# ON BINARY QUADRATIC EQUATION $2x^2 - 3y^2 = -4$

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**Abstract**- The binary quadratic Diophantine equation represented by the positive Pellian $2x^2 - 3y^2 = -4$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas and Pythagorean triangle.

***Key Words***: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions

## 1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of $a, b$ and $N$. In this context, one may refer [1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $2x^2 - 3y^2 = -4$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

## 2. METHOD OF ANALYSIS:

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$2x^2 - 3y^2 = -4 \quad (1)$$

Consider the linear transformations

$$x = X + 3T, \; y = X + 2T \quad (2)$$

From (1) and (2), we have

$$X^2 = 6T^2 + 4 \quad (3)$$

whose smallest positive integer solution is

$$X_0 = 10, T_0 = 4$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 6T^2 + 1 \quad (4)$$

Whose smallest positive integer solution is $(\tilde{X}_0, \tilde{T}_0) = (5, 2)$ the general solution of (4) is given by

$$\tilde{T}_n = \frac{1}{2\sqrt{6}} g_n, \; \tilde{X}_n = \frac{1}{2} f_n$$

Where

$$f_n = (5 + 2\sqrt{6})^{n+1} + (5 - 2\sqrt{6})^{n+1}$$

$$g_n = (5 + 2\sqrt{6})^{n+1} - (5 - 2\sqrt{6})^{n+1}, \quad n = -10, 1....$$

Applying Brahmagupta lemma between $(\tilde{x}_0, \tilde{y}_0)$ and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solutions of (3) are given by

$$x_{n+1} = 5 f_n + \frac{12}{\sqrt{6}} g_n \quad (5)$$

$$y_{n+1} = 2 f_n + \frac{5}{\sqrt{6}} g_n \quad (6)$$

From (2), (5) and (6) the values of $x$ and $y$ satisfying (1) are given by

$$x_{n+1} = 11 f_n + \frac{27}{\sqrt{6}} g_n$$

$$y_{n+1} = 9 f_n + \frac{22}{\sqrt{6}} g_n$$

The recurrence relations satisfied by $x$ and $y$ are given by

$$x_{n+3} - 10x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 10y_{n+2} + y_{n+1} = 0$$

Some numerical examples of $x_n$ and $y_n$ satisfying (1) are given in the Table: 1 below,

**Table: 1 Numerical Examples**

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 22 | 18 |
| 1 | 218 | 178 |
| 2 | 2158 | 1762 |
| 3 | 21362 | 17442 |
| 4 | 211462 | 172658 |
| 5 | 2093258 | 1709138 |

From the above table, we observe some interesting relations among the solutions which are presented below:

- Both $x_n$ and $y_n$ values are even.

- **Relations among the solutions are given below:**

  - $x_{n+3}-10x_{n+2}+x_{n+1}=0$
  - $5x_{n+1}-x_{n+2}+6y_{n+1}=0$
  - $x_{n+1}-5x_{n+2}+6y_{n+1}=0$
  - $5x_{n+1}-49x_{n+2}+6y_{n+3}=0$
  - $60y_{n+3}-49x_{n+3}+x_{n+1}=0$
  - $x_{n+3}-49x_{n+1}-60y_{n+1}=0$
  - $y_{n+2}-5y_{n+1}-4x_{n+1}=0$
  - $x_{n+3}-x_{n+1}-12y_{n+2}=0$
  - $5y_{n+3}-49y_{n+2}-4x_{n+1}=0$
  - $7x_{n+3}-1580y_{n+3}+125857x_{n+1}=0$
  - $49y_{n+1}-y_{n+3}+40x_{n+1}=0$
  - $6y_{n+1}-5x_{n+3}+49x_{n+2}=0$
  - $6y_{n+2}-21365x_{n+3}+211489x_{n+2}=0$
  - $6y_{n+3}-5x_{n+3}+x_{n+2}=0$
  - $44x_{n+2}+5x_{n+1}-539y_{n+1}=0$
  - $5y_{n+2}-y_{n+1}-4x_{n+2}=0$
  - $y_{n+3}-y_{n+1}-8x_{n+2}=0$
  - $x_{n+3}-5x_{n+2}-6y_{n+2}=0$
  - $y_{n+3}-5y_{n+2}-4x_{n+2}=0$
  - $49y_{n+2}-5y_{n+1}-4x_{n+3}=0$
  - $49y_{n+3}-y_{n+1}-40x_{n+3}=0$
  - $5y_{n+3}-y_{n+2}+4x_{n+3}=0$
  - $5y_{n+1}-y_{n+2}+4x_{n+1}=0$
  - $y_{n+3}-10y_{n+2}+y_{n+1}=0$
  - $4x_{n+1}+49y_{n+2}-5y_{n+3}=0$

- **Each of the following expressions represents a nasty number:**

  - $\left(12+27x_{2n+3}-267x_{2n+2}\right)$
  - $\frac{1}{10}\left(120+27x_{2n+4}-2643x_{2n+2}\right)$
  - $\left(12+162y_{2n+2}-132x_{2n+2}\right)$
  - $\frac{1}{5}\left(60+162y_{2n+3}-1308x_{2n+2}\right)$
  - $\frac{1}{49}\left(588+162y_{2n+4}-12948x_{2n+2}\right)$
  - $\left(12+267x_{2n+4}-2643x_{2n+3}\right)$
  - $\frac{1}{5}\left(60+1602y_{2n+2}-132x_{2n+3}\right)$
  - $\left(12+1602y_{2n+3}-1308x_{2n+3}\right)$
  - $\frac{1}{5}\left(60+1602y_{2n+4}-12948x_{2n+3}\right)$
  - $\frac{1}{49}\left(588+15858y_{2n+2}-132x_{2n+4}\right)$
  - $\frac{1}{5}\left(60+15858y_{2n+3}-1308x_{2n+4}\right)$
  - $\left(12+15858y_{2n+4}-12948x_{2n+4}\right)$
  - $\frac{1}{4}\left(48+1308y_{2n+2}-132y_{2n+3}\right)$
  - $\frac{1}{40}\left(480+12948y_{2n+2}-132y_{2n+4}\right)$
  - $\frac{1}{4}\left(48+12948y_{2n+3}-1308y_{2n+4}\right)$

- **Each of the following expressions represents a cubical integer:**

  - $\frac{1}{6}\left(\begin{matrix}27x_{3n+4}-267x_{3n+3}+81x_{n+2}\\-801x_{n+1}\end{matrix}\right)$
  - $\frac{1}{60}\left(\begin{matrix}27x_{3n+5}-2643x_{3n+3}+81x_{n+3}\\-7929x_{n+1}\end{matrix}\right)$
  - $\left(\begin{matrix}27y_{3n+3}-22x_{3n+3}+81y_{n+1}\\-66x_{n+1}\end{matrix}\right)$
  - $\frac{1}{5}\left(\begin{matrix}27y_{3n+4}-218x_{3n+3}+81y_{n+2}\\-654x_{n+1}\end{matrix}\right)$
  - $\frac{1}{49}\left(\begin{matrix}27y_{3n+5}-2158x_{3n+3}+81y_{n+3}\\-6474x_{n+1}\end{matrix}\right)$
  - $\frac{1}{6}\left(\begin{matrix}267x_{3n+5}-2643x_{3n+4}+801x_{n+3}\\-7929x_{n+2}\end{matrix}\right)$

- $\frac{1}{5}\begin{pmatrix}267y_{3n+3}-22x_{3n+4}+801y_{n+1}\\-66x_{n+2}\end{pmatrix}$
- $\begin{pmatrix}267y_{3n+4}-218x_{3n+4}+801y_{n+2}\\-654x_{n+2}\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}267y_{3n+5}-2158x_{3n+4}+801y_{n+3}\\-6474x_{n+2}\end{pmatrix}$
- $\frac{1}{49}\begin{pmatrix}2643y_{3n+3}-22x_{3n+5}+7929y_{n+1}\\-66x_{n+3}\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}2643y_{3n+4}-218x_{3n+5}+7929y_{n+2}\\-654x_{n+3}\end{pmatrix}$
- $\begin{pmatrix}2643y_{3n+5}-2158x_{3n+5}+7929y_{n+3}\\-6474x_{n+3}\end{pmatrix}$
- $\frac{1}{4}\begin{pmatrix}218y_{3n+3}-22y_{3n+4}+654y_{n+1}\\-66y_{n+2}\end{pmatrix}$
- $\frac{1}{40}\begin{pmatrix}2158y_{3n+3}-22y_{3n+5}+6474y_{n+1}\\-66y_{n+3}\end{pmatrix}$
- $\frac{1}{4}\begin{pmatrix}2158y_{3n+4}-218y_{3n+5}+6474y_{n+2}\\-654y_{n+3}\end{pmatrix}$

➢ **Each of the following expressions is a biquadratic integer:**

- $\frac{1}{6}\begin{pmatrix}27x_{4n+5}-267x_{4n+4}+108x_{2n+3}\\-1068x_{2n+2}+36\end{pmatrix}$
- $\frac{1}{60}\begin{pmatrix}27x_{4n+6}-2643x_{4n+4}+108x_{2n+4}\\-10572x_{2n+2}+360\end{pmatrix}$
- $\begin{pmatrix}27y_{4n+4}-22x_{4n+4}+108y_{2n+2}\\-88x_{2n+2}+6\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}27y_{4n+5}-218x_{4n+4}+108y_{2n+3}\\-872x_{2n+2}+30\end{pmatrix}$
- $\frac{1}{49}\begin{pmatrix}27y_{4n+6}-2158x_{4n+4}+108y_{2n+4}\\-8632x_{2n+2}+294\end{pmatrix}$
- $\frac{1}{6}\begin{pmatrix}267x_{4n+6}-2643x_{4n+5}+1068x_{2n+4}\\-10572x_{2n+3}+36\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}267y_{4n+4}-22x_{4n+5}+1068y_{2n+2}\\-88x_{2n+3}+30\end{pmatrix}$
- $\begin{pmatrix}267y_{4n+5}-218x_{4n+5}+1068y_{2n+3}\\-872x_{2n+3}+6\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}267y_{4n+6}-2158x_{4n+5}+1068y_{2n+4}\\-8632x_{2n+3}+30\end{pmatrix}$
- $\frac{1}{49}\begin{pmatrix}2643y_{4n+4}-22x_{4n+6}+10572y_{2n+2}\\-88x_{2n+4}+294\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}2643y_{4n+5}-218x_{4n+6}+10572y_{2n+3}\\-872x_{2n+4}+30\end{pmatrix}$
- $\begin{pmatrix}2643y_{4n+6}-2158x_{4n+6}+10572y_{2n+4}\\-8632x_{2n+4}+6\end{pmatrix}$
- $\frac{1}{4}\begin{pmatrix}218y_{4n+4}-22y_{4n+5}+872y_{2n+2}\\-88y_{2n+3}+24\end{pmatrix}$
- $\frac{1}{40}\begin{pmatrix}2158y_{4n+4}-22y_{4n+6}+8632y_{2n+2}\\-88y_{2n+4}+240\end{pmatrix}$
- $\frac{1}{4}\begin{pmatrix}2158y_{4n+5}-218y_{4n+6}+8632y_{2n+3}\\-872y_{2n+4}+24\end{pmatrix}$

➢ **Each of the following expression is a quintic integer:**

- $\frac{1}{6}\begin{pmatrix}27x_{5n+6}-267x_{5n+5}+135x_{3n+4}\\-1335x_{3n+3}+270x_{n+2}-2670x_{n+1}\end{pmatrix}$
- $\frac{1}{60}\begin{pmatrix}27x_{5n+7}-2643x_{5n+5}+135x_{3n+5}\\-13215x_{3n+3}+270x_{n+3}\\-26520x_{n+1}\end{pmatrix}$
- $\begin{pmatrix}27y_{5n+5}-22x_{5n+5}+135y_{3n+3}\\-110x_{3n+3}+270y_{n+1}-220x_{n+1}\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}27y_{5n+6}-218x_{5n+5}+135y_{3n+4}\\-1090x_{3n+3}+270y_{n+2}-2180x_{n+1}\end{pmatrix}$
- $\frac{1}{49}\begin{pmatrix}27y_{5n+7}-2158x_{5n+5}+135y_{3n+5}\\-10790x_{3n+3}+270y_{n+3}\\-21580x_{n+1}\end{pmatrix}$
- $\frac{1}{6}\begin{pmatrix}267x_{5n+7}-2643x_{5n+6}+1335x_{3n+5}\\-13215x_{3n+4}+2670x_{n+3}-26430x_{n+2}\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}267y_{5n+5}-22x_{5n+6}+1335y_{3n+3}\\-110x_{3n+4}+2670y_{n+1}-220x_{n+2}\end{pmatrix}$
- $\begin{pmatrix}267y_{5n+6}-218x_{5n+6}+1335y_{3n+4}\\-1090x_{3n+4}+2670y_{n+2}-2180x_{n+2}\end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix}267y_{5n+7}-2158x_{5n+6}+1335y_{3n+5}\\-10790x_{3n+4}++2670y_{n+3}-21580x_{n+2}\end{pmatrix}$

- $\frac{1}{49}\begin{pmatrix} 2643y_{5n+5}-22x_{5n+7}+13215y_{3n+3} \\ -110x_{3n+5}+26430y_{n+1}-220x_{n+3} \end{pmatrix}$
- $\frac{1}{5}\begin{pmatrix} 2643y_{5n+6}-218x_{5n+7}+13215y_{3n+4} \\ -1090x_{3n+5}+26430y_{n+2}-2180x_{n+3} \end{pmatrix}$
- $\begin{pmatrix} 2643y_{5n+7}-2158x_{5n+7}+13215y_{3n+5} \\ -10790x_{3n+5}+26430y_{n+3}-21580x_{n+3} \end{pmatrix}$
- $\frac{1}{4}\begin{pmatrix} 218y_{5n+5}-22y_{5n+6}+1090y_{3n+3} \\ -110y_{3n+4}+2180y_{n+1}-220y_{n+2} \end{pmatrix}$
- $\frac{1}{40}\begin{pmatrix} 2158y_{5n+5}-22y_{5n+7}+10790y_{3n+3} \\ -110y_{3n+5}+21580y_{n+1}-220y_{n+3} \end{pmatrix}$
- $\frac{1}{4}\begin{pmatrix} 2158y_{5n+6}-218y_{5n+7}+10790y_{3n+4} \\ -1090y_{3n+5}+21580y_{n+2}-2180y_{n+3} \end{pmatrix}$

## 3. REMARKABLE OBSERVATIONS

3.1 Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in Table 2 below:

**Table 2: Hyperbolas**

| S.No | Hyperbola | $(X,Y)$ |
|---|---|---|
| 1 | $X^2-6Y^2=144$ | $\begin{pmatrix} 27x_{n+2}-267x_{n+1}, \\ 109x_{n+1}-11x_{n+2} \end{pmatrix}$ |
| 2 | $X^2-6Y^2=14400$ | $\begin{pmatrix} 27x_{n+3}-2643x_{n+1}, \\ 1079x_{n+1}-11x_{n+3} \end{pmatrix}$ |
| 3 | $X^2-6Y^2=4$ | $\begin{pmatrix} 27y_{n+1}-22x_{n+1}, \\ 9x_{n+1}-11y_{n+1} \end{pmatrix}$ |
| 4 | $X^2-6Y^2=100$ | $\begin{pmatrix} 27y_{n+2}-218x_{n+1}, \\ 89x_{n+1}-11y_{n+2} \end{pmatrix}$ |
| 5 | $X^2-6Y^2=9604$ | $\begin{pmatrix} 27y_{n+3}-2158x_{n+1}, \\ 881x_{n+1}-11y_{n+3} \end{pmatrix}$ |
| 6 | $X^2-6Y^2=144$ | $\begin{pmatrix} 267x_{n+3}-2643x_{n+2}, \\ 1079x_{n+2}-109x_{n+3} \end{pmatrix}$ |
| 7 | $X^2-6Y^2=100$ | $\begin{pmatrix} 267y_{n+1}-22x_{n+2}, \\ 9x_{n+2}-109y_{n+1} \end{pmatrix}$ |
| 8 | $X^2-6Y^2=4$ | $\begin{pmatrix} 267y_{n+2}-218x_{n+2}, \\ 89x_{n+2}-109y_{n+2} \end{pmatrix}$ |
| 9 | $X^2-6Y^2=100$ | $\begin{pmatrix} 267y_{n+3}-2158x_{n+2}, \\ 881x_{n+2}-109y_{n+3} \end{pmatrix}$ |
| 10 | $X^2-6Y^2=9604$ | $\begin{pmatrix} 2643y_{n+1}-22x_{n+3}, \\ 9x_{n+3}-1079y_{n+1} \end{pmatrix}$ |
| 11 | $X^2-6Y^2=100$ | $\begin{pmatrix} 2643y_{n+2}-218x_{n+3}, \\ 89x_{n+3}-1079y_{n+2} \end{pmatrix}$ |
| 12 | $X^2-6Y^2=4$ | $\begin{pmatrix} 2643y_{n+3}-2158x_{n+3}, \\ 881x_{n+3}-1079y_{n+3} \end{pmatrix}$ |
| 13 | $X^2-6Y^2=64$ | $\begin{pmatrix} 218y_{n+1}-22y_{n+2}, \\ 9y_{n+2}-89y_{n+1} \end{pmatrix}$ |
| 14 | $X^2-6Y^2=6400$ | $\begin{pmatrix} 2158y_{n+1}-22y_{n+3}, \\ 9y_{n+3}-88y_{n+1} \end{pmatrix}$ |
| 15 | $X^2-6Y^2=64$ | $\begin{pmatrix} 2158y_{n+2}-218y_{n+3}, \\ 89y_{n+3}-881y_{n+2} \end{pmatrix}$ |

3.2 Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in Table: 3 below,

**Table 3: Parabolas**

| S.No | Parabola | $(X,Y)$ |
|---|---|---|
| 1 | $Y^2=X-12$ | $\begin{pmatrix} 27x_{2n+3}-267x_{2n+2}, \\ 109x_{n+1}-11x_{n+2} \end{pmatrix}$ |
| 2 | $Y^2=10X-1200$ | $\begin{pmatrix} 27x_{2n+4}-2643x_{2n+2}, \\ 1079x_{n+1}-11x_{n+3} \end{pmatrix}$ |

| | | |
|---|---|---|
| 3 | $6Y^2 = X - 2$ | $\begin{pmatrix} 27y_{2n+2} - 22x_{2n+2}, \\ 9x_{n+1} - 11y_{n+1} \end{pmatrix}$ |
| 4 | $6Y^2 = 5X - 50$ | $\begin{pmatrix} 27y_{2n+3} - 218x_{2n+2}, \\ 89x_{n+1} - 11y_{n+2} \end{pmatrix}$ |
| 5 | $6Y^2 = 49X - 4802$ | $\begin{pmatrix} 27y_{2n+4} - 2158x_{2n+2}, \\ 881x_{n+1} - 11y_{n+3} \end{pmatrix}$ |
| 6 | $Y^2 = X - 12$ | $\begin{pmatrix} 267x_{2n+4} - 2643x_{2n+3}, \\ 1079x_{n+2} - 109x_{n+3} \end{pmatrix}$ |
| 7 | $6Y^2 = 5X - 50$ | $\begin{pmatrix} 267y_{2n+2} - 22x_{2n+3}, \\ 9x_{n+2} - 109y_{n+1} \end{pmatrix}$ |
| 8 | $6Y^2 = X - 2$ | $\begin{pmatrix} 267y_{2n+3} - 218x_{2n+3}, \\ 89x_{n+2} - 109y_{n+2} \end{pmatrix}$ |
| 9 | $6Y^2 = 5X - 50$ | $\begin{pmatrix} 267y_{2n+4} - 2158x_{2n+3}, \\ 881x_{n+2} - 109y_{n+3} \end{pmatrix}$ |
| 10 | $6Y^2 = 49X - 4802$ | $\begin{pmatrix} 2643y_{2n+2} - 22x_{2n+4}, \\ 9x_{n+3} - 1079y_{n+1} \end{pmatrix}$ |
| 11 | $6Y^2 = 5X - 50$ | $\begin{pmatrix} 2643y_{2n+3} - 218x_{2n+4}, \\ 89x_{n+3} - 1079y_{n+2} \end{pmatrix}$ |
| 12 | $6Y^2 = X - 2$ | $\begin{pmatrix} 2643y_{2n+4} - 2158x_{2n+4}, \\ 881x_{n+3} - 1079y_{n+3} \end{pmatrix}$ |
| 13 | $6Y^2 = 4X - 32$ | $\begin{pmatrix} 218y_{2n+2} - 22y_{2n+3}, \\ 9y_{n+2} - 89y_{n+1} \end{pmatrix}$ |
| 14 | $6Y^2 = 40X - 3200$ | $\begin{pmatrix} 2158y_{2n+2} - 22y_{2n+4}, \\ 9y_{n+3} - 88y_{n+1} \end{pmatrix}$ |
| 15 | $6Y^2 = 4X - 32$ | $\begin{pmatrix} 2158y_{2n+3} - 218y_{2n+4}, \\ 89y_{n+3} - 881y_{n+2} \end{pmatrix}$ |

3.3 Let $p,q$ be two non-zero distinct integers such that $p > q > 0$. Treat $p,q$ as the generators of the Pythagorean triangle $T(\alpha,\beta,\gamma)$ where $\alpha = 2pq, \beta = p^2 - q^2, \gamma = p^2 + q^2, p > q > 0$.

Taking $p = x_{n+1} + y_{n+1}, q = x_{n+1}$, it is observed that $T(\alpha,\beta,\gamma)$ is satisfied by the following relations:

- $3\alpha - \beta - 2\gamma = -4$
- $4\frac{A}{P} = \alpha + \beta - \gamma$
- $2\frac{A}{P} = x_{n+1}y_{n+1}$

where A, P represent the area and perimeter of the Pythagorean triangle.

## 4. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the positive Pell equation $2x^2 - 3y^2 = -4$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of Pell equations and determine their integer solutions along with suitable properties.

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