SOFT HYPERFILTERS IN HYPERLATTICES

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Abstract - *Firstly, hyper filters of hyperlattices are introduced and several interesting examples of them are given. Secondly, soft hyper filters are proposed, which are generalizations of hyper filter sand soft hyper filters in hyperlattices. Finally, under the soft homomorphism of hyperlattices, the image and pre-image of soft hyper filters are studied.*

Key Words: Filters, Softset, Hyperfilters and Softhyperfilters

1. INTRODUCTION

The concept of hyperstructure was introduced in 1934 by a French mathematician, Marty [1]. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. There appeared many components of hyperalgebras such as hypergroups in [2], hyperrings in [3] etc. As soft set theory, the theory of fuzzy soft sets turned out to have applications. Roy and Maji [4] presented some applications of this notion to decision-making problems, we introduce hyperfilters soft hyperfilters in hyperlattices, and study some properties of them

Definition 1.1:

Let X be a universe set and E be a set of parameters. Let P(X) be the power set of X and $A \subseteq E$. A pair (F, A) is called a soft set over X, where A is a subset of the set of parameters E and $F : A \to P(X)$ is a set-valued mapping.

Example 1.2:

Let I = [0, 1] and E be all convenient parameter sets for the universe X. Let X denote the set of all fuzzy sets on X and $A \subseteq E$.

Definition 1.3:

A pair (f, A) is called a fuzzy soft set over X, where A is a subset of the set of parameters E and $f : A \to I X$ is a mapping. That is, for all $a \in A$, $f(a) = fa : X \to I$ is a fuzzy set on X.

Definition 1.4:

Let $(L,\leq)$ be a non empty partial ordered set and $\vee : L\times L \to \rho(L)*$ be a hyperoperation, where $\rho(L)$ is a power set of L and $\rho(L)* = \rho(L)\backslash\{\emptyset\}$ and $\wedge : L\times L \to L$ be an operation. Then $(L,\vee,\wedge)$ is a hyperlattice if for all $a,b,c \in L$,

(i)$a \in a\vee a, a\wedge a = a$;

(ii)$a\vee b = b\vee a, a\wedge b = b\wedge a$;

(iii)$(a\vee b)\vee c = a\vee(b\vee c);(a\wedge b)\wedge c = a\wedge(b\wedge c)$;

(iv) $a \in [a\wedge(a\vee b)]\cap[a\vee(a\wedge b)]$;

(v) $a \in a\vee b \Rightarrow a\wedge b = b$;

where for all non empty subsets A and B of L, $A\wedge B = \{a\wedge b \mid a \in A, b \in B\}$ and $A\vee B = \cup\{a\vee b \mid a \in A, b \in B\}$.

Definition 1.5:

Let $(L,\vee,\wedge)$ is a hyperlattice. A Partial ordering relation $\leq$ is defined on L by $x \leq y$ f and only if $x \wedge y = x$ and $x \vee y = y$.

Definition 1.6:

A Nonempty subset F of a Hyperlattice L is called a Filter of L if (i)$a \wedge b \in F$ and $a\vee x \in F$

(ii $a \in F$ and $a \leq b$ then $b \in F$

Properties of filters 1.7:

A Filter F of L is a subset $F \subseteq L$ with the following properties:

(i) $1 \in F$

(ii) $a \in F$ and $a \leq b$, $b \in L$ then $b \in F$

(iii)If $a , b \in F$ then $ab \in F$

Definition 1.8:

Let $(L,\vee,\wedge)$ is a hyperlattice. For any $x \in L$ the set $\{x\in L \mid a \leq x\}$ is a filter ,which is called as a principal filter generated by a.

2. SOFT HYPERFILTERS IN HYPERLATTICES

In this section, we will introduce soft hyperfilters in hyperlattices and give several interesting examples of them.

Definition 2.1:

Let L be a nonempty set and $P*(L)$ be the set of all nonempty subsets of L. A hyperoperation on L is a map $\circ : L\times L \to P*(L)$, which associates a nonempty subset $a\circ b$ with any pair (a,b) of elements of $L\times L$. The couple $(L,\circ)$ is called a hypergroupoid.

Definition 2.2:

Let L be a nonempty set endowed with two hyperoperations "⊗" and "⊕". The triple (L,⊗,⊕) is called a hyperlattice if the following relations hold: for all a,b,c ∈ L,

(1) a ∈ a⊗a, a ∈ a⊕a;

(2) a⊗b = b⊗a, a⊕b = b⊕a;

(3) (a⊗b)⊗c = a⊗(b⊗c), (a⊕b)⊕c = a⊕(b⊕c)

(4) a ∈ a⊗(a⊕b), a ∈ a⊕(a⊗b).

Definition 2.3:

Let (L,⊗,⊕) be a hyperlattice and A be a non-empty subset of L. A is called a ⊕-hyperfilter of L if for all a,b ∈ A and x ∈ L,

(i) a⊕b ⊆ A and a⊗x ⊆ A

(ii) a ∈ A and a ≤ b then b ∈ A

Definition 2.4:

Let (L,⊗,⊕) be a hyperlattice and A be a non-empty subset of L. A is called a ⊗-hyperfilter of L if for all a,b ∈ A and x ∈ L,

(i) a⊗b ⊆ A and a⊕x ⊆ A

(ii) a ∈ A and a ≤ b then b ∈ A

We now introduce theorems of hyperfilters.

THEOREM 2.5:

Any ⊕ hyperfilter A of a Hyperlattice L satisfies,If a ∈ A and a ≤ b then b ∈ A

Proof:

Given (L,⊗,⊕) be a hyperlattice and A is a ⊕hyperflter of L .Assume that for all a∈A and a≤b .

Take (ab)= 1 ∈ A imples ab ∈ A implies ab ∈ A so that b ∈ A when a ∈ A.Hence proved.

Theorem 2.6:

In a hyperlattice(L,⊗,⊕) ,Every filter is a ⊕hyperfilter

Proof:

Given (L,⊗,⊕) be a hyperlattice and A be any filter of L .Let a,b ∈ A .Take b(a⊕b)=ba ⊕ bb=ba⊕1=ba≥a

Implies that b(a⊕b)≥a and b(a⊕b) ∈ A implies that a ⊕ b ∈ A ;similarly when x∈ L implies a⊕ x ⊆ A(By the previous theorem) For all a ∈ A and a≤b implies thar b ∈ A.

From the above two result A is a ⊕ hyperfilter .Hence proved

Definition 2.7:

Let (L,⊗,⊕) be a hyperfilter and (F,A) be a softset over L,(F,A) is called a soft ⊕ hyperfilter over L,if F(x) is ⊕hyperfilter of L for all x ∈ sup (F,A)

Definition 2.8:

Let (L,⊗,⊕) be a hyperfilter and (F,A) be a softset over L,(F,A) is called a soft ⊗ hyperfilter over L,if F(x) is ⊗ hyperfilter of L for all x ∈ sup (F,A)

Example2.9:

Let μ be a fuzzy ⊕hyperfilter of a hyperlattice (L, ⊕,⊗) ,the fuzzyset of μ satisfies the following condition:

For all x ,y ∈ L (i) ⋀ μ(z) ≥ μ(x)∧μ(y)

(ii) ⋀ μ(z) ≥ μ(x)∨μ(y)

Clearly μ is a fuzzy ⊕ hyperfilter of L.if and only if for all t ∈ [0,1] with $\mu_t \neq 0$ .Let μ_t = { x ∈ L |μ(x) ≥t } is a ⊕ hyperfilter of L. and F(t) = { x ∈ L |μ(x) ≥t } for all t ∈ [0,1] and F(t) is a ⊕ hyperfilter of L.

Note:

Every fuzzy ⊕ hyperflter(⊗ hyperilter) can be intrepreted as soft ⊕ hyperfilter(⊗ hyperfilter)

Theorem 2.10:

If (L,⊗,⊕) is a hyperlattice and (F,L) denote a softset over L,then the following conditions hold:

(i) (F,L) is a soft ⊕hyperfilter of L

(ii) (F' , L) is a soft ⊗ hyperfilter of L

Proof:

(i) By hypothesis, (L,⊗,⊕) be a hyperlattice .so clealy (L,∧,∨)be a lattice.Now define two hyperoperations on L .For all a , b ∈ L therefore,a ⊗ b = { x ∈L | a ∨ x = b ∨x = a ∨b } and a⊕ b = { x ∈ L | x ≤ a ∧b }

For all a ∈ L ,define a principal filter generated by a ,I(a) = { x ∈ L | x ≤ a } = a. Hence I(a) is a ⊕hyperfilter of the hyperlattice L. Now define a map F: L → P(L) by , F(a) = I(a) = ↓a for all a ∈ L(By definition of Softset)then (F,L) becomes a soft ⊕ hyperfilter over L.

(ii) By hypothesis, (L,⊗,⊕) be a hyperlattice .so clealy (L,∧,∨)be a lattice.Now define two hyperoperations on L .For all a , b ∈ L a ⊗ b = { x ∈ L | a ∨ b ≤ x } and a ⊕ b = { x ∈ L | a∧x= b∧x=a∧b}

For all a ∈ L , define a principal filter generated by a ,F(a) = { x ∈ L | x ≥ a } = ↑a. Hence F(a) is a ⊗ hyperfilter of the hyperlattice L. Now define a map F': L → P(L) by ,F'(a) = F(a) = ↑a for all a ∈ L(By definition of Softset)then (F',L) becomes a soft ⊗ hyperfilter over L.

Hence proved.

3. CONCLUSIONS

In this paper, we apply the notion of soft sets to the theory of hyperlattices. We introduce hyperfilters and soft hyperfilters, and study some properties of them. This study is just at the begining and it can be continuated in many directions:

(1) To do some further work on the properties of soft hyperfilters, which may be useful to characterize the structure of hyperlattices;

(2) To study the construction the quotient hyperlattices in the mean of soft structures and soft hyperfilters theorems of hyperlattices;

(3) To apply the soft set theory of hyperlattices to some applied fields, such as decision making, data analysis and forecasting and so on.

REFERENCES

[1] F. Marty, Sur une generalization de la notion de groupe, in: 8th Congress Math. Scandinaves, Stockholm, 1934, pp. 45-49.

[2] J. Jantosciak, Transposition hypergroups: noncommutative join spaces, Journal of Algebra, 187 (1977) 97-119.

[3] R. Rosaria, Hyperaffine planes over hyperrings, Discrete Mathematics, 155 (1996) 215-223.

[4] A.R. Roy, P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math., 203 (2007) 412-418.