2y_{n+3}, \\ 16\sqrt{35}y_{n+3}-1976\sqrt{35}y_{n+1}\end{pmatrix}$
10	$Y^2-35X^2=117740$	$\begin{pmatrix}166x_{n+3}-1976x_{n+2}, \\ 334\sqrt{35}x_{n+2}-28\sqrt{35}x_{n+3}\end{pmatrix}$
11	$Y^2-35X^2=117740$	$\begin{pmatrix}166y_{n+2}-980x_{n+2}, \\ 166\sqrt{35}x_{n+2}-28\sqrt{35}y_{n+2}\end{pmatrix}$

12	$Y^2-35X^2=4238640$	$\begin{pmatrix}166y_{n+3}-11690x_{n+2}, \\ 1976\sqrt{35}x_{n+2}-28\sqrt{35}y_{n+3}\end{pmatrix}$
13	$Y^2-35X^2=4238640$	$\begin{pmatrix}1976y_{n+2}-980x_{n+3}, \\ 166\sqrt{35}x_{n+3}-334\sqrt{35}y_{n+2}\end{pmatrix}$
14	$Y^2-35X^2=117740$	$\begin{pmatrix}1976y_{n+3}-11690x_{n+3}, \\ 1976\sqrt{35}x_{n+3}-334\sqrt{35}y_{n+3}\end{pmatrix}$
15	$Y^2-35X^2=144231500$	$\begin{pmatrix}11690y_{n+2}-980y_{n+3}, \\ 166\sqrt{35}y_{n+3}-1976\sqrt{35}y_{n+2}\end{pmatrix}$

II. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in Table:3 below:

Table: 3 Parabolas

S.NO	Parabolas	(Y,X)
1	$29Y-X^2=3364$	$\begin{pmatrix}16y_{2n+2}-70x_{2n+2}+58, \\ 16\sqrt{35}x_{n+1}-2\sqrt{35}y_{n+1}\end{pmatrix}$
2	$29Y-X^2=3364$	$\begin{pmatrix}16x_{2n+3}-166x_{2n+2}+58, \\ 28\sqrt{35}x_{n+1}-2\sqrt{35}x_{n+2}\end{pmatrix}$
3	$348Y-X^2=484416$	$\begin{pmatrix}16x_{2n+4}-1976x_{2n+2}+696, \\ 334\sqrt{35}x_{n+1}-2\sqrt{35}x_{n+3}\end{pmatrix}$
4	$174Y-X^2=121104$	$\begin{pmatrix}16y_{2n+3}-980x_{2n+2}+348, \\ 166\sqrt{35}x_{n+1}-2\sqrt{35}y_{n+2}\end{pmatrix}$
5	$2059Y-X^2=16957924$	$\begin{pmatrix}16y_{2n+4}-11690x_{2n+2}+4118, \\ 1976\sqrt{35}x_{n+1}-2\sqrt{35}y_{n+3}\end{pmatrix}$
6	$174Y-X^2=121104$	$\begin{pmatrix}166y_{2n+2}-70x_{2n+3}+348, \\ 16\sqrt{35}x_{n+2}-28\sqrt{35}y_{n+1}\end{pmatrix}$
7	$2059Y-X^2=16957924$	$\begin{pmatrix}1976y_{2n+2}-70x_{2n+4}+4118, \\ 16\sqrt{35}x_{n+3}-334\sqrt{35}y_{n+1}\end{pmatrix}$
8	$35525Y-X^2=4120900$	$\begin{pmatrix}28y_{2n+2}-2y_{2n+3}+58, \\ 16\sqrt{35}y_{n+2}-166\sqrt{35}y_{n+1}\end{pmatrix}$

9	$426300Y - X^2 = 593409600$	$\begin{pmatrix} 334y_{2n+2} - 2y_{2n+4} + 696, \\ 16\sqrt{35}y_{n+3} - 1976\sqrt{35}y_{n+1} \end{pmatrix}$
10	$29Y - X^2 = 3364$	$\begin{pmatrix} 166x_{2n+4} - 1976x_{2n+3} + 58, \\ 334\sqrt{35}x_{n+2} - 28\sqrt{35}x_{n+3} \end{pmatrix}$
11	$29Y - X^2 = 3364$	$\begin{pmatrix} 166y_{2n+3} - 980x_{2n+3} + 58, \\ 166\sqrt{35}x_{n+2} - 28\sqrt{35}y_{n+2} \end{pmatrix}$
12	$174Y - X^2 = 121104$	$\begin{pmatrix} 166y_{2n+4} - 11690x_{2n+3} + 348, \\ 1976\sqrt{35}x_{n+2} - 28\sqrt{35}y_{n+3} \end{pmatrix}$
13	$174Y - X^2 = 121104$	$\begin{pmatrix} 1976y_{2n+3} - 980x_{2n+4} + 348, \\ 166\sqrt{35}x_{n+3} - 334\sqrt{35}y_{n+2} \end{pmatrix}$
14	$29Y - X^2 = 3364$	$\begin{pmatrix} 1976y_{2n+4} - 11690x_{2n+4} + 58, \\ 1976\sqrt{35}x_{n+3} - 334\sqrt{35}y_{n+3} \end{pmatrix}$
15	$1015Y - X^2 = 4120900$	$\begin{pmatrix} 11690y_{2n+3} - 980y_{2n+4} + 2030, \\ 166\sqrt{35}y_{n+3} - 1976\sqrt{35}y_{n+2} \end{pmatrix}$

3. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the positive Pell equation $y^2 = 35x^2 + 29$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of Pell equations and determine their integer solutions along with suitable properties.

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