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INTUITIONISTIC FUZZY DIVISOR CORDIAL GRAPH

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Abstract:- In this paper we introduce the concept of Intuitionistic fuzzy divisor cordial labeling. A graph with Intuitionistic fuzzy divisor cordial labeling is called Intu- itionistic fuzzy divisor cordial graph. As part of this paper we proved path, cycle, star graph and wheel graph are Intuitionistic fuzzy divisor cordial graphs.

Key word: Intuitionistic Fuzzy Divisor Cordial Graph

1 Introduction

Intuitionistic Fuzzy is a newly emerging mathematical framework to deal uncertainty. In 1975, Rosenfield introduced the concept of fuzzy graph. As an extension of fuzzy graph Sovan and Attanassov introduced the concept of Intuitionistic fuzzy graph. One of the most important area in graph theory is graph labeling, which have been introduced so far and many researchers are still working on it. It has wide applications within Mathematics as well as to several areas of computer science and communication networks. M. Sumathi and A. Agwin Charles introduced Fuzzy divisor cordial graph [1]. Motivated by Fuzzy divisor cordial labeling of graphs, we introduced the concept of Intuitionistic fuzzy divisor cordial graph through this paper.

This paper is structured as: Section 2, contains basic definitions that are required for the succeeding section. In Section 3, we introduce Intuitionistic fuzzy divisor cordial graph, and some theorems based on it, and section 4 concludes the paper.

2 Basic Definitions

Definition 2.1 [2]

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition 2.2 [2]
Let G = (V(G),E(G)) be a simple graph and f : V(G) $\rightarrow$ {1 ,2, . . ., | V(G) | } be a function. For each edge uv assign the label 1 if f(u) divides f(v) denoted by f(u) | f(v) or f(v) | f(u) and the label 0, otherwise. Then the function f is called a divisor cordial labeling if $|\ e_f(0) - e_f(1)\ | \leq 1$, Where $e_f(0)$ denotes the number of edges with label 0 and $e_f(d)$ denotes the number of edges with label 1. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition 2.3 [1]
A graph G = $(\sigma, \mu)$is said to be a fuzzy divisor labeling graph if σ : V $\rightarrow$ [0,1] and μ : V $\rightarrow$ [0,1] is bijective such that the membership value of edges and vertices are distinct and μ(u,v) $\leq$ σ(u) Λ σ(v) $\forall$ u ,v ε V.

Definition 2.4 [1]
Let G = (σ, μ) be a simple graph and σ : V $\rightarrow$ [0,1] be a bijection. For each edge uv assign the label d if either σ(u) | σ(v) or σ(v) | σ(u) and the label 0 otherwise. σ is called a fuzzy divisor cordial labeling if $|\ e_f(0) - e_f(1)\ | \leq 1$, where d is a very small positive quantity which is close to 0 and d ε (0,1). A graph with a fuzzy divisor cordial labeling is called a fuzzy divisor cordial graph.

Definition 2.5 [3]
A graph G = (A,B) is said to be Intuitionistic fuzzy labeling graph if $\mu_A : V \rightarrow [0,1]$, $\nu_A : V \rightarrow [0,1]$, $\mu_B : V \times V \rightarrow [0,1]$ and $\nu_B : V \times V \rightarrow [0,1]$ are bijective such that $\mu_B(x,y)$, $\nu_B(x,y)$ ε [0,1] all are distinct for each nodes and edges, where μ_A is the degree of membership and ν_A is degree of non-membership of nodes, similarly μ_B and ν_B are degree of membership and non-membership of edges.

3 Intuitionistic Fuzzy Divisor Cordial Labeling

Definition 3.1 Let G = (V,E) be a simple graph, where E $\subseteq V \times V$ and $\mu_V : V \rightarrow [0,1]$ and $\nu_V : V \rightarrow [0,1]$ are injective such that, for each edge uv, assign the membership value d if either μ_V(u) | $\mu_V(v)$ or μ_V(v) | $\mu_V(u)$ and assign 0 if $\mu_V(u) \nmid \mu_V(v)$. And assign the non-membership value d$'$ if either ν_V(u) | $\nu_V(v)$ or ν_V(u) | $\nu_V(v)$ and assign 0 if ν_V(u)$\nmid$ $\mu_V(v)$. Where d, d' are very small quantity ε (0,1). Then this labeling is called an Intuitionistic fuzzy divisor cordial labeling if

$$|e_{\mu_E}(0) - e_{\mu_E}(d)| \leq 1 \tag{1}$$

and

$$|e_{\nu_E}(0) - e_{\nu_E}(d')| \leq 1 \tag{2}$$

Remark 3.1 A Graph with an intuitionistic fuzzy cordial labeling is called an Intuitionistic Fuzzy Divisor Cordial Graph.

Theorem 3.1

Every path P_n is an Intuitionistic fuzzy divisor cordial graph.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of a path P_n. Take two values say m, $m^{'}$ ε (0,1).

Case 1:

When n is even,

Label $\frac{n}{2}$ vertices v_j by a membership value $\mu_V(v_j) = \frac{m}{10^j}$, where j = 1, 2, 3, ... up to $\frac{n}{2}$. Then label the remaining $\frac{n}{2}$ vertices by any membership value which is not a multiple of the membership value of adjacent vertices, without repeating any $\mu_V(v_j)$, $\forall$ j

$\therefore$ e_{μ_E} (d) = $\frac{n}{2} - 1$

e_{μ_E} (0) = $\frac{n}{2}$

$\therefore$ | e_{μ_E} (0)$-$ e_{μ_E} (d) | = | $\frac{n}{2}-$ $\frac{n}{2}$ +1 | = 1.

Hence | e_{μ_E} (0) $-$ e_{μ_E} (d) | $\leq$ 1, holds.

Also label the same $\frac{n}{2}$ vertices v_j by a non-membership value $\nu_V(v_j) = \frac{m^{'}}{10^j}$ where j = 1, 2, ... upto $\frac{n}{2}$. Then label the remaining vertices by any non-membership value which is not a multiple of the non-membership value of adjacent vertices without repeating any $\nu_V(v_j)$, $\forall j$

$\therefore$ e_{ν_E} ($d^{'}$) = $\frac{n}{2} - 1$

e_{μ_E} (0) = $\frac{n}{2}$

$\therefore$ | e_{ν_E} (0) $-$ e_{ν_E} ($d^{'}$) | = | $\frac{n}{2}$ $-$ $\frac{n}{2}$ + 1 | = 1

$\therefore$ | e_{ν_E} (0) $-e_{\nu_E}$ ($d^{'}$) | $\leq$ 1, hold.

Case 2:

When n is odd,

Then label $\frac{n+1}{2}$ vertices v_j by a membership value $\mu_V(v_j) = \frac{m}{10^j}$, where j = 1, 2, ... up to $\frac{n+1}{2}$. Then label the remaining $\frac{n-1}{2}$ vertices by any membership value which is not a multiple of the membership value of the adjacent vertices without repeating any $\mu_V(v_j)$, $\forall$ j

$\therefore e_{\mu_E}$ (d) $= \frac{n-1}{2}$

e_{μ_E} (0) $= \frac{n-1}{2}$

$\therefore |\ e_{\mu_E}$ (0) $- e_{\mu_E}$ (d) $| = |\ \frac{n-1}{2} - \frac{n-1}{2}\ | = 0$.

Hence $|\ e_{\mu_E}$ (0) $- e_{\mu_E}$ (d) $| \leq 1$, holds.

Also label the same $\frac{n+1}{2}$ vertices v_j by a non-membership value $\nu_V(v_j) = \frac{m'}{10^j}$ where j = 1, 2, ... upto $\frac{n+1}{2}$. Then label the remaining vertices by a non-membership value which is not a multiple of the non-membership value of adjacent vertices without repeating any $\nu_V(v_j)$, $\forall j$

$\therefore e_{\nu_E}$ (d') $= \frac{n-1}{2}$

e_{μ_E} (0) $= \frac{n-1}{2}$

$\therefore |\ e_{\nu_E}$ (0) $- e_{\nu_E}$ (d') $| = |\ \frac{n-1}{2} - \frac{n-1}{2}\ | = 0$

$\therefore |\ e_{\nu_E}$ (0) $- e_{\nu_E}$ (d') $| \leq 1$, hold.

Therefore every path P_n is an Intuitionistic fuzzy divisor cordial graph.

Illustration 3.1

When n = 4

Take m $= \frac{2}{10}$,$m' = \frac{3}{10}$

$\mu_V(V_1) = \frac{m}{10^1} = \frac{2}{10^2}$, $\nu_V(V_1) = \frac{m'}{10} = \frac{3}{10^2}$ e_{μ_E} (d) = 1, e_{μ_E} (0) = 2

e_{ν_E} (d') = 1, e_{ν_E} (0) = 2

$\therefore |\ e_{\mu_E}$ (d) $- e_{\mu_E}$ (0) $| = |1-2| = 1$, $|\ e_{\nu_E}$ (d') $- e_{\nu_E}$ (0)$| = |1-2| = 1$

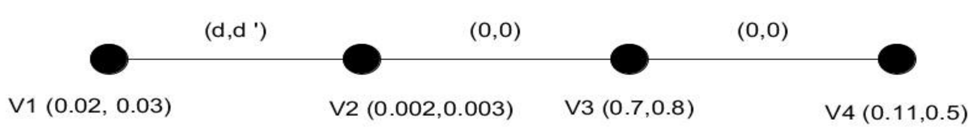

Figure 1:

When n = 5

Take m $= \frac{5}{10}$, $m' = \frac{4}{10}$

$|\ e_{\mu_E}$ (d) $- e_{\mu_E}$ (0) $| = |2-2| = 0$, $|\ e_{\nu_E}$ (d') $- e_{\nu_E}$ (0) $| = |2-2| = 0$

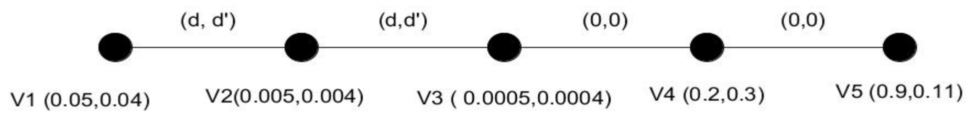

Figure 2:

Theorem 3.2

Every Cycle C_n (n $\geq$ 3)is an Intuitionistic fuzzy divisor cordial graph.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of a path C_n. Take two values say m, $m^{'}$ ε (0,1).

Case 1:

When n is even,

Label $\frac{n}{2}+1$ vertices v_j by a membership value $\mu_V(v_j) = \frac{m}{10^j}$, where j = 1, 2, ... up to $\frac{n}{2}+1$. Then label the remaining vertices by any membership value which is not a multiple of the membership value of adjacent vertices, without repeating any $\mu_V(v_j)$, $\forall$ j

$\therefore$ e_{μ_E} (d) = $\frac{n}{2}$

e_{μ_E} (0) = $\frac{n}{2}$

$\therefore$|e_{μ_E} (0) $-$ e_{μ_E} (d)| = | $\frac{n}{2} - \frac{n}{2}$ | = 0.

Hence | e_{μ_E} (0) $-$ e_{μ_E} (d) | $\leq$ 1, holds.

Also label the same $\frac{n}{2}+1$ vertices v_j by a non-membership value $\nu_V(v_j) = \frac{m^{'}}{10^j}$ where j = 1, 2, ... upto $\frac{n}{2}+1$. Then label the remaining vertices by any non-membership value which is not a multiple of the non-membership value of adjacent vertices, without repeating any $\nu_V(v_j)$, $\forall j$

$\therefore$ e_{ν_E} (0) = $\frac{n}{2}$

e_{μ_E} $(d^{'})$ = $\frac{n}{2}$

$\therefore$ | e_{ν_E} (0) $-$ e_{ν_E} $(d^{'})$ | = | $\frac{n}{2} - \frac{n}{2}$ | = 0

$\therefore$ | e_{ν_E} (0) $-$ e_{ν_E} $(d^{'})$ | $\leq$ 1, hold.

Case 2:

When n is odd,

Then label $\frac{n+1}{2}$ vertices v_j by a membership value $\mu_V(v_j) = \frac{m}{10^j}$, where j = 1, 2, ... up to $\frac{n+1}{2}$. Then label the remaining $\frac{n-1}{2}$ vertices by any membership value which is not a multiple of the membership value of the adjacent vertices, without repeating any $\mu_V(v_j)$, $\forall$ j

$\therefore e_{\mu_E}$ (d) $= \frac{n+1}{2} - 1$

e_{μ_E} (0) $= \frac{n+1}{2}$

$\therefore \mid e_{\mu_E}$ (0) $- e_{\mu_E}$ (d) $\mid = \mid \frac{n+1}{2} - \frac{n+1}{2} - 1 \mid = 1$.

Hence $\mid e_{\mu_E}$ (0) $- e_{\mu_E}$ (d) $\mid \leq 1$, holds.

Also label the same $\frac{n+1}{2}$ vertices v_j by a non-membership value $\nu_V(v_j) = \frac{m'}{10^j}$ where j = 1,2, ... upto $\frac{n+1}{2}$. Then label the remaining vertices by any non-membership value which is not a multiple of the non-membership value of adjacent vertices, without repeating any $\nu_V(v_j)$, $\forall j$

$\therefore e_{\nu_E}$ (d') $= \frac{n+1}{2} - 1$

e_{μ_E} (0) $= \frac{n+1}{2}$

$\therefore \mid e_{\nu_E}$ (0) $- e_{\nu_E}$ (d') $\mid = \mid \frac{n+1}{2} - \frac{n+1}{2} - 1 \mid = 1$

$\therefore \mid e_{\nu_E}$ (0) $- e_{\nu_E}$ (d') $\mid \leq 1$, hold.

Therefore every path C_n is an Intuitionistic fuzzy divisor cordial graph.

Theorem 3.3

Every Star graph $S_{1,n}$ admits Intuitionistic Fuzzy divisor cordial labeling.

Proof:

Let v,u_1,u_2, ..., u_n be the vertices of a star graph $S_{1,n}$.

Take m,m' ε (0,1), label $\mu_V(v) = \frac{m}{10}$ and $\nu_V(v) = \frac{m'}{10}$

Case 1: When n is even.

Label $\frac{n}{2}$ vertices u_j by a membership value $\frac{\mu_V(v)}{10^j}$ and a non-membership value $\frac{\nu_V(v)}{10^j}$. And label the remaining vertices by a membership value which is not a multiple of $\mu_V(v)$ and a non-membership value which is not a multiple of $\nu_V(v)$.

$\therefore e_{\mu_E}$ (d) $= \frac{n}{2}$, e_{μ_E} (0) $= \frac{n}{2}$

and e_{ν_E} (d') $= \frac{n}{2}$, e_{ν_E} (0) $= \frac{n}{2}$

Case 2: When n is odd.

Label $\frac{n+1}{2}$ vertices in the similar way, then e_{μ_E} (d) $= \frac{n+1}{2}$, e_{μ_E} (0) $= \frac{n+1}{2} - 1$

and e_{ν_E} (d') $= \frac{n+1}{2}$, e_{ν_E} (0) $= \frac{n+1}{2} - 1$

In both cases, conditions (1) and (2) of definition 3.1 holds.

Hence Every star graph is an Intuitionistic fuzzy divisor cordial graph.

Illustration 3.2

This Illustration shows the Intuitionistic fuzzy divisor cordial labeling of star graph $S_{1,5}$, it can be easily verified by the following figure.

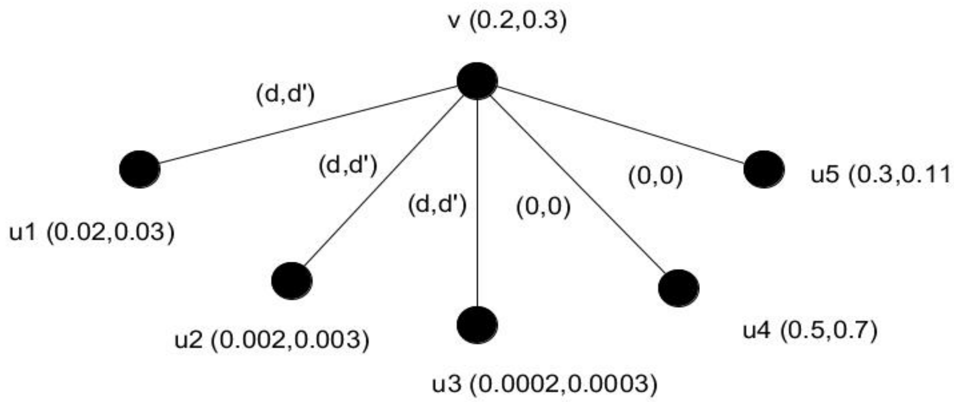

Figure 3:

Theorem 3.4

Wheel graph W_n admits Intuitionistic Fuzzy Divisor Cordial labeling.

Proof:

Let v, u_1, u_2, ..., u_{n-1} be the vertices of a wheel graph W_n.

Take m, m' ε (0,1).

Fix v as the central vertex. A wheel graph with n vertices have 2(n − 1) edges. Label the vertices u_1, u_2, ..., u_{n-1} by a membership value $\mu_V(u_j) = \frac{m}{10^j}$, for j = 1, 2, ... $n-1$. And label the vertex v by a non-membership value which is not a multiple of any of $\mu_V(u_j)$, ∀j.

Also label the vertices u_1,u_2, ..., u_{n-1} by a non-membership value $\nu_V(u_j) = \frac{m'}{10^j}$ for j = 1, 2, ... n − 1. And label the central vertex v by a non-membership value which is not a multiple of any of $\nu_V(uj)$, ∀j. $\therefore$ e_{μ_E} (0) = n − 1, e_{μ_E} (d) = n − 1.

e_{ν_E} (0) = n − 1 and e_{ν_E} $(d^{'})$ = n − 1.

$\therefore$ | e_{μ_E} (0) − e_{μ_E} (d) | $\leq$ 1 and | e_{μ_E} (0) − e_{μ_E} $(d^{'})$ | $\leq$ 1

Therefore Wheel graph W_n is an Intuitionistic Fuzzy divisor cordial graph.

Illustration 3.3

This Illustration shows the Intuitionistic fuzzy divisor cordial labeling of wheel graph W_5. It is clear from the following figure.

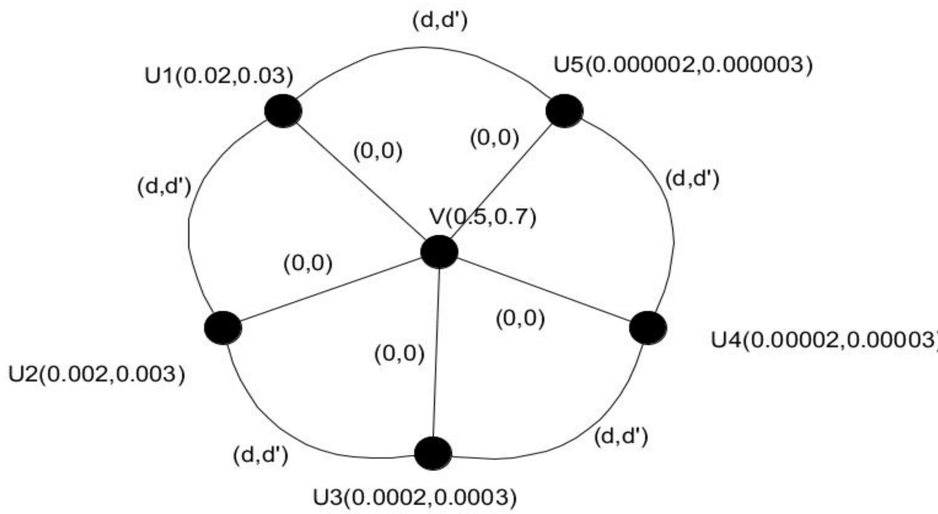

Figure 4:

4 Conclusion

In this paper we introduced the concept of intuitionistic fuzzy divisor cordial labeling of graphs. And we discussed some theorems that shows both paths and cycles obey intuitionistic fuzzy divisor cordial labeling. In the upcoming papers we planned to extend our study on some more special graphs.

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