GLOBAL NON SPLIT DOMINATION IN JUMP GRAPHS

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ABSTRACT - Let both J(G) and J($\overline{G}$) be connected graphs. A set D of vertices in a graph J(G)=(V(J(G)), E(J(G))) is said to be a global non split dominating set. If D is non split dominating set of both J(G) and J($\overline{G}$). The global non split domination number γ_{gns} (J(G)) of J(G) is the minimum cardinality of a global non split dominating set. Beside bounds in γ_{gns} (J(G)), its relationship with other domination parameters is investigated. Also some properties of global non split dominating sets are given.

Keywords; jump graphs, complement of a jump graph, non split domination, global non split domination.

Mathematical classification: 05C69.

INTRODUCTION:

All graphs J(G) considered here are finite, undirected and connected (i,e, both J(G) and J($\bar{G}$) are connected) with no loops and multiple edges. Any undefined term in this communication may be found in Harary[1]

A set D of vertices in a graph J(G)= (V , E) is said to be a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ (J(G)) of J(G) is the minimum cardinality of a dominating set of J(G).

Different type of domination parameters have been defined by many authors [2]

N.Pratap Babu Rao and Sweta.N introduced the concept of non split domination in jump graphs as follows.

A dominating set D of a connected jump graph J(G)= (V, E) is said to be a non split dominating set (nsd-set) if the induced sub graph <V- D> is connected. The non split domination number γ_{ns}(J(G)) of J(G) is he minimum cardinality of a nsd-set of J(G).

A dominating set D of a jump graph J(G) is said to be a global dominating set (gd-set) if D is also a dominating set of J($\bar{G}$). The global domination number γ_g(J(G)) of J(G) is the minimum cardinality of a gd-set of J(G).

In this paper, we combine these two concept and introduced the concept of global non split domination as follows.

A dominating set D of a jump graph J(G) is said to be a global non split dominating set (gnsd-set) if D is a nsd-set of both J(G) and J($\bar{G}$). The global non split domination number of J(G).The γ_{gns}-set is a minimum gnsd-set, similarly other sets can be expected.

It is easy to observe the following.

Theorem 1; For any graph J(G)

i) γ_{gns}(J(G)) ≥max γ_{ns}(J(G))
ii) $\frac{(\gamma_{ns}(J(G)) + \gamma_{ns}(J(\bar{G})))}{2} \leq \gamma_{ns}(J(G)) \leq \gamma_{ns}(J(\bar{G}))$

Next we give sufficient conditions for an nsd-set to be a gnsd-set.

Theorem 2: An nsd-set D of G is a gnsd-set if the following conditions are satisfied.

i) Every vertex in V-D is not adjacent to some vertex in D

ii) <V-D> is K_1 or other exists a set S⊆ V-D such that diam(<S>) ≥3 and for every vertex v ∈ V-D there exists a vertex u in S with u not adjacent to v.

Proof: By (i) D is a dominating set of $J(\bar{G})$
By (ii) ,V-D> is connected in $J(\bar{G})$. This implies that D is a gnsd-set.

Corollary: 2.1: Let D be a γ_{ns} –set of J(G) such that <V-D > is a tree with at least from cut vertices Then,
$\gamma_{gns}(J(G)) \le \gamma_{ns}(J(G)) + 2$
Next we characterize nsd-set which are gsd-sets.

Theorem 3: Let J(G) be a connected graph with at least two adjacent non adjacent vertices. An nsd-set D of J(G) is a gnsd-set, if and only if the following conditions are satisfied.

- **i)** Every vertex in V-D is not adjacent to some vertex in D.
- **ii)** D is not a vertex connecting of $J(\bar{G})$.

Proof: Suppose d satisfies the given conditions. Then by (i) D is a dominating set of $J(\bar{G})$. By (ii) <V-D> is connected in $J(\bar{G})$. This implies that D is a gnsd-set of J(G).

Conversely, let D be a gnsd-set of J(G). On the contrary, suppose one of the given conditions, say i) is not satisfied. Then there exists a vertex v in V-D adjacent to every vertex of D. This implies that w, $J(\bar{G})$. V is not adjacent to any vertex of D and hence D is not a dominating set of $J(\bar{G})$., a contradiction. Suppose (ii) does no hold. Then <V-D> is disconnected in $J(\bar{G})$., Once again a contradiction. Hence the given conditions are satisfied.

A dominating set D of z connected jump graph J(G)= (V, E) is said to be a split dominating set if the induced sub graph <V-D> is disconnected in J(G). The split domination number ⍰$_s$(J(G)) of J(G) is he minimum cardinality of a split dominating set of J(G).

Theorem A [4] Let G be a graph with diam(G) ≥ 5 then $\gamma(G) \le \gamma_g(G)$.

Theorem 4: Let G be a jump graph J(G) with diam(J(G)) ≥ 5 if $\gamma(J(G)) < \gamma_s(J(G))$,
$\gamma_{ns}(J(G)) = \gamma_{gns}(J(G))$.

Proof: Let D be a γ-set of J(G). Clearly <V-D> is connected and hence D is a γ_{ns}-set of J(G). Also by Theorem A it is a γ_s-set of J(G). Since D is minimal, each vertex in D is adjacent to some vertex in V-D, suppose diam(<V-D>)≤2.

Then it follows that diam(J(G)) ≤4 a contradiction. Thus diam (<V-D>) ≥ 3 and by theorem 2 D is a gnsd-set of J(G).

Next we obtain a lower bound for $\gamma_{gns}(J(G))$

Let ┌ x ┐ denotes least integer greater than or equal to x.

Theorem 5: For any graph J(G) $\frac{p(q-p)-6}{6} \le \gamma_{gns} \lceil J(G)) \rceil$

Further the bound is attained if and only if there exists a γ_{gns}-set D satisfying the following conditions.

i. D has exactly two vertices.
ii. Every vertex in V-D is adjacent to exactly one vertex in D and <V-D> is self complementary.

Proof: Let D be a γ_{gns}-set of J(G) and q_1 denotes the number of edges in $J(G) \cup J(\bar{G})$ incident to the vertices of V-D. Similarly q_2 denotes the number of edges of $J(G) \cup J(\bar{G})$ incident to the vertices of D only. Then $\frac{(p(p-1))}{2} \ge q_1 - q_2 \ge 4\,|-D| - 2 + |D-1|$

This implies that $\gamma_{gns}(J(G)) \geq \frac{p(q-p-6)}{6}$ gives $\gamma_{gns}(J(G))$ is a whole number

Hence the theorem holds.

Now we can prove the second part,
Suppose the bounds is attained, then $6 \leq p \leq 8$ and $q_1 = 4|V-D|-2$ and $q_2 = |D-1|$

if p=8 then either D contains three vertices or V-D has six vertices. In both cases either

$q_2 > 1$, $|D-1|$ or $q_1 > 4|V-D|-2$ contradiction. Hence p = 6 or 7 and D has exactly two vertices. Since D is a global dominating set, each vertex in V-D is adjacent to exactly one vertex in D. As $4 \leq |V-D| \leq 5$ and <V-D > has same number of edges in both J(G) and $J(\bar{G})$. <V-D> is a self-complementary graph.

Now we obtain an upper bound for $\gamma_{gns}(J(G))$.

Theorem 6: For any graph J(G) $\gamma_{gns}(J(G)) \leq p-4$. If and only if J(G) contains a path P_4 such that for any vertex v in P_4 there exists two vertices u and w not in P_4 such that v is adjacent to u but not w.

Proof: Suppose above inequality hold on the contrary for every set S with four vertices either <S> is not in P_4(I,e, path of four vertices) or there exists a vertex v∈S adjacent to every vertex of V-S is either J(G) or $J(\bar{G})$. This implies that <V-S> is not gnsd-set of J(G) and hence $\gamma_{gns}(J(G)) = p-1$ a contradiction. This proves necessity.

Sufficiency is straight forward

The next result establishes relationship between $\gamma_{gns}(J(G))$ and $\gamma_{gns}(J(H))$ for every spanning sub graphs J(H) of J(G).

Theorem7:Let J(H) be a spanning sub graph of J(G) if $e(J(T)) \leq \gamma_{gns}(J(H))$ then

$$\boldsymbol{\gamma_{gns}(J(G)) \leq \gamma_{gns}(J(H))}$$

where e(J(T)) is the maximum number of end vertices in a spanning tree J(T) of J(G)

Proof: Let D be a γ_{gns}-set of J(H). Then obviously <V-D> is connected in both J(G) and $J(\bar{G})$.Suppose there exists a vertex in V-D adjacent to every vertex of D in J(G) then it is easy to see that $e(T) \geq \gamma_{gns}J(H)$ a contradiction.

Hence each vertex of V-D is not adjacent to some vertex of D in J(G). This implies that D is also a gnsd-set of J(G)

Hence above inequality holds.

Similarly we can prove

Theorem 8: if e(J(T)) ¸p - $\gamma_{gns}(J(G))$ then

$\gamma_c(J(\bar{G})) + \gamma_{gns}(J(G)) \leq p$ when $\gamma_c(J(G))$ is the connected domination number of J(G).

Theorem 9: Let J(G) be a block graph with diam(J(G))≥ 5 if every cut vertex is adjacent to a non-cut vertex then $\gamma_{gns}(J(G)) = k$ where k is the number of blocks in J(G) containing non-cut vertices.

Proof: Let $B_1, B_2, B_3, \ldots B_k$ be the k number of blocks in J(G) containing non-cut vertices $u_1, u_2, u_3, \ldots u_k$ respectively. Then clearly $\{u_1, u_2, u_3, \ldots u_k\}$ is a γ_{gns}-set of J(G).

Further each vertex in V - $\{u_1, u_2, u_3, \ldots u_k\}$ is not adjacent to some vertex in $\{u_1, u_2, u_3, \ldots u_k\}$ and < V - $\{u_1, u_2, u_3, \ldots u_k\}$ > is connected in $J(\bar{G})$ also as diam (J(G)) ≥ 5, This implies that $\{u_1, u_2, u_3, \ldots u_k\}$ is a γ_{gns}-set of J(G)

Hence the proof.

A dominating set D of J(G) is said to be a co-total dominating set (ctd-set) if <V-D> has no isolates. The co total domination number γ_{ct}(J(G)) of jump graph J(G) is the minimum cardinality of a ctd-set of J(G). This concept was also studied as restrained domination in graphs[7]

Theorem 10: For any graph J(G)

$$\gamma_{cr}J(G)) \leq \gamma_{gns}(J(G)).$$

Proof : Let D be a γ_{gns}-set of J(G).

We consider the following cases.

Case (i) Suppose γ_{gns}(J(G) = p – 1. Since J(G) has at least two non adjacent end vertices u and v. V – {u, v} is a ctd-set of J(G).

Case (ii) Suppose γ_{gns}(J(G) = p – 2 Since <V-D> has no isolate, D is a ctd-set of J(G).

Hence from case (i) and (ii) Theorem holds.

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