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ON STRONG DOMINATION NUMBER OF JUMPGRAPHS

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ABSTRACT - *A subset S of a vertex set V is called a dominating set of jump graph J(G) if every vertex of V-S is senoted by some element of set S. If e is edge with end vertices u and v as deg u ≥ deg v then we say u strongly dominates v. If every vertex of V-S is dominated by some vertex of S then S is called strongly dominated set. The minimum cardinality of a strong dominating set is called the strongly domination number of jump graph J(G).We investigate strong domination numbers of some graphs and study related parameters.*

Keywords: Domination number, Independent domination number strong domination MS classification2010 No.05C69, 05C76.

1. Introduction

We consider the simple, finite connected and undirected graph J(G) with vertex set V and edge set E for all standard terminology and notations we follow west(2003) while the terms related to the theory of domination in graphs are used in the sense of Haynes et. al., (1998).The domination number is a well studied parameter as observed by Hedetiniemi and Laskar (1990)

Definition 1; A set S⊆ V of vertices in a graph J(G)is called dominating set, if every vertex v in V is either an element of S or is adjacent to an element of S. A dom9inating set S is a minimal dominating set (MDS) if no proper subset S'⊂ S is a dominating set. The set of all minimal dominating set of jump graph J(G) is denoted by MDS(J(G)). The domination number γ(J(G)) of a graph J(G) is equals the minimum cardinality a set MDS(J(G))

Definition2; A set S⊆ V is an independent set of J(G), if for any u, v

N(u) ∩ {v}=ϕ. A dominating set which is independent is called an independent dominating set. The minimum cardinality of an independent dominating set in J(G) is called the independent domination number γ_i (J(G)) of a graph J(G).

The theory of independent domination was formalized by Berge C (1962) and Ore(1962). Allan and Laskar (1978) have discussed some results for which γ(J(G)) = γ_i(J(G)) where as bounds on the independent domination number are determined by Goddarl and Henning (2013) Vaidya and Pandit (2016) have investigated the exact value of independent domination number of some wheel related jump graphs.

Definition 3: Let J(G) be a graph and uv in E, then u strongly dominates v

(v weakly dominates u)if deg(u) ≥ deg(v). It is obvious that every vertex of V can strongly dominates itself. A set S is astrong dominating set (weakly dominatein set) if every vertex v in V-S is strongly(weakly) dominated by some u in Sstrong dominating set and weak dominating set are abbreviated as sd-set and wd-set respectively. The strong domination number of J(G) is denoted by γ_{st}(J(G)) and the strong domination number γ_{st}(J(G)) and weak domination number γ_{wt}(J(G)) of J(G) are the minimum cardinality of an Sd-set and wd-set respectively.

The concept of strong domination and weak domination were introduced by Sampath kumar and Puspalatha (1996) many researchers like Rautenbach(1999).Meena et.al., (2014) and Domke et. al.,(2002) have explored these concepts Bounds on strong domination number are also explored by Desai and Gangadharappa(2011) and also by Rautenbach(2000)

Definition 4: The independent strong domination number γ_{ist}(J(G)) of a graph J(G) is the minimum cardinality of a strong dominating set which is independent.

Definition 5: The centre C(J(G)) of J(G) s the set of vertices of minimum eccentricity namely C(J(G))={ v ∈ V(J(G)) : ecc (v) ≤ ecc (u) for all u∈V(J(G)))}. The eccentricity of vertex v is ecc (v)= max {d(u, v); w ∈V} For any tree J(T0 the centre C(J(T)) consists of one vertex or two adjacent vertices.

2. Main Results.

Theorem 6: If $S \subseteq V(J(G))$ is a strong domination set and $v \in V(J(G))$ is only vertex of maximum degree in J(G) then $v \in S$.

Proof: Let v be the vertex of maximum degree In J(G) and S be a strong dominating set

To prove: $v \in S$

Suppose $v \notin S$, which implies that $v \in V-S$. As v is the only vertex with maximum degree it will be strongly dominated by itself only. Therefore if $v \notin S$ then there is no vertex n S which strongly dominates v. That is S is not an sd-set which contradicts to our assumption that S is a strong dominating set, Hence $v \in S$

Theorem 7: Let v be a vertex with degree (v) = $\Delta(J(G))$ and v is not adjacent to any other vertex of degree k then v must be in sd-set.

Proof: Let v be any ertex of maximum degree k in J(G) which is not adjacent to any vertex of the same degree k that is,m deg (v) > deg (w) for any w in N(v) so, the vertex v is strongly dominated by itself only.

Hence v must be an sd-set.

Theorem 8: Let J(G) be a graph of order n such that $\Delta(J(G))$ = kand there are mutually non-adjacent vertices with degree k such that there is no vertex which is strongly dominated by any two or more vertices of degree k then

$\gamma \leq \gamma_{st}(J(G)) \leq n - \gamma(\Delta(J(G)))$.

Proof: By Theorem 7. All the mutually nn-adjacent vertices of degree k must be in every sd-set Therefore $\gamma \leq \gamma_{st}(J(G)$.

Let v be any vertex of maximum degree k in G which is not adjacent to any two or more vertices of degree k. Therefore each vertex of degree k strongly dominates k+1 distinct vertices from V(J(G)). If we consider ‘r’ vertices of maximum degree in an sd-set then $r + r\,\Delta(J(G))$ vertices are strongly dominated. If $r + r\,\Delta(J(G)) \leq n$ then $n - (r + r\,\Delta(J(G)))$ number of vertices are not strongly dominated. To strongly dominate all the vertices of V(J(G)) we need to consider at least

$r + n - (r + r\,\Delta(J(G)))$ vertices from V(J(G)) Hence $\gamma_{st}(J(G)) \leq n - r\,\Delta(J(G))$.

Proposition 9 (Boutrig and Chellai(2012))

$$\gamma_{st}(J(G)) = \lceil \frac{n}{3} \rceil$$

Definition 10: Let J(G) + (V, E) be a jump graph and $D \subset V$ then

i) D is full if every u in d is adjacent to some v in V-D
ii) D is S-full (w-full) if every u in D strongly (weakly)dominates some v in V-D.

Definition 11: A graph J(G) is domination balanced (d-balanced) if there exists a sd-set D_1 and a wd-set D_2 such that $D_1 \cap D_2 = \phi$.

Theorem 13: If there exists an isolated vertex in jumpo graph J(G) then J(G) is not d-balanced.

Proof: By definition of d-balanced graph J(G) is d-balanced if there exists sd-set D_1 and wd-set D_2 such that $D_1 \cap D_2 = \phi$. If there is an isolated vertex In J(G) then it must be in every sd-set and every wd-set So, there is neither the set D_1 (sd-set) nr the set D_2(wd-set) such that $D_1 \cap D_2 = \phi$. Therefore the graph J(G) is not d-balanced. Hence the result.

3. Concluding Remark:

The Concept of strong domination is a variant of usual domination. This concept is useful to deploy the security troops and their transition from one place to another. This work cn be applied to rearrange the existing security network in the case of high alert situation and to beep up the surveillance.

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