A STUDY ON MULTI FUZZY GRAPH

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ABSTRACT: In this paper, some properties of multi fuzzy graph are studied and proved. Fuzzy graph is the generalization of the crisp graph and multi fuzzy graph is the generalization of fuzzy graph. A new structure of a multi fuzzy graph is introduced.

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KEY WORDS: Multi fuzzy subset, multi fuzzy relation, multi fuzzy graph, Level set, Degree of multi fuzzy vertex, order of the multi fuzzy graph, size of the multi fuzzy graph, multi fuzzy regular graph, multi fuzzy complete graph.

1.INTRODUCTION:

In 1965, Zadeh [11] introduced the notion of fuzzy set as a method of presenting uncertainty. Since complete information in science and technology is not always available. Thus we need mathematical models to handle various types of systems containing elements of uncertainty. After that Rosenfeld [9] introduced fuzzy graphs. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graph. It has numerous applications to problems in computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc. NagoorGani.A [7, 8] introduced a fuzzy graph and regular fuzzy graph. Multi fuzzy set was introduced by Sabu Sebastian, T.V. Ramakrishnan [10]. After that the fuzzy sets have been generalized with fuzzy loop and fuzzy multiple edges, this type of concepts was introduced by K. Arjunan and C. Subramani [1,2]. In this paper a new structure is introduced that is multi fuzzy graph and some results of multi fuzzy graph are stated and proved.

2. PRELIMINARIES:

2.1[8] Definition. Let X be any nonempty set. A mapping $M: X \to [0,1]$ is called a fuzzy subset of X.

2.2 Example. A fuzzy subset A={ (a, 0.4), (b, 0.7), (c, 0.2) }of a set X = {a, b, c}.

2.3[7] Definition. A **multi fuzzy subset** A of a set X is defined as an object of the form $A = \{ \langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to [0, 1]$ for all i. Here A is called multi fuzzy subset of X with dimension n. It is denoted as

$A = \langle A_1, A_2, A_3, \ldots, A_n \rangle$.

2.4 Example. Let X = { a, b, c } be a set. Then A = {⟨a, 0.4, 0.3, 0.7⟩, ⟨b, 0.2, 0.7, 0.8⟩, ⟨ c, 0.4, 0.1, 0.5⟩ } is a multi fuzzy subset of X with the dimension three.

2.5[7] Definition. Let A and B be any two multi fuzzy subsets of a set X with dimension n. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $A_i(x) \leq B_i(x)$ for all i and for all $x \in X$.

(ii) A = B if and only if $A_i(x) = B_i(x)$ for all i and for all $x \in X$.

(iii) $A^c = 1-A = \langle 1-A_1, 1-A_2, 1-A_3, \ldots, 1-A_n \rangle$, $A_i^c(x) = 1-A_i(x)$ for all i, for all $x \in X$.

(iv) $A \cap B = \{\langle x, \min\{A_1(x), B_1(x)\}, \min\{A_2(x), B_2(x)\}, \ldots, \min\{A_n(x), B_n(x)\}\rangle / x \in X\}$.

(v) $A \cup B = \{\langle x, \max\{A_1(x), B_1(x)\}, \max\{A_2(x), B_2(x)\}, \ldots, \max\{A_n(x), B_n(x)\}\rangle / x \in X\}$.

2.6 Example. A= { < a, 0.6, 0.2, 0.4 >, < b, 0.3, 0.4, 0.5 >, < c, 0.2, 0.4, 0.6 > } and B = { < a, 0.5, 0.3, 0.8 >, < b, 0.1, 0.3, 0.7>, < c, 0.2, 0.1, 0.6 > } are multi fuzzy subsets of X = { a, b, c }. Then

(i) $A \cap B$ = { < a, 0.5, 0.2, 0.4 >, < b, 0.1, 0.3, 0.5 >, < c, 0.2, 0.1, 0.6 > }.

(ii) $A \cup B$ = { < a, 0.6, 0.3, 0.8 >, < b, 0.3, 0.4, 0.7 >, < c, 0.2, 0.4, 0.6> }.

(iii) A^C = { < a, 0.4, 0.8, 0.6 >, < b, 0.7, 0.6, 0.5 >, < c, 0.8, 0.6, 0.4 > }.

2.7 Definition. Let M be a multi fuzzy subset in a set S, the **strongest multi fuzzy relation** on S, that is a multi fuzzy relation V with respect to M given by $V_i(x,y) = \min\{ M_i(x), M_i(y) \}$ for all x and y in S and for all i.

2.8 Definition. Let V be any nonempty set, E be any set and $f: E \longrightarrow V \times V$ be any function. Then $A = \langle A_1, A_2, A_3, \ldots, A_n \rangle$ is a multi fuzzy subset of V, S is a multi fuzzy relation on V with respect to A and $B = \langle B_1, B_2, B_3, \ldots, B_n \rangle$ is a multi fuzzy subset of E such that $B_i(e) \leq \underset{e \in f^{-1}(x,y)}{S_i(x,y)}$ for all i. Then the ordered triple F = (A, B, f) is called a **multi fuzzy graph**, where the elements of A are called **multi fuzzy points** or **multi fuzzy vertices** and the elements of B are called **multi fuzzy lines** or **multi fuzzy edges** of the multi fuzzy graph F. If f(e) = (x, y), then the multi fuzzy points $(x, A_i(x))$, $(y, A_i(y))$ for all i, are called **multi fuzzyadjacent points** and multi fuzzy points $(x, A_i(x))$ for all i, multi fuzzy line $(e, B_i(e))$ for all i, are called **incident** with each other. If two district multi fuzzy lines $(e_1, B_i(e_1))$ and $(e_2, B_i(e_2))$ for all i, are incident with a common multi fuzzy point, then they are called **multi fuzzy adjacent lines.**

2.9 Definition. A multi fuzzy line joining, a multi fuzzy point to itself is called a **multi fuzzy loop**.

2.10 Definition. Let F = (A, B, f) be a multi fuzzy graph. If both multi fuzzy loops and multi fuzzy multiple edges, then the multi fuzzy graph F is called a **multi fuzzy pseudo graph**.

2.11 Definition. F = (A, B, f) is called a **multi fuzzy simple graph** if it has neither multi fuzzy multiple lines nor multi fuzzy loops.

2.12 Example. F = (A, B, f), where $V = \{v_1, v_2, v_3, v_4, v_5\}$, E = {a, b, c, d, e, h, g }and $f: E \longrightarrow V \times V$ is defined by $f(a) = (v_1, v_2)$, $f(b) = (v_2, v_2)$, $f(c) = (v_2, v_3)$, $f(d) = (v_3, v_4)$, $f(e) = (v_3, v_4)$, $f(h) = (v_4, v_5)$, $f(g) = (v_1, v_5)$. A multi fuzzy subset A = {(v_1, 0.2, 0.4, 0.7), (v_2, 0.3, 0.5, 0.8), (v_3, 0.4, 0.6, 0.8), (v_4, 0.2, 0.5, 0.8), (v_5, 0.5, 0.6, 0.8) } of V. A multi fuzzy relation S = { ((v_1, v_1), 0.2, 0.4, 0.7), ((v_1, v_2), 0.2, 0.4, 0.7), ((v_1, v_3), 0.2, 0.4, 0.7), ((v_1, v_4), 0.2, 0.4, 0.7), ((v_1, v_5), 0.2, 0.4, 0.7), ((v_2, v_1), 0.2, 0.4, 0.7), ((v_2, v_2), 0.3, 0.5, 0.8), ((v_2, v_3),0.3, 0.5, 0.8), ((v_2, v_4), 0.2, 0.5, 0.8), ((v_2, v_5), 0.3, 0.5, 0.8), ((v_3, v_1), 0.2, 0.4, 0.7), ((v_3, v_2), 0.3, 0.5, 0.8), ((v_3, v_3) ,0.4, 0.6, 0.8), ((v_3, v_4), 0.2, 0.5, 0.8), ((v_3, v_5), 0.4, 0.6, 0.8), ((v_4, v_1), 0.2, 0.4, 0.7), ((v_4, v_2), 0.2, 0.5, 0.8), ((v_4, v_3), 0.2, 0.5, 0.8), ((v_4, v_4), 0.2, 0.5, 0.8), ((v_4, v_5), 0.2, 0.5, 0.8), ((v_5, v_1),0.2, 0.4, 0.7), ((v_5, v_2), 0.3, 0.5, 0.8), ((v_5, v_3), 0.4, 0.6, 0.8), ((v_5, v_4), 0.2, 0.5, 0.8), ((v_5, v_5), 0.5, 0.6, 0.8) } on V with respect to $A = \langle A_1, A_2, A_3, \ldots, A_n \rangle$ and a multi fuzzy subset B = { (a, 0.2, 0.4, 0.6), (b, 0.2, 0.3, 0.4), (c, 0.3, 0.5, 0.7), (d, 0.2, 0.3, 0.6), (e, 0.1, 0.3, 0.5), (h, 0.2, 0.2, 0.5), (g, 0.2, 0.4, 0.7) }of E.

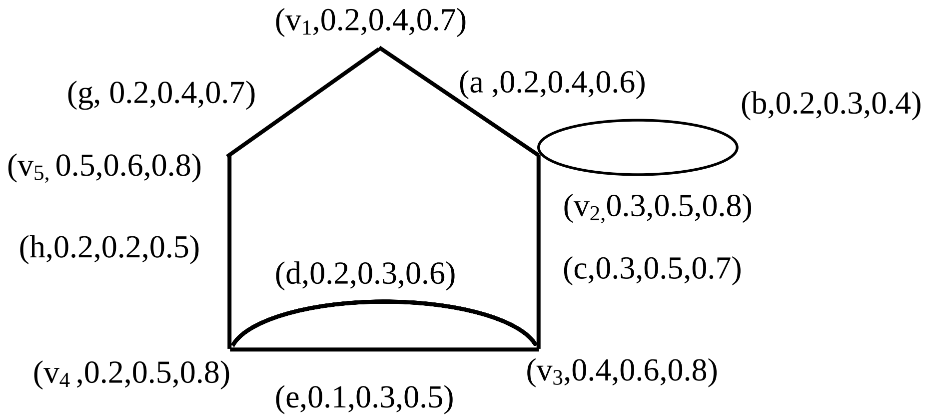

Fig. 2.1

In figure 1.1, (i) (v_1, 0.2, 0.4, 0.7) is a multi fuzzy point. (ii) (a, 0.2, 0.4, 0.6) is a multi fuzzy edge. (iii) (v_1, 0.2, 0.4, 0.7) and (v_2, 0.3 ,0.5, 0.8) are multi fuzzy adjacent points. (iv) (a, 0.2, 0.4, 0.6) join with (v_1, 0.2, 0.4, 0.7) and (v_2, 0.3, 0.5, 0.8) and therefore it is incident with (v_1, 0.2, 0.4, 0.7) and (v_2, 0.3, 0.5, 0.8). (v) (a, 0.2, 0.4, 0.6) and (g, 0.2, 0.4, 0.7) are multi fuzzy adjacent lines. (vi) (b, 0.2, 0.3, 0.4) is a multi fuzzy loop. (vii) (d, 0.2, 0.3, 0.6) and (e, 0.1, 0.3, 0.5) are multi fuzzy multiple edges. (viii) It is not a multi fuzzy simple graph. (ix) It is a multi fuzzy pseudo graph.

2.13 Definition. The multi fuzzy graph H = (C, D, f) is called a **multi fuzzy subgraph** of F = (A, B, f) if $C \subseteq A$ and $D \subseteq B$.

2.14 Definition. The multi fuzzy subgraph H = (C, D, f) is said to be a **multi fuzzy spanning subgraph** of F = (A, B, f) if C= A.

2.15 Definition. The multi fuzzy subgraph H = (C, D, f) is said to be a **multi fuzzy induced sub graph** of F = (A, B, f) if H is the maximal multi fuzzy subgraph of F with multi fuzzy point set C.

2.16 Definition. Let F = (A, B, f) be a multi fuzzy graph with respect to the sets V and E. Let C be a multi fuzzy subset of V, the multi fuzzy subset D of E is defined as $D_i(e) = C_i(u) \cap C_i(v) \cap B_i(e)$ for all i, where f(e) = (u, v) for all e in E. Then H = (C, D, f) is called **multi fuzzy partial subgraph** of F.

2.17 Definition. Let F = (A, B, f) be a multi fuzzy graph. Let $(x, A_i(x)) \in A$ for all i. The multi fuzzy sub graph of F obtained by removing the multi fuzzy point $(x, A_i(x))$ for all i and all the multi fuzzy lines incident with $(x, A_i(x))$ for all i, is called the multi fuzzy subgraph obtained by the removal of the multi fuzzy point $(x, A_i(x))$ for all i and is denoted F–(x, A(x)). Thus if F–(x, A(x)) = (C, D, f) then C = A–{ (x, A(x)) } and D = { (e, B(e)) /(e, B(e)) $\in$ B and (x, A(x)) is not incident with (e, B(e)) }. Clearly F–(x, A(x)) is **multi fuzzy induced subgraph** of F. Let (e, B(e)) $\in$ B. Then F – (e, B(e)) = (A, D, f) is called multi fuzzy sub graph of F obtained by the removal of the multi fuzzy line (e, B(e)), where D = B–{(e, B(e))}. Clearly F– (e, B(e)) is an multi fuzzy spanning sub graph of F which contains all the lines of F except (e, B(e)). Here B(e) means ($B_1(e), B_2(e), \ldots, B_n(e)$) and A(x) means ($A_1(x), A_2(x), \ldots, A_n(x)$).

2.18 Definition. By deleting from a multi fuzzy graph F all multi fuzzy loops and in each collection of multi fuzzy multiple edges all multi fuzzy edge but one multi fuzzy edge in the collection we obtain a multi fuzzy simple spanning subgraph F, called **multifuzzy underling simple graph of F.**

2.19 Example.

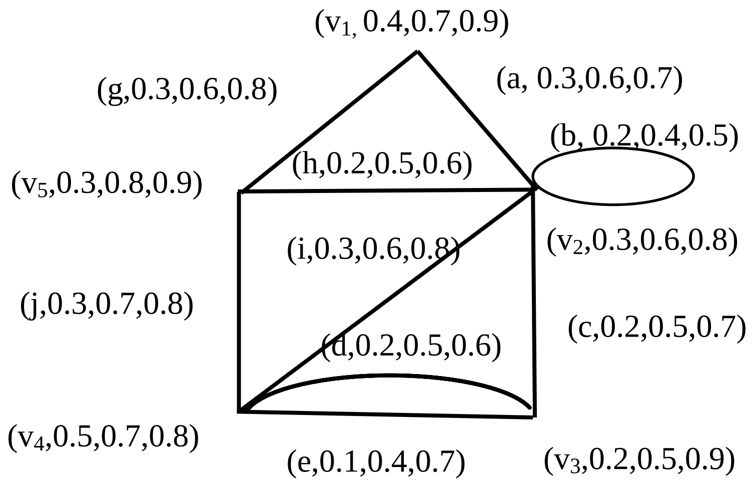

Fig. 2.2 A multi fuzzy pseudo graph F

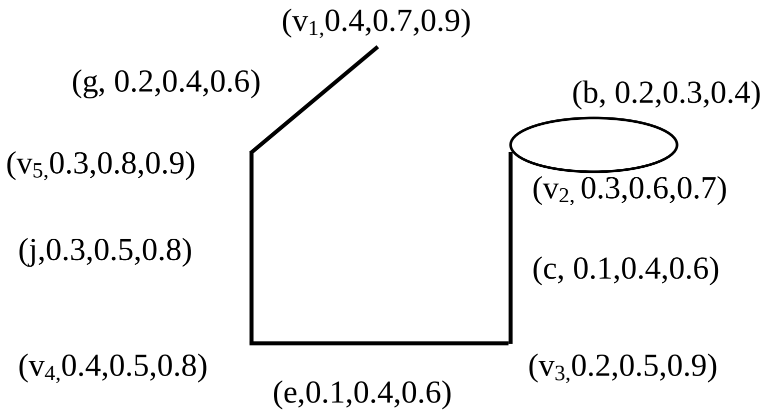

Fig. 2.3 A multi fuzzy subgraph of F

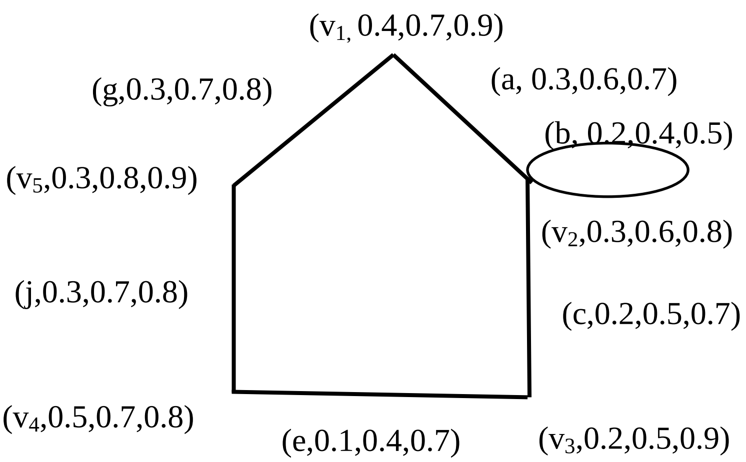

Fig. 2.4 A multi fuzzy spanning subgraph of F

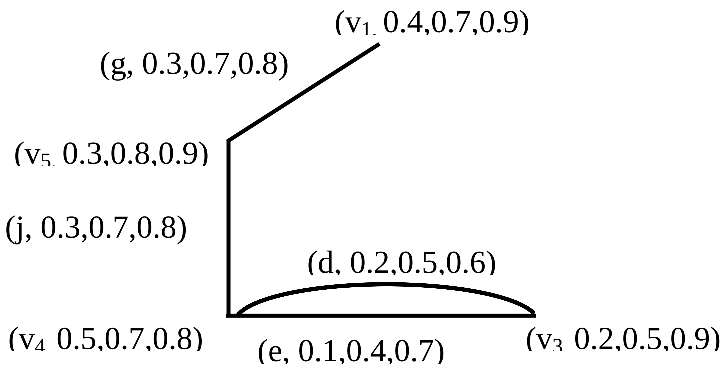

Fig. 2.5

A multi fuzzy subgraph induced by P = {v_1, v_3, v_4, v_5}

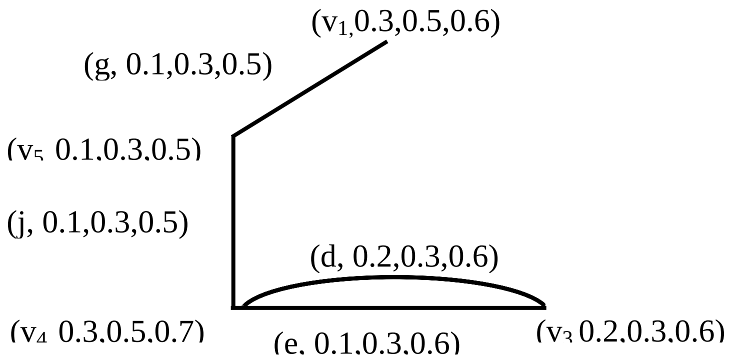

Fig. 2.6

A multi fuzzy partial subgraph induced by C, where C(v_1) = (0.3, 0.5, 0.6), C(v_3) = (0.2, 0.3, 0.6), C(v_4) = (0.3, 0.5, 0.7), C(v_5) = (0.1, 0.3, 0.5)

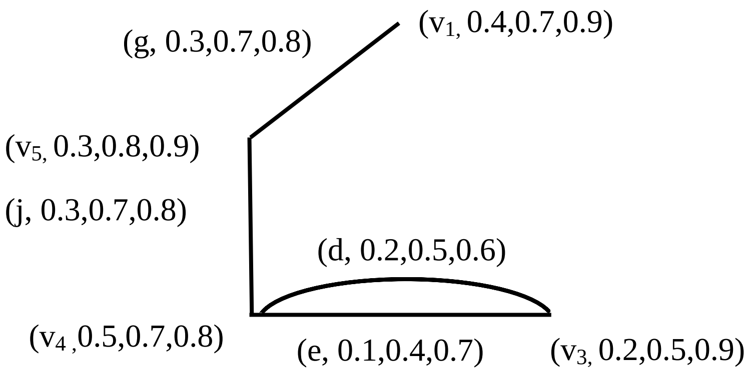

Fig. 2.7 F – (v_2 , 0.3, 0.6, 0.8)

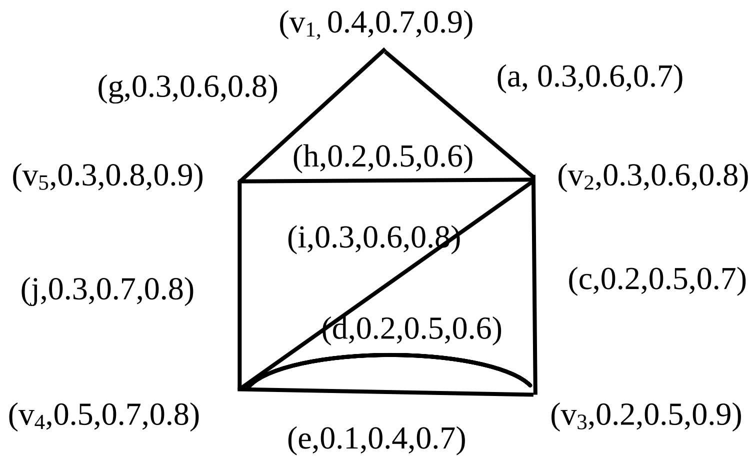

Fig. 2.8 F– (b, 0.2, 0.4, 0.5)

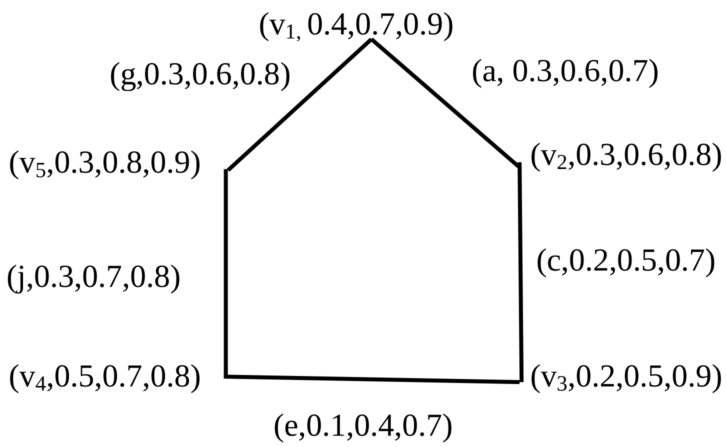

Fig. 2.9 Underling multi fuzzy simple graph of F.

2.20 Definition. Let A be a multi fuzzy subset of X then the **level subset** or α **– cut** of A is $A_\alpha = \{x \in A / A_i(x) \geq \alpha_i\}$, where $\alpha_i \in [0,1]$ for all i. Here α means $(\alpha_1, \alpha_2, ..., \alpha_n)$.

Note: α means $(\alpha_1, \alpha_2, ..., \alpha_n)$ and β means $(\beta_1, \beta_2, ..., \beta_n)$.

2.21 Theorem. Let F = (A, B, f) be a multi fuzzy graph with respect to the set V and E. Let $\alpha, \beta \in [0,1]$ and $\alpha \leq \beta$ .Then (A_β , B_β , f) is a subgraph of (A_α , B_α ,f).

Proof. The proof follows from definition 2.20.

2.22 Theorem. Let F = (A, B, f) be a multi fuzzy graph with respect to the set V and E, the level subsets A_α , B_α of A and B subset of V and E respectively. Then $F_\alpha = (A_\alpha, B_\alpha, f)$ is a subgraph of G = (V, E, f).

Proof. The proof follows from definition 2.20.

2.23 Theorem. Let H = (C, D, f) be a multi fuzzy subgraph of F = (A, B, f) and $\alpha \in [0,1]$. Then $H_\alpha = (C_\alpha, D_\alpha, f)$ is a subgraph of $F_\alpha = (A_\alpha, B_\alpha, f)$.

Proof. Let H = (C, D, f) be a multi fuzzy subgraph of F = (A, B, f) and $\alpha \in [0,1]$. Suppose $u \in C_\alpha \Rightarrow C_i(u) \geq \alpha_i \Rightarrow A_i(u) \geq C_i(u) \geq \alpha_i \Rightarrow A_i(u) \geq \alpha_i$ for all i $\Rightarrow u \in A_\alpha$. Therefore $C_\alpha \leq A_\alpha$. Let $e \in D_\alpha \Rightarrow D_i(e) \geq \alpha_i \Rightarrow B_i(e) \geq D_i(e) \geq \alpha_i \Rightarrow B_i(e) \geq \alpha_i$ for all i. Therefore $D_\alpha \leq B_\alpha$. Thus H_α is a subgraph of F_α.

2.24 Definition. Let A be a multi fuzzy subset of X. Then the **strong level subset** or **strong α-cut** of A is $A_{\alpha+} = \{ x \in A \,/\, A_i(x) > \alpha_i \}$, where $\alpha_i \in [0,1]$.

2.25 Theorem. Let F = (A, B, f) be a multi fuzzy graph with respect to the set V and E. Let $\alpha, \beta \in [0,1]$ and $\alpha \leq \beta$. Then $(A_{\beta+}, B_{\beta+}, f)$ is a subgraph of $(A_{\alpha+}, B_{\alpha+}, f)$.

Proof. The proof follows from definition 2.24 and Theorem 2.23.

2.26 Theorem. Let F = (A, B, f) be a multi fuzzy subgraph with respect to the set V and E, the strong level subsets $A_{\alpha+}$, $B_{\alpha+}$ of A and B subset of V and E respectively. Then $F_{\alpha+} = (A_{\alpha+}, B_{\alpha+}, f)$ is a subgraph of G = (V, E, f).

Proof. The proof follows from definition 2.24 and Theorem 2.25.

2.27 Theorem. Let H = (C, D, f) be a multi fuzzy subgraph of F = (A, B, f) and $\alpha \in [0,1]$. Then $H_{\alpha+} = (C_{\alpha+}, D_{\alpha+}, f)$ is a subgraph of $F_{\alpha+} = (A_{\alpha+}, B_{\alpha+}, f)$.

Proof. Let H = (C, D, f) be a multi fuzzy subgraph of F = (A, B, f) and $\alpha \in [0,1]$. Suppose $u \in C_{\alpha+} \Rightarrow C_i(u) > \alpha_i \Rightarrow A_i(u) \geq C_i(u) > \alpha_i \Rightarrow A_i(u) > \alpha_i$ for all I $\Rightarrow u \in A_{\alpha+}$. Therefore $C_{\alpha+} \leq A_{\alpha+}$. Let $e \in D_{\alpha+} \Rightarrow D_i(e) > \alpha_i \Rightarrow B_i(e) \geq D_i(e) > \alpha_i \Rightarrow B_i(e) > \alpha_i$ for all i. Therefore $D_{\alpha+} \leq B_{\alpha+}$. Hence $H_{\alpha+}$ is a subgraph of $F_{\alpha+}$.

2.28 Theorem. Let F = (A, B, f) be a multi fuzzy subgraph with respect to the set V and E, let $\alpha, \beta \in [0,1]$ and F_α and F_β be two subgraphs of G. Then (i) $F_\alpha \cap F_\beta$ is a subgraph of G. (ii) $F_\alpha \cup F_\beta$ is a subgraph of G.

Proof. Since A_α and A_β are subset of V. Clearly $F_\alpha \cap F_\beta$ is a subgraph of G. Also $F_\alpha \cup F_\beta$ is a subgraph of G.

2.29 Definition. Let F = (A, B, f) be a multi fuzzy graph. Then the **degree of a multi fuzzy vertex** is defined by $d(v) = \sum_{e \in f^{-1}(u,v)} B(e) + 2 \sum_{e \in f^{-1}(v,v)} B(e)$. Where B(e) means ($B_1(e), B_2(e), \ldots, B_n(e)$) and d(v) means ($d_1(v), d_2(v), \ldots, d_n(v)$).

2.30 Definition. The **minimum degree** of the multi fuzzy graph F = (A, B, f) is $\delta(F) = \wedge\{d(v)/v \in V\}$ and the **maximum degree** of F is $\Delta(F) = \vee\{d(v)/v \in V\}$.

2.31 Definition. Let F = (A, B, f) be a multi fuzzy graph. Then the **order of multi fuzzy graph** F is defined to be $o(F) = \sum_{v \in V} A(v)$, where A(v) means ($A_1(v), A_2(v), \ldots, A_n(v)$).

2.32 Definition. Let F = (A, B, f) be a multi fuzzy graph. Then the **size of the multi fuzzy graph F** is defined to be $S(F) = \sum_{e \in f^{-1}(u,v)} B(e)$. Where B(e) means ($B_1$(e), B_2(e), ..., B_n(e)).

2.33 Example.

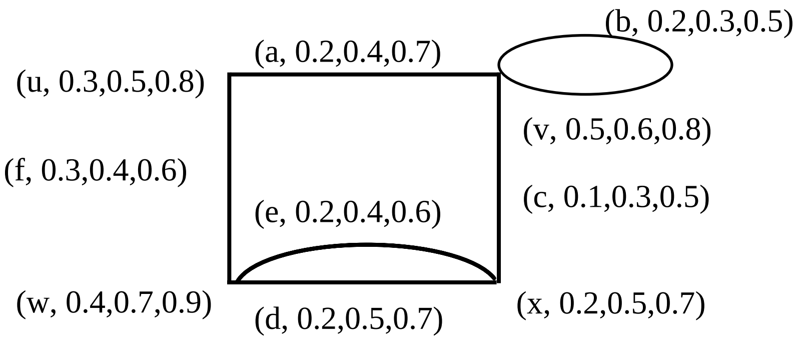

Fig.2.10 Multi fuzzy graph F

Here d(u) = (0.5, 0.8, 1.3), d(v) = (0.7, 1.3, 2.2), d(w) = (0.7, 1.3, 1.9), d(x) = (0.5, 1.2, 1.8), δ(F) = (0.5, 0.8, 1.3), Δ(F) = (0.7, 1.3, 2.2), o(F) = (1.4, 2.3, 3.2), S(F) = (1.2, 2.3, 3.6).

2.34 Remark. 0 ≤ δ(F) ≤ Δ(F), where 0 means (0, 0,...,0).

2.35 Theorem. The sum of the degree of all multi fuzzy vertices in a multi fuzzy graph is equal to twice the sum of the membership value of all multi fuzzy edges. i.e., $\sum_{v \in V} d(v) = 2S(F)$.

Proof. Let F = (A, B, f) be a multi fuzzy graph with respect to the set Vand E. Since degree of a multi fuzzy vertex denote sum of the membership values of all multi fuzzy edges incident on it. Each multi fuzzy edges of F is incident with two multi fuzzy vertices. Hence membership value of each multi fuzzy edge contributes two to the sum of degrees of multi fuzzy vertices. Hence the sum of the degree of all multi fuzzy vertices in a multi fuzzy graph is equal to twice the sum of the membership value of all multi fuzzy edges. i.e., $\sum_{v \in V} d(v) = 2S(F)$.

2.36 Theorem. Let F be any multi fuzzy graph and P be the number of multi fuzzy vertices. Then $\delta(F) \le \frac{2S(F)}{P} \le \Delta(F)$.

Proof. Suppose F = (A, B, f) any multi fuzzy graph with P–vertices. If every multi fuzzy vertex has degree δ, then $\sum_{v \in V} d(v) = \sum_{v \in V} \delta = P\delta$. If every multi fuzzy vertex has degree Δ, then $\sum_{v \in V} d(v) = \sum_{v \in V} \Delta = P\Delta$. But $\sum_{v \in V} \delta \le \sum_{v \in V} d(v) \le \sum_{v \in V} \Delta$

$\Rightarrow P\delta \le 2S(F) \le P\Delta$. Hence $\delta(F) \le \frac{2S(F)}{P} \le \Delta(F)$.

2.37 Theorem. Let F = (A, B, f) be a multi fuzzy graph with number of multi fuzzy vertices n, all of whose multi fuzzy vertices have degree s or t. If F has p-multi fuzzy vertices of degree s and n–p multi fuzzy vertices of degree t then $S(F) = \frac{p(s-t)+nt}{2}$.

Proof. Let V_1 be the set of all multi fuzzy vertices with degree s. Let V_2 be the set of all multi fuzzy vertices with degree t.

Then $\sum_{v\in V} d(v) = \sum_{v\in V_1} d(v) + \sum_{v\in V_2} d(v)$ which implies that 2S(F) = ps + (n–p)t which implies that S(F) = $\frac{p(s-t)+nt}{2}$.

3. MULTI FUZZY REGULAR GRAPH:

3.1 Definition. A multi fuzzy graph F = (A, B, f) is called **multi fuzzy regular graph** if d(v) = k for all v in V, where d(v) means ($d_1(v), d_2(v), \ldots, d_n(v)$) and k means ($k_1, k_2, \ldots, k_n$).

3.2 Remark. F is a **multi fuzzy k-regular graph** if and only if δ = Δ = k.

3.3 Example.

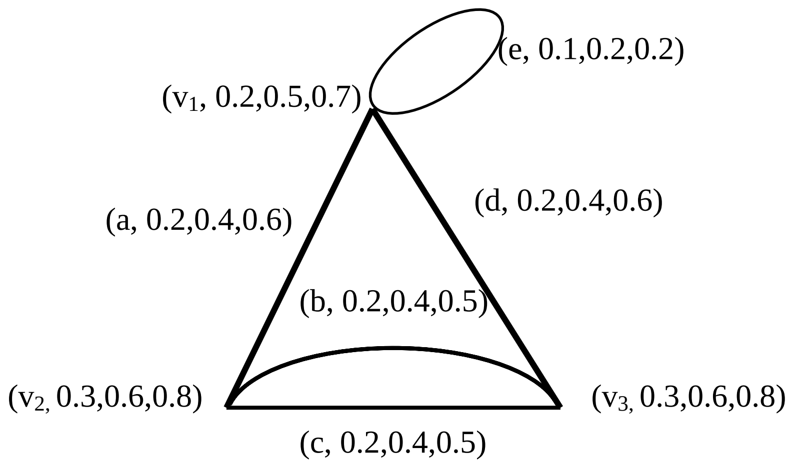

Fig.3.1

Here $d(v_1)$ = (0.6, 1.2, 1.6), $d(v_2)$ = (0.6, 1.2, 1.6), $d(v_3)$ = (0.6, 1.2, 1.6), δ = (0.6, 1.2, 1.6), Δ = (0.6, 1.2, 1.6). Clearly it is a multi fuzzy (0.6, 1.2, 1.6)- regular graph.

3.4 Definition. A multi fuzzy graph F = (A, B, f) is called a **multi fuzzy complete graph** if every pair of distinct multi fuzzy vertices are multi fuzzy adjacent and $B_i(e) = \underset{e\in f^{-1}(x,y)}{s_i}(x,y)$ for all x, y in V and for all i.

3.5Definition. A multi fuzzy graph F = (A, B, f) is a **multi fuzzy strong graph** if $B_i(e) = \underset{e\in f^{-1}(x,y)}{s_i}(x,y)$ for all e in E and for all i.

3.6 Example.

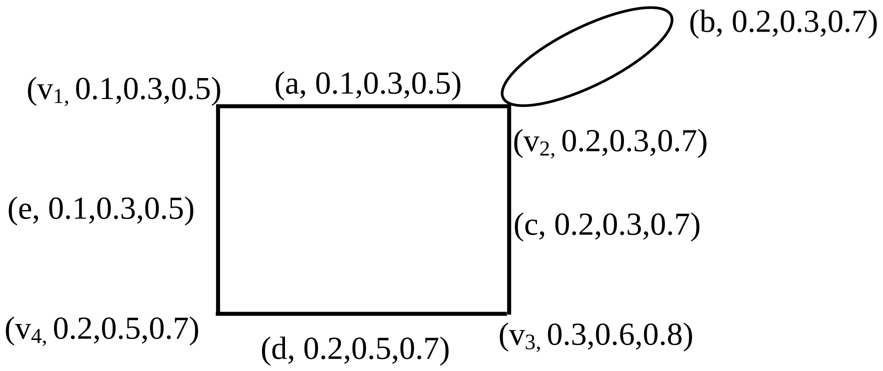

Fig. 3.2 A multi fuzzy strong graph

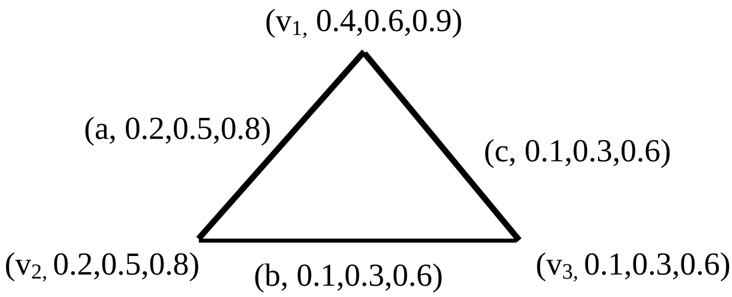

Fig. 3.3 A multi fuzzy complete graph

3.7 Remark. Every multi fuzzy complete graph is a multi fuzzy strong graph. A multi fuzzy strong graph need not be multi fuzzy complete graph from the fig.3.2.

3.8 Theorem. If F is a multi fuzzy k-regular graph with p-multi fuzzy vertices. Then $S(F) = \frac{pk}{2}$.

Proof. Given that the multi fuzzy graph is a multi fuzzy k-regular graph, so d(v) = k for all v in V. Here there are p-multi fuzzy vertices, so $\sum_{v \in V} d(v) = \sum_{v \in V} k$ = pk which implies that $S(F) = \frac{pk}{2}$.

3.9 Remark. In a crisp graph theory any complete graph is regular. But in this multi fuzzy graph, every multi fuzzy complete graph need not be multi fuzzy regular graph. In fig. 3.3, it is a multi fuzzy complete graph but not a multi fuzzy regular graph since $d(v_1)=(0.3, 0.8, 1.4)$, $d(v_2)=(0.3, 0.8, 1.4)$, $d(v_3)=(0.2, 0.6, 1.2)$.

3.10 Theorem. Let F = (A, B, f) be multi fuzzy complete graph and A is constant function. Then F is a multi fuzzy regular graph.

Proof. Since A is a constant function, so A(v) = k (say) for all v in V and F is a multi fuzzy complete graph, so $B_i(e) = \underset{e \in f^{-1}(x,y)}{s_i}(x, y)$ for all x and y in V and for all i and x≠y. Therefore membership values of all multi fuzzy edges are k.

Hence d(v) = (p–1) k for all v in V.

3.11 Remark. Let F = (A, B, f) be multi fuzzy complete graph with p-multi fuzzy vertices and A(v) = k for all v in V. Then F is a multi fuzzy (p–1)k -regular graph.

3.12 Theorem. If F = (A, B, f) is multi fuzzy complete graph with p-multi fuzzy vertices and A is constant function then sum of the membership values of all multi fuzzy edges is $\frac{p(p-1)}{2}$A(v) for all v in V. i.e., S(F) = ${}^pC_2$ A(v), for all v in V.

Proof. Suppose F is a multi fuzzy complete graph and A is a constant function.

Let A(v) = k for all v in V and d(v) = (p–1) k for all v in V. Then $\sum_{v\in V} d(v) = \sum_{v\in V}(p-1)k$ which implies that 2S(F) = p(p–1) k. Hence S(F) = $\frac{p(p-1)}{2}$ k.

i.e., S(F) = ${}^pC_2$ A(v) for all v in V.

3.13 Definition. Let F = (A, B, f) be a multi fuzzy graph. The **total degree of multi fuzzy vertex v** is defined by $d_T(v)$ = $\sum_{e_i\in f^{-1}(u_i,v)} B(e_i) + 2\sum_{e_i\in f^{-1}(v,v)} B(e_i) + A(v)$ = d(v) + A(v) for all v in V.

3.14 Definition. A fuzzy graph F is multi **fuzzy k-totally regular graph** if each vertex of F has the same total degree k.

3.15 Example.

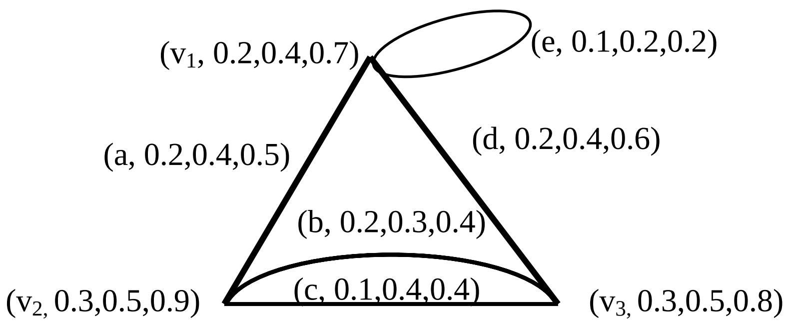

Fig .3.4

Here $d_T(v_1)$ = (0.8, 1.6, 2.2), $d_T(v_2)$ = (0.8, 1.6, 2.2), $d_T(v_3)$ = (0.8, 1.6, 2.2), it is multi fuzzy (0.8, 1.6, 2.2)-totally regular graph.

3.16 Example. Fig 3.1 it is a multi fuzzy regular graph, but it is not a multi fuzzy totally regular graph since $d_T(v_1)$ = (0.8, 1.7, 2.3), $d_T(v_2)$ = (0.9, 1.8, 2.4) and $d_T(v_1) \neq d_T(v_2)$.

3.17 Example. Fig 3.4, it is a multi fuzzy totally regular graph but it is not a multi fuzzy regular graph since $d(v_1)$ = (0.6, 1.2, 1.5), $d(v_2)$ = (0.5, 1.1, 1.4) and $d(v_1) \neq d(v_2)$.

3.18 Example.

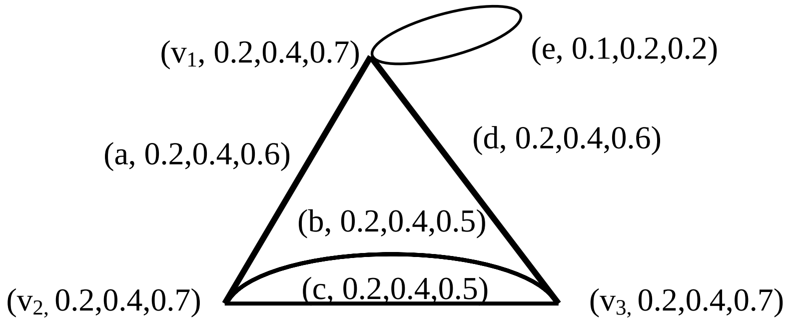

Fig.3.5

Here $d(v_i) = (0.6, 1.2, 1.6)$ for all i, $d_T(v_i) = (0.8, 1.6, 2.3)$ for all i. It is both multi fuzzy regular graph and multi fuzzy totally regular graph.

3.19 Example.

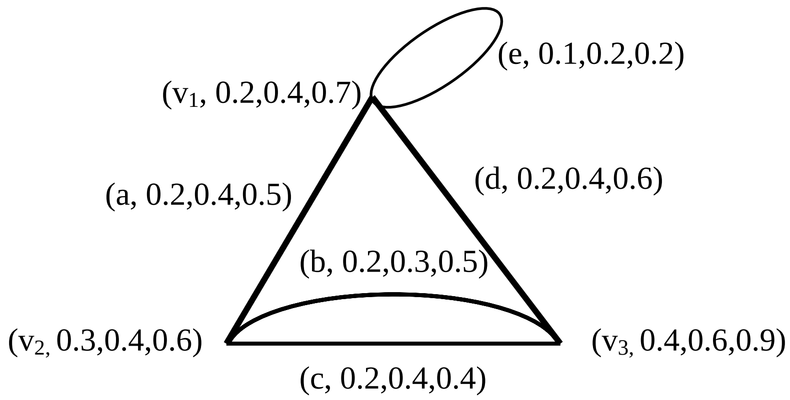

Fig.3.6

Here $d(v_1) = (0.6, 1.2, 1.5)$, $d(v_2) = (0.6, 1.1, 1.4)$, $d(v_3) = (0.6, 1.1, 1.5)$, $d_T(v_1) = (0.8, 1.6, 2.2)$, $d_T(v_2) = (0.9, 1.5, 2.0)$, $d_T(v_3) = (1.0, 1.7, 2.4)$, it is neither multi fuzzy regular graph nor multi fuzzy totally regular graph.

3.20 Theorem. Let F= (A, B, f) be multi fuzzy complete graph and A is constant function. Then F is a multi fuzzy totally regular graph.

Proof. By theorem 3.10, clearly F is multi fuzzy regular graph. i.e., $d(v) = (p-1)k$ for all v in V. Also given A is constant function. i.e., $A(v) = k$ for all v in V. Then $d_T(v) = d(v)+ A(v) = (p-1)k + k = pk - k + k = pk$ for all v in V. Hence F is multi fuzzy totally regular graph.

3.21 Remark. Let F be a multi fuzzy complete graph with p-multi fuzzy vertices and $A(v) = k$ for all v in V. Then F is a multi fuzzy pk-totally regular graph.

3.22 Theorem. Let F = (A, B, f) be a multi fuzzy regular graph. Then H = (C, B, f) is a multi fuzzy totally regular graph if C(v) $= \sum_{i=1}^{n} A(V_i) \leq 1$ for all v_i in V.

Proof. Assume that F = (A, B, f) is a multi fuzzy k- regular graph. i.e., $d(v_i) = k$ for all v_i in V. Given $C(v) = \sum_{i=1}^{n} A(V_i) \leq 1$ for all v_i in V. Then $C(v_i) = k_1$(say) for all v_i in V and $d_{T(H)}(v_i) = d(v_i) + C(v_i) = k + k_1$ for all v_i in V. Hence H is multi fuzzy totally regular graph.

3.23 Theorem. Let F = (A, B, f) be a multi fuzzy graph and A is a constant function (ie. A(v) = c (say) for all $v \in V$). Then F is multi fuzzy k-regular graph if and only if F is multi fuzzy (k + c)-totally regular graph.

Proof. Assume that F is a multi fuzzy k-regular graph and A(v) = c for all v in V, so d(v) = k for all v in V. Then $d_T(v) = d(v) + A(v) = k + c$ for all v in V. Hence F is multi fuzzy (k + c)-totally regular graph. Conversely, Assume that F is multi fuzzy (k + c)-totally regular graph. ie., $d_T(v) = k + c$ for all v in V which implies that d(v) + A(v) = k + c for all v in V implies that A(v) = c for all v in V implies that d(v)+c = k + c for all v in V. Therefore d(v) = k for all v in V. Hence F is multi fuzzy k-regular graph.

3.24 Theorem. If F = (A, B, f) is both multi fuzzy regular graph and multi fuzzy totally regular graph then A is a constant function.

Proof. Assume that F is a both multi fuzzy regular graph and multi fuzzy totally regular graph.

Suppose that A is not constant function. Then $A(u) \neq A(v)$ for some u, v in V. Since F is a multi fuzzy k-regular graph. Then d(u) = d(v) = k. Then $d_T(u) \neq d_T(v)$ which is a contradiction to our assumption. Hence A is a constant function.

3.25 Remark. Converse of the above theorem need not be true.

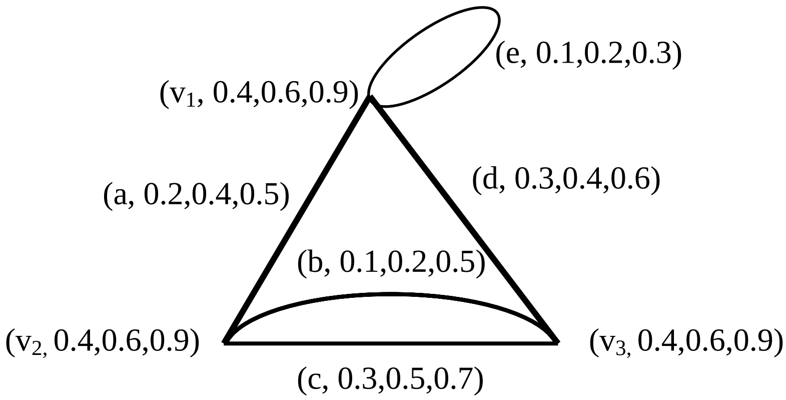

Fig.3.7

Here $A(v_i) = (0.4, 0.6, 0.9)$ for all i, $d(v_1) = (0.7, 1.2, 1.7)$, $d(v_2) = (0.6, 1.1, 1.7)$, $d(v_3) = (0.7, 1.1, 1.8)$, $d_T(v_1) = (1.1, 1.8, 2.6)$, $d_T(v_2) = (1.0, 1.7, 2.6)$, $d_T(v_3) = (1.1, 1.7, 2.7)$. Hence F is neither multi fuzzy regular graph nor multi fuzzy totally regular graph.

3.27 Theorem. If F = (A, B, f) is a multi fuzzy c-totally regular graph with p-multi fuzzy vertices. Then $S(F) = \frac{pc - o(F)}{2}$.

Proof. Assume that F is a multi fuzzy c-totally regular graph with p-multi fuzzy vertices. Then $d_T(v) = c$ for all v in V implies that d(v) + A(v) = c for all v in V which implies that $\sum d(v) + \sum A(v) = \sum c$ for all v in V which implies that 2S(F) + o(F) = pc. Hence $S(F) = \frac{pc - o(F)}{2}$

3.27 Theorem. If F = (A, B, f) is both multi fuzzy k-regular graph and multi fuzzy c-totally regular graph with p-multi fuzzy vertices. Then o(F) = p(c–k).

Proof. Assume that F is multi fuzzy k-regular graph with p-multi fuzzy vertices. Then 2S(F) = pk. By theorem 3.26, 2S(F) + o(F) = pc implies that o(F) = p(c–k).

4. CONCLUSION:

Multi fuzzy graph is a generalized form of fuzzy graph that is fuzzy graph is a particular case of multi fuzzy graph. In this paper, some basic definitions with examples are given and multi fuzzy regular, multi fuzzy totally regular is also defined. Using these definitions, some theorems are derived. Based on this paper, we can find new results and theorems.

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