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INTEGRAL SOLUTIONS OF THE DIOPHANTINE EQUATION Y²=20x²+4

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Abstract:- *The binary quadratic equation $y^2 = 20x^2 + 4$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under considerations a few patterns of Pythagorean triangles are observed.*

Key Words: Binary, Quadratic, Pyramidal numbers, integral solutions

1. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non –square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-5]. In this context one may also refer [4, 10]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 20x^2 + 4$ representing a hyperbola. A few interesting properties among the solution are presented. Employing the integral solutions of the equation consideration a few patterns of Pythagorean triangles are obtained.

1.1 Notations

$t_{m,n}$: Polygonal number of rank n with size m

P_n^m : Pyramidal number of rank n with size m

$\Pr_n$: Pronic number of rank n

S_n : Star number of rank n

$Ct_{m,n}$: Centered Pyramidal number of rank n with size m

$GF_n(k,s)$: Generalized Fibonacci sequence of rank n

$GL_n(k,s)$: Generalized Lucas sequence of rank n

2. METHOD OF ANALYSIS

Consider the binary quadratic Diophantine equation is

$$y^2 = 20x^2 + 4 \qquad (1)$$

Whose smallest positive integer solutions of (x_0, y_0) is,

$$x_0 = 4,\ y_0 = 18 \qquad (2)$$

To obtain the other solutions of (1), Consider pellian equation is

$$y^2 = 20x^2 + 1 \qquad (3)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 2,\ \tilde{y}_0 = 9$$

Whose general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{20}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (9 + 2\sqrt{20})^{n+1} + (9 - 2\sqrt{20})^{n+1}$$

$$g_n = (9 + 2\sqrt{20})^{n+1} - (9 - 2\sqrt{20})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = 2f_n + \frac{9}{\sqrt{20}} g_n \qquad (4)$$

$$y_{n+1} = 9f_n + \frac{40}{\sqrt{20}} g_n \qquad (5)$$

Therefore (3) becomes

$$\sqrt{20}x_{n+1} = 2\sqrt{20} f_n + 9g_n \qquad (6)$$

Replace n by $n+1$ in (6), we get

$$\sqrt{20}x_{n+2} = 2\sqrt{20} f_{n+1} + 9g_{n+1}$$

$$= 2\sqrt{20}(9f_n + 2\sqrt{20}g_n) + 9(9g_n + 2\sqrt{20} f_n)$$

$$\sqrt{20}x_{n+2}=36\sqrt{20}f_n+161g_n \quad (7)$$

Replace n by $n+1$ in (7), we get

$$\sqrt{20}x_{n+3}=36\sqrt{20}f_{n+1}+161g_{n+1}$$

$$=36\sqrt{20}(9f_n+2\sqrt{20}g_n)+161(9g_n+2\sqrt{20}g_n)$$

$$\sqrt{20}x_{n+3}=646\sqrt{20}f_n+2889g_n \quad (8)$$

For simplicity and clear understanding the other choices of integer solutions are presented below:

$$\sqrt{20}y_{n+1}=9\sqrt{20}f_n+9g_n \quad (9)$$

$$\sqrt{20}y_{n+2}=161\sqrt{20}f_n+720g_n \quad (10)$$

$$\sqrt{20}y_{n+3}=2889\sqrt{20}f_n+12920g_n \quad (11)$$

These are representing the non-zero distinct integer solutions of (1)

Some few numerical examples are given in the following table

Table 1: Numeric examples

n	x_{n+1}	y_{n+1}
-1	4	18
0	72	322
1	1292	5778
2	23184	103682
3	416020	1860498
4	7465176	33385282

The recurrence relation satisfied by the values of x_{n+1} and y_{n+1} are respectively

$$x_{n+3}-18x_{n+2}+x_{n+1}=0, \quad n=-1,0,1,......$$

$$y_{n+3}-18y_{n+2}+y_{n+1}=0, \quad n=-1,0,1,.....$$

A few interesting relations among the solutions are presented below:

1. $x_{n+3}-18x_{n+2}+x_{n+1}=0$
2. $2y_{n+1}+9x_{n+1}-x_{n+2}=0$
3. $2y_{n+2}+x_{n+1}-9x_{n+2}=0$
4. $2y_{n+3}+9x_{n+1}-161x_{n+2}=0$
5. $36y_{n+1}+161x_{n+1}-x_{n+3}=0$
6. $36y_{n+3}+x_{n+1}-161x_{n+3}=0$
7. $y_{n+2}-40x_{n+1}-9y_{n+1}=0$
8. $y_{n+3}-720x_{n+1}-161y_{n+1}=0$
9. $9y_{n+3}-40x_{n+1}-161y_{n+2}=0$
10. $2y_{n+1}+161x_{n+2}-9x_{n+3}=0$
11. $2y_{n+2}+9x_{n+2}-x_{n+3}=0$
12. $2y_{n+3}+x_{n+2}-9x_{n+3}=0$
13. $9y_{n+3}-720x_{n+2}-9y_{n+1}=0$
14. $9y_{n+2}-40x_{n+2}-y_{n+1}=0$
15. $y_{n+3}-40x_{n+2}-y_{n+1}=0$
16. $161y_{n+2}-40x_{n+3}+9y_{n+1}=0$
17. $x_{n+1}-x_{n+3}+4y_{n+2}=0$
18. $161y_{n+3}-720x_{n+3}-y_{n+1}=0$
19. $40x_{n+3}+y_{n+2}-9y_{n+3}=0$
20. $40y_{n+1}-720y_{n+2}+40y_{n+3}=0$

2.1. Each of the following expression represents a cubical integer:

- $\frac{1}{4}[x_{3n+5}-321x_{3n+3}+3x_{n+3}-963x_{n+1})]$
- $\frac{1}{2}[9x_{3n+4}-161x_{3n+3}-483x_{n+1}+27x_{n+2}]$
- $9y_{3n+3}-40x_{3n+3}+27y_{n+1}-120x_{n+1}$
- $y_{3n+4}-80x_{3n+3}+3y_{n+2}-240x_{n+1}$
- $\frac{1}{161}\left[\begin{matrix}(y_{3n+5}-12920x_{3n+3}+27y_{n+3}\\-38760x_{n+1}\end{matrix}\right]$
- $\frac{1}{2}\left[\begin{matrix}161x_{3n+5}-2889x_{3n+4}+483x_{n+3}\\-8667x_{n+2}\end{matrix}\right]$
- $\frac{1}{9}[161y_{3n+3}-40x_{3n+4}+483y_{n+1}-120x_{n+2}]$
- $161y_{3n+4}-720x_{3n+4}+783y_{n+2}2160x_{n+2}$

- $\frac{1}{9}\begin{bmatrix}161y_{3n+5}-12920x_{3n+4}+783y_{n+3}\\ -38760x_{n+2}\end{bmatrix}$
- $\frac{1}{161}\begin{bmatrix}2889y_{3n+3}-40x_{3n+5}+y_{n+1}\\ -40x_{n+3}\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}2889y_{3n+4}-720x_{3n+5}+8667y_{n+2}\\ -2160x_{n+3}\end{bmatrix}$
- $\frac{1}{18}\begin{bmatrix}323y_{3n+3}-y_{3n+5}+949y_{n+1}\\ -3y_{n+3}\end{bmatrix}$
- $\begin{bmatrix}2889y_{3n+5}-12920x_{3n+5}+8667y_{n+3}\\ -38760x_{n+3}\end{bmatrix}$
- $18y_{3n+3}-y_{3n+4}+54y_{n+1}-3y_{n+2}$
- $\begin{bmatrix}323y_{3n+4}-18y_{3n+5}+969y_{n+2}\\ -54y_{n+3}\end{bmatrix}$

2.2. Each of the following expression represents bi-quadratic integer:

- $\frac{1}{4}\left[x_{4n+6}-321x_{4n+4}-1284x_{2n+2}+4x_{2n+4}-24\right]$
- $\frac{1}{2}\left[9x_{4n+5}-161x_{4n+4}-644x_{2n+2}+36x_{2n+3}+12\right]$
- $9y_{4n+4}-40x_{4n+4}+36y_{2n+2}-160x_{2n+2}+6$
- $y_{4n+5}-80x_{4n+4}+4y_{2n+3}-320x_{2n+2}+6$
- $\frac{1}{161}\begin{bmatrix}9y_{4n+6}-12920x_{4n+4}-36y_{2n+4}\\ -51680x_{2n+2}+322\end{bmatrix}$
- $\frac{1}{2}\begin{bmatrix}161x_{4n+6}-2889y_{4n+5}+644x_{2n+4}-\\ 11556x_{2n+3}-12\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}161y_{4n+4}-40x_{4n+5}+644y_{2n+2}\\ -160x_{2n+3}-54\end{bmatrix}$
- $\begin{bmatrix}161y_{4n+5}-720x_{4n+5}+644y_{2n+3}\\ -2880x_{2n+3}+6\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}161y_{4n+6}-12920x_{4n+5}+644y_{2n+4}-51680x_{2n+3}\\ -54\end{bmatrix}$
- $\frac{1}{161}\begin{bmatrix}2889y_{4n+4}-40x_{4n+6}+11556y_{2n+2}\\ -160x_{2n+4}+322\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}2889y_{4n+5}-720x_{4n+6}+11556y_{2n+3}\\ -2880x_{2n+4}\\ -54\end{bmatrix}$
- $\begin{bmatrix}2889y_{4n+6}-12920x_{4n+6}+11556y_{2n+4}\\ -51680x_{2n+4}+6\end{bmatrix}$
- $18y_{4n+4}-y_{4n+5}+72y_{2n+2}-4y_{2n+3}+6$
- $\frac{1}{18}\left[323y_{4n+4}-y_{4n+6}+1292y_{2n+2}-4y_{2n+4}-36\right]$
- $232y_{4n+5}-18y_{4n+6}+1292y_{2n+3}-72y_{2n+4}+6$

2.3. Each of the following expression represents quintic integer:

- $\frac{1}{4}\begin{bmatrix}x_{5n+7}-321x_{5n+5}+5x_{3n+5}-1505x_{3n+3}\\ +10x_{n+3}-3210x_{n+1}\end{bmatrix}$
- $\frac{1}{2}\begin{bmatrix}9x_{5n+6}-161x_{5n+5}-805x_{3n+3}+45x_{3n+4}\\ +1610x_{n+1}+90x_{n+2}\end{bmatrix}$
- $\begin{bmatrix}9y_{5n+5}-40x_{5n+5}+45y_{3n+3}-200x_{3n+3}\\ -400x_{n+1}+90y_{n+1}\end{bmatrix}$
- $\begin{bmatrix}y_{5n+6}-80x_{5n+5}+5y_{3n+4}-400x_{3n+3}\\ -800x_{n+1}+10y_{n+2}\end{bmatrix}$
- $\frac{1}{161}\begin{bmatrix}9y_{5n+7}-12920x_{5n+5}-64600x_{3n+3}\\ +45y_{3n+5}-129200x_{n+1}+90y_{n+3}\end{bmatrix}$
- $\frac{1}{2}\begin{bmatrix}161x_{5n+7}-2889x_{5n+6}+805y_{3n+3}\\ -14445x_{3n+4}-400x_{n+2}+1610y_{n+1}\end{bmatrix}$
- $\begin{bmatrix}161y_{5n+6}-720x_{5n+6}-3600x_{3n+4}+805y_{3n+4}\\ -7200x_{n+2}+10y_{n+2}\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}161y_{5n+7}-12920x_{5n+6}-64600x_{3n+4}\\ +805y_{3n+5}-129200x_{n+2}+1610y_{n+3}\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}161y_{5n+5}-40x_{5n+6}-200x_{3n+4}\\ +805y_{3n+3}-400x_{n+2}+1610y_{n+1}\end{bmatrix}$
- $\frac{1}{161}\begin{bmatrix}2889y_{5n+5}-40x_{5n+7}-200x_{3n+5}\\ +14445y_{3n+3}-400x_{n+3}+28890y_{n+1}\end{bmatrix}$
- $\frac{1}{9}\begin{bmatrix}2889y_{5n+6}-720x_{5n+7}-3600x_{3n+5}\\ +14445y_{3n+4}-7200x_{n+3}+28890y_{n+2}\end{bmatrix}$
- $\begin{bmatrix}2889y_{5n+7}-12920x_{5n+7}-64600x_{3n+5}\\ +14445y_{3n+5}-129200x_{n+3}+28890y_{n+3}\end{bmatrix}$
- $\begin{bmatrix}18y_{5n+5}-y_{5n+7}+90y_{3n+3}-5y_{3n+4}\\ -10y_{n+2}+180y_{n+1}\end{bmatrix}$
- $\frac{1}{18}\begin{bmatrix}323y_{5n+5}-y_{5n+7}+1615y_{3n+3}\\ -5y_{3n+5}-10y_{n+3}+3230y_{n+1}\end{bmatrix}$
- $\begin{bmatrix}323y_{5n+6}-18y_{5n+7}+1615y_{3n+4}\\ -90y_{3n+5}-180y_{n+3}+3230y_{n+2}\end{bmatrix}$

2.4. Each of the following expression represents Nasty number:

- $\frac{1}{4}\left[48+x_{2n+4}-321x_{2n+2}\right]$
- $12+54y_{2n+2}-240x_{2n+2}$

- $12+6y_{2n+3}-480x_{2n+2}$
- $\frac{1}{161}[1932+54y_{2n+4}-77520x_{2n+2}]$
- $\frac{1}{2}[12+161x_{2n+4}-2889x_{2n+3}]$
- $\frac{1}{9}[108+966y_{2n+2}-240x_{2n+3}]$
- $12+966y_{2n+3}-4320x_{2n+3}$
- $\frac{1}{9}[108+966y_{2n+4}-77520x_{2n+3}]$
- $\frac{1}{161}[1932+17334y_{2n+2}-240x_{2n+4}]$
- $\frac{1}{9}[108+17334y_{2n+3}-4320x_{2n+4}]$
- $12+17334y_{2n+4}-77520x_{2n+4}$
- $12+108y_{2n+2}-6y_{2n+3}$
- $\frac{1}{18}[216+1938y_{2n+2}-6y_{2n+4}]$

2.5. REMARKABLE OBSERVATIONS

I. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in table 2 below:

Table 2: Hyperbola

S.NO	Hyperbola	(X_n, Y_n)
1	$X_n^2-40Y_n^2=4$	$(9x_{n+2}-161x_{n+1},\ 18x_{n+1}-x_{n+2})$
2	$81X_n^2-80Y_n^2=5184$	$(x_{n+1}-321x_{n+3},\ 323x_{n+1}-x_{n+3})$
3	$X_n^2-20Y_n^2=4$	$(9y_{n+1}-40x_{n+1},\ 9x_{n+1}-2y_{n+1})$
4	$81X_n^2-20Y_n^2=324$	$(y_{n+2}-80x_{n+1},\ 161x_{n+1}-2y_{n+2})$
5	$X_n^2-20Y_n^2=103684$	$(9y_{n+3}-12920x_{n+1},\ 2889x_{n+1}-2y_{n+3})$
6	$X_n^2-80Y_n^2=16$	$(161x_{n+3}-2889x_{n+2},\ 323x_{n+2}-18x_{n+3})$
7	$X_n^2-162Y_n^2=324$	$(161y_{n+1}-40x_{n+2},\ x_{n+2}-4y_{n+1})$
8	$X_n^2-20Y_n^2=4$	$(161y_{n+2}-720x_{n+2},\ 161x_{n+2}-36y_{n+2})$
9	$X_n^2-20Y_n^2=324$	$(161y_{n+3}-12920x_{n+2},\ 2889x_{n+2}-36y_{n+3})$
10	$X_n^2-20Y_n^2=103684$	$(2889y_{n+1}-40x_{n+3},\ 9x_{n+3}-646y_{n+1})$
11	$X_n^2-20Y_n^2=324$	$(2889y_{n+2}-720x_{n+3},\ 161x_{n+3}-646y_{n+2})$
12	$X_n^2-20Y_n^2=4$	$(2889y_{n+3}-12920x_{n+3},\ 2889x_{n+3}-646y_{n+3})$
13	$80X_n^2-Y_n^2=320$	$(18y_{n+1}-y_{n+2},\ 9y_{n+2}-161y_{n+1})$
14	$80X_n^2-81Y_n^2=103680$	$(323y_{n+1}-y_{n+3},\ y_{n+3}-321y_{n+1})$
15	$80X_n^2-Y_n^2=320$	$(323y_{n+2}-18y_{n+3},\ 161y_{n+3}-2889y_{n+2})$

II Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in table 3 below:

Table 3: Parabola

S.NO	Parabola	(X_n, Y_n)
1	$X_n-40Y_n^2=8$	$(8+9x_{2n+3}-161x_{2n+2},\ 18x_{2n+2}-x_{2n+3})$
2	$9X_n-40Y_n^2=144$	$(8+x_{2n+4}-321x_{2n+2},\ 323x_{2n+2}-x_{2n+4})$
3	$X_n-20Y_n^2=4$	$(2+9y_{2n+2}-40x_{2n+2},\ 9x_{2n+2}-2y_{2n+2})$
4	$9X_n-20Y_n^2=4$	$(8+y_{2n+3}-80x_{2n+2},\ 161x_{2n+2}-2y_{2n+3})$
5	$161X_n-20Y_n^2=101164$	$(322+9y_{2n+4}-12920x_{2n+2},\ 2889x_{2n+2}-2y_{2n+4})$
6	$X_n-40Y_n^2=8$	$(4+161x_{2n+4}-2889x_{2n+3},\ 323x_{2n+3}-18x_{2n+4})$

7	$X_n - 180Y_n^2 = 36$	$\begin{pmatrix}18+161y_{2n+2}-40x_{2n+3}, \\ x_{2n+3}-4y_{2n+2}\end{pmatrix}$
8	$X_n - 20Y_n^2 = 4$	$\begin{pmatrix}2+161y_{2n+3}-720x_{2n+3}, \\ 161x_{2n+3}-36y_{2n+3}\end{pmatrix}$
9	$9X_n - 20Y_n^2 = 324$	$\begin{pmatrix}18+161y_{2n+4}-12920x_{2n+3}, \\ 2889x_{2n+3}-36y_{2n+4}\end{pmatrix}$
10	$161X_n - 20Y_n^2 = 103684$	$\begin{pmatrix}322+2889y_{2n+2}-40x_{2n+4}, \\ 9x_{2n+4}-646y_{2n+2}\end{pmatrix}$
11	$X_n - 20Y_n^2 = 36$	$\begin{pmatrix}18+2889y_{2n+3}-720x_{2n+4}, \\ 161x_{2n+4}-646y_{2n+3}\end{pmatrix}$
12	$X_n - 20Y_n^2 = 4$	$\begin{pmatrix}2+2889y_{2n+4}-12920x_{2n+4}, \\ 2889x_{2n+4}-646y_{2n+4}\end{pmatrix}$
13	$20X_n - Y_n^2 = 320$	$\begin{pmatrix}2+18y_{2n+2}-y_{2n+3}, \\ 161y_{2n+2}-9y_{2n+3}\end{pmatrix}$
14	$160X_n - 20Y_n^2 = 11520$	$\begin{pmatrix}36+323y_{2n+2}-y_{2n+4}, \\ 321y_{2n+2}-y_{2n+4}\end{pmatrix}$
15	$80X_n - Y_n^2 = 320$	$\begin{pmatrix}2+323y_{2n+3}-18y_{2n+4}, \\ 2889y_{2n+3}-161y_{2n+4}\end{pmatrix}$

III Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of straight line which are presented in the table below:

Table: 4 Straight lines

S.NO	Straight line	(X,Y)
1	$Y = 2X$	$\begin{pmatrix}9x_{n+2}-161x_{n+1}, \\ x_{n+3}-321x_{n+1}\end{pmatrix}$
2	$Y = 4X$	$\begin{pmatrix}9y_{n+1}-40x_{n+1}, \\ x_{n+3}-321x_{n+1}\end{pmatrix}$
3	$Y = X$	$\begin{pmatrix}y_{n+2}-80x_{n+1}, \\ 9y_{n+1}-40x_{n+1}\end{pmatrix}$
4	$Y = 161X$	$\begin{pmatrix}y_{n+2}-80x_{n+1}, \\ 9y_{n+3}-12920x_{n+1}\end{pmatrix}$
5	$Y = 18X$	$\begin{pmatrix}323y_{n+2}-18y_{n+3}, \\ 323y_{n+1}-y_{n+3}\end{pmatrix}$
6	$Y = 9X$	$\begin{pmatrix}323y_{n+2}-18y_{n+3}, \\ 2889y_{n+2}-720x_{n+3}\end{pmatrix}$
7	$Y = 2X$	$\begin{pmatrix}2889y_{n+2}-720x_{n+3}, \\ 323y_{n+1}-y_{n+3}\end{pmatrix}$
8	$Y = 2X$	$\begin{pmatrix}9y_{n+1}-40x_{n+1}, \\ 9x_{n+2}-161x_{n+1}\end{pmatrix}$
9	$Y = 4X$	$\begin{pmatrix}y_{n+2}-80x_{n+1}, \\ x_{n+3}-321x_{n+1}\end{pmatrix}$
10	$Y = 161X$	$\begin{pmatrix}161y_{n+2}-720x_{n+2}, \\ 9y_{n+3}-12920x_{n+1}\end{pmatrix}$
11	$Y = X$	$\begin{pmatrix}161y_{n+2}-12920x_{n+2}, \\ 161y_{n+1}-40x_{n+2}\end{pmatrix}$
12	$Y = 2X$	$\begin{pmatrix}2889y_{n+2}-720x_{n+3}, \\ 323y_{n+1}-y_{n+3}\end{pmatrix}$
13	$Y = 9X$	$\begin{pmatrix}y_{n+2}-80x_{n+1}, \\ 161y_{n+1}-40x_{n+2}\end{pmatrix}$
14	$Y = 9X$	$\begin{pmatrix}y_{n+1}-40x_{n+1}, \\ 161y_{n+1}-40x_{n+2}\end{pmatrix}$
15	$Y = 161X$	$\begin{pmatrix}161y_{n+2}-720x_{n+2}, \\ 2889y_{n+1}-40x_{n+3}\end{pmatrix}$

IV Consider $p = x_{n+1} + y_{n+1}, q = x_{n+1}$

Note that $p > q > 0$,treat p, q as the generators of the Pythagorean triangle $T(X, Y, Z)$

Where $X = 2pq, Y = p^2 - q^2, Z = p^2 + q^2, p > q > 0$

It is observed that $T(X, Y, Z)$ is satisfied by the following relations:

i. $X - 10Y + 9Z = 4$

ii. $\frac{2A}{P} = x_{n+1}y_{n+1}$

iii. $X - \frac{4A}{P} + Y = Z$

iv. $3(Z - Y)$ is a Nasty number

v. $X-\frac{4A}{P}+Y$ is written as the sum of two squares

Where A,P represents the area and perimeter of $T(X,Y,Z)$

V Employing the solutions in terms of special integers sequence namely, generalized Fibonacci sequence, $GF_n(k,s)$ and the generalized Lucas sequence $GL_n(k,s)$ are exhibited below:

$$x_{n+1}=2GL_{n+1}(18,-1)+36GF_{n+1}(18,-1)$$

$$y_{n+1}=9GL_{n+1}(18,-1)+160GF_{n+1}(18,-1)$$

VI Employing the solutions of (1),each of the following among the special polygonal, pyramidal, star numbers, pronic numbers, centered pyramidal number is a congruent to under modulo 4.

- $\left(\frac{12p^5{}_y}{s_{y+1}-1}\right)^2-20\left(\frac{36p^3{}_{x-2}}{s_{x+1}-1}\right)^2$
- $\left(\frac{6p^4{}_{y-1}}{t_{3,2y-1}}\right)^2-20\left(\frac{4p^5{}_x}{ct_{4,x}-1}\right)^2$
- $\left(\frac{4p^5{}_y}{ct_{4,y}-1}\right)^2-20\left(\frac{6p^5{}_x}{ct_{6,x}-1}\right)^2$
- $\left(\frac{3p^3{}_{y-2}}{t_{3,y-2}}\right)^2-20\left(\frac{4p^5{}_x}{ct_{4,x}-1}\right)^2$
- $\left(\frac{2p^5{}_{y-1}}{t_{4,y-1}}\right)^2-20\left(\frac{3p^3{}_x}{t_{3,x+1}}\right)^2$
- $\left(\frac{p^5{}_y}{t_{3,y}}\right)^2-20\left(\frac{12p^5{}_x}{s_{x+1}-1}\right)^2$
- $\left(\frac{3p^3{}_{y-2}}{t_{3,y-2}}\right)^2-20\left(\frac{6p^3{}_x}{pr_{x+1}}\right)^2$
- $\left(\frac{6p^4{}_{y-1}}{t_{3,2y-1}}\right)^2-20\left(\frac{4pt_{x-3}}{p^3{}_{x-3}}\right)^2$
- $\left(\frac{3p^3{}_{y-2}}{t_{3,y-2}}\right)^2-20\left(\frac{p^5{}_x}{t_{3,x}}\right)^2$
- $\left(\frac{6p^4{}_{y-1}}{t_{3,2y-1}}\right)^2-20\left(\frac{3(p^4{}_{x+1}-p^3{}_{x+1})}{t_{4,x+1}}\right)^2$

CONCLUSION

In this dissertation , we have presented infinitely many integer solutions of the binary quadratic Diophantine equation are rich in variety, one may search for the other choices of binary quadratic Diophantine equation and determine their integral solutions along with suitable properties.

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