VIBRATION ANALYSIS OF A TELECOMMUNICATION TOWER UNDER LOADING

Anthony Kpegele Le-ol[1], Charles B. Kpina[2]

[1]Department of Mechanical Engineering, Rivers State University Port Harcourt, Nigeria.
[2]Department of Mechanical Engineering, Nigeria Maritime University Okerenkoro Delta State

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Abstract - *This research evaluates the vibrational effect of telecommunication tower under loading. An integrated analytical and numerical methodology was used to evaluate the structural characteristic at different loading nodal points. Six critical points analysis were carried out to establish critical structural parameters such as the strain-life parameters and isotropic elasticity parameters. The results showed that the case structure has an optimum strength coefficient of 950 MPa beyond which yielding, and buckling may occur, with maximum deformation increment towards the nodal point with higher structural member interconnectivity. The results further showed a compressive yield strength and tensile ultimate strength of 250 MPa and 460 MPa respectively for the structure and a maximum young modulus of 2X10⁵ MPa. Therefore, this study provides an insight into structural response of such infrastructures under loading as a guide against failure and for effective monitoring.*

Key Words:- **Vibration, loading, structure, telecommunication and strength**

1. INTRODUCTION

The telecommunication and electrical power transmission systems expansion were the main reasons for efficient and cost-effective transmission and telecommunication steel towers. Steel truss towers have been used to support transmission antennas or to enable electrical power transmission lines to be built, interconnecting a vast territory. However, structural collapses, mainly associated with the wind action, are not uncommon to this structural solution [1]. The two types of telecommunication towers mainly known to engineers are guyed towers and self-supporting towers. The self-supporting towers are categorized into two groups of 4-legged and 3-legged towers. Most researches to date have been performed on 3-legged self-supporting towers and very limited attention has been paid to the dynamic behavior of the 4-legged self-supporting telecommunication towers.

It is worthy of note that most of the traditional structural analysis methods for telecommunication and transmission steel towers still assume simple truss behaviour, where all connections are considered hinged. On the other hand, structural mechanisms that could compromise the assumed structural response, can be present in various commonly used tower geometries whenever truss type models are adopted [2]. A usual solution to overcome this problem is the use of dummy structural bars to prevent the occurrence of the unwanted degrees of freedom. These bars, possessing a small axial stiffness, are generally employed to prevent the occurrence of structural mechanisms, enabling the use of standard finite element programs. A possible explanation for the structure stability is related to the semi-rigid, instead of the assumed hinged joint behaviour. In fact, most major steel tower constructors still rely on full-scale tests to determine which design and fabrication details can provide a good test correlation with the assumed simple truss model results. Cohen and Pemn [3] made the earliest contributions to the study of guyed towers. The result of the investigation on wind loading was presented as a set of charts that could be used to predict the drag loads produced by wind on various types of structures. They further presented a model that described the behaviour of a guyed tower. The mast was treated as a cantilever beam-column on elastic supports and the guy cables were considered to follow a parabolic profile. Dean [4] investigated the amplification of stresses and displacements in guyed towers when changes in geometry are included. Modelling the guys as bars, charts were developed that could be used to determine when advanced methods of structural analysis are required in the design and what modifications could be made to the analysis to obtain reasonable results.

Hull [5] expressed the critical moment of inertia corresponding to a critical buckling wind load and conducted a stability analysis of guyed towers. It was suggested that increasing the stiffness of guys is the most efficient means of increasing the buckling capacity of a tower. It was shown that the buckling capacity could be increased up to the limit where it begins to buckle into several sine waves. He further showed that once this point has been reached, a further increase of guy stiffness does not increase the buckling load of a tower. At this stage, it was found that the only way to further increase the bucking capacity of a tower was to increase the moment of inertia of mast. Goldberg and Gaunt [6] investigated the importance of including wind effects on guy cables. A method of analysis was presented for guyed towers that considered non-linear behaviour and the effect of wind on guy cable stiffness. The study also reported that neglecting wind effects on guy cables resulted in discrepancies in the end moments, shear, goy tensions and lateral displacements. Following the assumption that an inclined guy cable follows a parabolic profile.

Bathe [7] presented a simplified systematic procedure for the design of guyed towers by use of interaction diagrams. A tower is first analyzed following the assumption that it is a continuous beam on elastic supports while secondary effects are included. The tower is then re-analyzed including amplification stresses from axial loads. The interaction diagrams provided a designer with a graphical visualization of the design range thereby preventing a trial and error procedure. Goldberg and Gaunt [6] presented a method that could be used to determine the instability of guyed towers. In their analysis, lateral load increments are applied until a tower reaches instability. The criterion used to define buckling in the analysis is the occurrence of a large increase in the tower deformations for a small increase in the applied load. A parametric study was also included which showed the influence of certain system parameters on the critical load of a tower. They showed that increasing the moment of inertia of the shaft was a less effective way of increasing the critical load. Based on the assumption that the static profile of a cable followed the shape of a parabola.

Kahla [8] presented a dynamic guy modulus which was meant to consider the effects of the dynamic nature of wind loading on a guy and a mast. Dean [4] introduced catenary equations for the static profile of a guy cable. Dean argued that due to the availability of computers there is no need to use a parabolic approximation in the analysis of a cable. Irvine and Caughey [9] presented a linear theory for the free-vibration of a uniform horizontally suspended cable for ratios of sag-to-span of 1:8 or less. They showed that if the sag is small enough for the static geometry to be described by a parabola, the theory provided good results. These authors also developed expressions for the natural frequencies of a horizontal cable fixed at both ends for in-plane and out-of-plane motions. The expressions were presented as functions of the cable's axial stiffness, horizontal tension, self-weight and cable effective length. In addition to a horizontal cable, consideration was also given to the analysis of inclined cables. The effects of inextensible cable assumption were also discussed.

Irvine and Caughey [10] presented solutions for free vibrations of an inclined cable hanging under self-weight. Non-dimensional natural frequencies of symmetric in-plane modes were shown to depend only on one dimensionless system parameter while the remaining frequencies were shown to be independent of any parameters. In many studies, guyed towers have been assumed to oscillate linearly about their static equilibrium position. Kahla [8], dynamically modelled the rupture of a cable present in guyed steel towers. The analysis indicated that the guyed steel towers cable rupture, disregarding the wind actions, was one of the most severe critical load hypotheses for the investigated structures.

Khedr and McClure [10] performed additional studies to introduce simplifying methods for the seismic analysis of telecommunication towers. Galvez investigated three different numerical models of 3-legged lattice steel towers with heights ranging from 90 to 121 meters that were subjected to 45 earthquake records. It was concluded that contribution of second and third transversal modes of vibration on the maximum acceleration at the top of the towers, depending on the tower type, varies from 15% to 50%. One of the main disadvantages of the Galvez method was the bilinear shape of the acceleration profile, which did not thoroughly include the towers with different geometries. They later introduced a modified method for the horizontal acceleration profile, so that for every specified tower a separate acceleration profile be obtained. Moreover, in the latest edition of TIA/EIA code, provisions for seismic design of towers have been included.

Konno and Kimura [11] performed an investigation of the numerical models used in steel telecommunication masts. The authors stressed the relevance of considering the non-linear effects present even at service load levels. Mikus, [12] evaluated telecommunication guyed steel towers from their static and dynamical structural responses. The static analysis compared linear and non-linear mathematical models. The dynamical analysis employed the Monte Carlo simulation method including the wind load floating parcel producing interesting results. Daryl, [13] used the finite element method by means of a geometrical and physical non-linear analysis to simulate the structural response of telecommunication and transmission steel towers. Recently, Reddy [14] tend to investigate the structural behaviour of the guyed steel towers, preventing the occurrence of spurious structural mechanisms that could lead to uneconomic or unsafe structure. The towers investigated in the present paper (50m, 70m and 90m), have truss type geometry with a square cross section. Hot rolled angle sections connected by bolts compose the main structure as well as the bracing system. Pre-stressed cables support the main structure, which must be always in tension. Some of these cables are linked to a specific set of bars arranged to improve the system torsion stiffness.

Due to the relevance and wide application of telecommunication, several researches have attempted to understand the static and dynamic behaviour of the communication mast under loading. The objective of this work therefore, is to apply the technique recommended by Rayleigh to demonstrate on structures and find a single equation and correction factor that can be used to resolve practical problems in Engineering.

2. MATERIALS AND METHODS

A beam is used for demonstration of the analysis. A beam is a structural element that primarily resists loads applied laterally to the beam's axis. Its mode of deflection is primarily by bending. The loads applied to the beam result in reaction forces at the beam's support points. The total effect of all the forces acting on the beam is to produce shear forces and bending moments within the beam, that in turn induce internal stresses, strains and deflections of the beam. Beams are characterized by their manner of support, profile (shape of cross-section), length, and their material. Euler–Bernoulli beam theory (also known as engineer's beam theory or classical beam theory) is a simplification of the linear theory of elasticity which provides a means of calculating the load-carrying and deflection characteristics of beams. It covers the case for small deflections of a beam that are subjected to lateral loads only. It is thus a special case of Timoshenko beam theory.

Additional analysis tools have been developed such as plate theory and finite element analysis, but the simplicity of beam theory makes it an important tool in the sciences, especially structural and mechanical engineering.

2.1 Static Beam Equation

The Euler–Bernoulli equation describes the relationship between the beam's deflection and the applied load:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = q \qquad (1)$$

E is the elastic modulus and I is the mass moment of inertia.

2.2 Dynamic beam equation.

The dynamic beam equation is the Euler–Lagrange equation for the following action

$$S = \int_0^L \left[\frac{1}{2}\mu\left(\frac{\partial w}{\partial t}\right)^2 - \frac{1}{2}EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + q(x)w(x,t)\right] dx \qquad (2)$$

The first term represents the kinetic energy where μ is the mass per unit length; the second term represents the potential energy due to internal forces (when considered with a negative sign) and the third term represents the potential energy due to the external load $q(x)$.
The Euler–Lagrange equation is used to determine the function that minimizes the functional$S$.
For a dynamic Euler–Bernoulli beam, the Euler–Lagrange equation is

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = -\mu\frac{\partial^2 w}{\partial t^2} + q \qquad (3)$$

2.3 Newton's Laws of motion

Newton's laws of motion are three physical laws that, together laid the foundation for classical mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces. More precisely, the first law defines the force qualitatively, the second law offers a quantitative measure of the force, and the third asserts that a single isolated force doesn't exist.

$$F = ma \qquad (4)$$

2.4 Superposition Principle

In physics and systems theory, the superposition principle, also known as superposition property, states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually. So that if input A produces response X and input B produces response Y then input $(A + B)$ produces response $(X + Y)$.
The homogeneity and additivity properties together are called the superposition principle. A linear function is one that satisfies the properties of superposition. It is defined as:

$$F(x_1 + x_2) = F(x_1) + F(x_2) \qquad (5)$$
$$F(ax) = aF(x) \qquad (6)$$

2.4.1 The Strain-Displacement Relations

- **Normal Strain**

Consider a line element of length Δx emanating from position (x, y) and lying in the $x - direction$, denoted by AB in the figure below. After deformation, the line element occupies $A'B'$ having undergone a translation, extension and rotation.

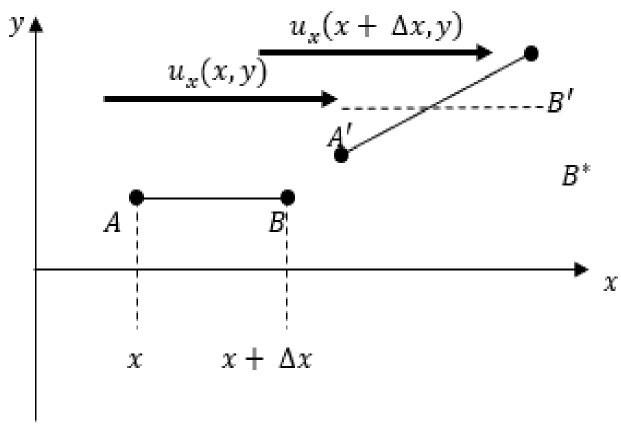

Fig. 1: Deformation of a line element

The particle that was originally at x has undergone a displacement $u_x(x, y)$ and the other end of the line element has undergone a displacement $u_x(x+\Delta x, y)$. By the definition of (small) normal strain;

$$\varepsilon_{xx} = \frac{A'B^* - AB}{AB} = \frac{u_x(x+\Delta x,y) - u_x(x,y)}{\Delta x} \quad (7)$$

In the *limit* $\Delta x \rightarrow 0$ one has

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \quad (8)$$

- **Vibration Analysis (VA)**

Vibrational analysis (VA) is a key component of a Condition Monitoring (CM) program and is often referred to as Predictive Maintenance (PdM). It is used to detect faults in rotating equipment (Fans, Motors, Pumps, and Gearboxes etc.) such as Unbalance, Misalignment, rolling element bearing faults and resonance conditions. Vibration Analysis can use the units of Displacement, Velocity and Acceleration displayed as a Time Waveform (TWF), but most commonly the spectrum is used, derived from a Fast Fourier Transform of the TWF. The vibration spectrum provides important frequency information that can pinpoint the faulty component.

The fundamentals of vibration analysis can be understood by studying the simple Mass-spring-damper model. Indeed, even a complex structure such as an automobile body can be modelled as a "summation" of simple mass–spring–damper models.

$$F_s = -kx \quad (9)$$

The force generated by the mass is proportional to the acceleration of the mass as given by Newton's second law of motion:

$$\sum F = ma = m\ddot{x} = m\frac{d^2x}{dt^2} \quad (10)$$

The sum of the forces on the mass then generates this ordinary differential equation:

$$m\ddot{x} + kx = 0 \quad (11)$$

If the initiation of vibration begins by stretching the spring by the distance of A and releasing, the solution to the above equation that describes the motion of mass is:

$$x(t) = A\cos(2\pi f_n t) \quad (11)$$

This solution says that it will oscillate with simple harmonic motion that has an amplitude of A and a frequency of f_n. The number f_n is called the **undamped natural frequency**. For the simple mass–spring system, f_n is defined as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (12)$$

3. RESULTS AND DISCUSSION

The proposed analytical and numerical vibrational model used was demonstrated on a simple beam loading as shown to represent the deck loading effect of the structure.

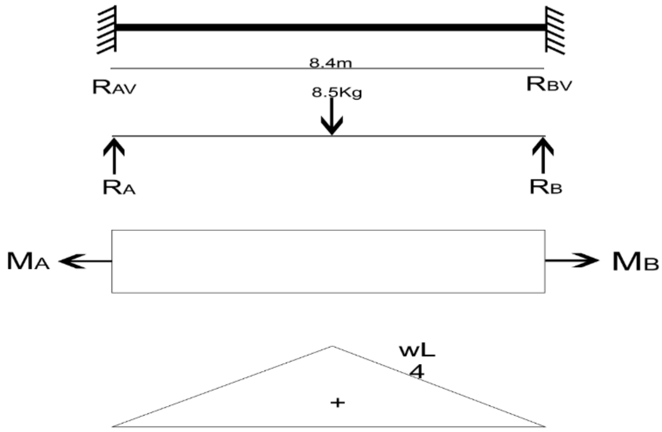

Fig. 2: A simple analytical demonstration

A beam length of 8.4 m fixed at both ends was used to demonstrate the structural loading under vibrational effect. The result of the analysis using ANSYS software at different loading mode is shown in Table 1.

Table 1. Result of Different Nodal Loading Effect on the Structure

Mode	Frequency (Hz)	Maximum deformation(mm)	Average deformation(mm)
1	0.77377	0.4231	0.1351
2	0.77425	0.4317	0.1349
3	1.8013	0.8190	5.54×10^{-2}
4	2.0511	0.8129	6.013×10^{-2}
5	2.2014	1.1058	5.566×10^{-2}
6	2.2585	1.4209	3.45×10^{-2}

Given the result as demonstrated on the nodel loading point on the structures, as the natural frequency of the structural increase, it causes an increase in the deformation of the structural member across the mode. The deformation is optimized at the 6-nodal point with an average maximum value at point 1 considering the recorded minimum deformations. The average deformation is crucial in decision making because it considered the various deformations observed at different loading effect. This provide an operational and technical envelope for structural designer and maintenance engineers for effective monitoring.

For structural strain-life parameter as shown in Table 2, the maximum strength coefficient after which structural yielding and bucking was reached. The result shows a maximum of 950Mpa. The elasticity characteristic of the structure was evaluated numerically (ANSYS) and the result is shown in Table 3 respectively. Technically, the result of the analysis provides a limiting loading criteria consideration for the area of location and the environmental parameters to ensure safety and loading resistive capability.

Table 2. Result of Strain Life Parameter of the Structure

Strength Coefficient MPa	Strength Exponent	Ductility Coefficient	Ductility Exponent	Cyclic Strength Coeff. MPa	Cyclic Strain Hardening exponent
950	-0.106	0.213	-0.47	1000	0.2

Table 3. Result of Isotropic Elasticity Parameters of the Structure

Young modulus (MPa)	Poisson's ratio	Bulk modulus (MPa)	Shear modulus (MPa)	Temperature ^{0}C
2X10^5	0.3	1.667X10^5	76923	4.27

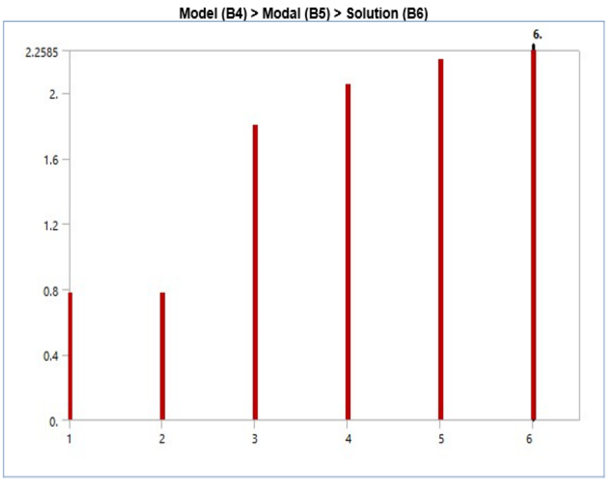

Fig. 3: Result of Numerical Analysis in ANSYS

Figure 3 shows the modal point in relation to the deformation profile. It shows that the profile increases from modal point 1 to modal point 6. This is influenced by the loading characteristic and the joint loads members. It was observed that high member loading joints, show higher maximum deformation characteristics compare with lower joint loads. Figure 4 show the loading characteristic with white, red and blue colour for different region.

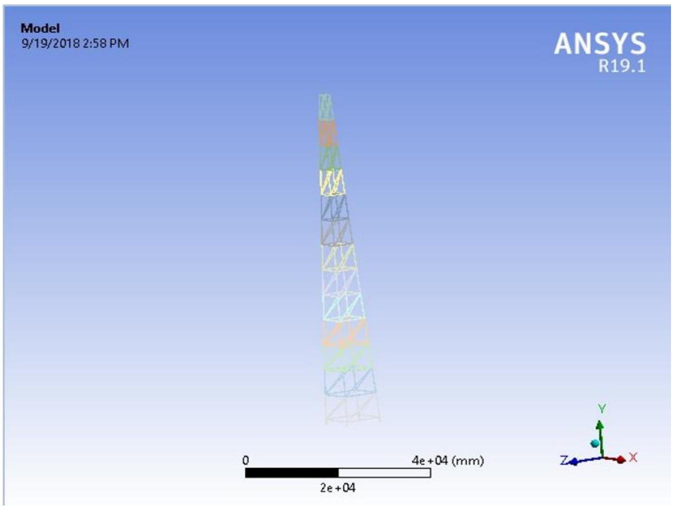

Fig. 4: ANSYS Screen Shot of the Analysis

4. CONCLUSION

The analytical and numerical-based methodology was successfully applied to determine natural frequencies and mode shapes of telecommunication towers. Using a three-dimensional beam-column element and mass matrices for the tower mast were computed. The global mass and stiffness matrices were used in an Eigen analysis to determine the natural frequencies of the telecommunication tower system. Natural frequencies of a communication mast were found to change significantly when variations in beam tensions were included in the analysis.

In the analysis of the vibration of telecommunication masts, the beams in trusses format were noticed to have been subject to wind force causing them to vibrate about their fixed positions. The model was initially designed using the analytical equation and then analysis was made using ANSYS. This choice of the software was based on recommendations from previous work and the ease of use and great user interface and experience of the software. The software was designed to meet the more intrinsic solutions of modern engineering problems. Free vibration analysis is a great starting point to understand dynamic behaviour of telecommunication masts, because it considers the natural frequency towers.

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