International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 06 Issue: 11 | Nov 2019 www.irjet.net p-ISSN: 2395-0072

Nano Generalized Delta Semi Closed Sets in Nano Topological Spaces

Umadevi I Neeli[1], Roopa S Algur[2]

[1]Associate Professor, Department of Mathematics, BVB College of Engineering and Technology, Hubballi Karnataka, India 580031
[2]Department of Mathematics, BVB College of Engineering and Technology, Hubballi, Karnataka, India 580031

Abstract -*The aim of this paper is to introduce a new class of sets called Nano generalized delta semi closed sets and to study some of their properties and relationships. Several examples are provided to illustrate the behavior of new set*

Key Words: *Nano topology, Nano open sets, Nano closure, Nano semiopen set, Nano delta open sets.*

1. INTRODUCTION

The concept of generalized closed sets as a generalization of closed sets in Topological Spaces was introduced by Levine[4] in 1970.This concept was found to be useful and many results in general topology were improved. One of the generalizations of closed set is generalized δ_semiclosed sets which was defined byS.S.Benchalli and UmadeviI. Neeli [5], investigated some of its applications andrelated topological properties regarding generalized δ_semiclosed sets.Lellis Thivagar[2] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nanotopological space are called Nano open sets .He has also defined Nano closed sets, Nano-interior and Nano closure of a set. Bhuvaneswari K et.al[1] introduced and investigated Nano generalized_closed sets in Nanotopologicalspaces. The purpose of this paper is to introduce the concept of Nano generalized δ semi_closed sets(briefly$Ng\delta s_closed$) *and study their basic properties in Nano topological spaces.*

2.1 PRELIMANARIES

Definition2.1[2] Let U be a non-empty set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let$X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by

$L_R(X) = \underbrace{U}_{x\in U} \{R(x): R(x) \subseteq X\}$ Where $R(x)$ denotes the equivalence class determined by $x \in U$

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by

$$U_R(X) = \underbrace{U}_{x\in U} \{R(x): R(x) \cap X \neq \emptyset\}$$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. This is $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2[2] If (U, R) is an approximation space and, $Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$

(iii) $L_R(U) = U_R(U) = U$

(iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(vi)$L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(viii) $L_R(X) \subseteq L_R(Y) and U_R(X) \subseteq U_R(Y)$ Whenever $X \subseteq Y$

(ix)$U_R(X^c) = [L_R(X)]^c$and$L_R(X^c) = [U_R(X)]^c$

(x) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$

(xi) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition2.3[2] Let$U$ be a non-empty, finite universe of objects and R be an equivalence relation on$U$. Let$X \subseteq U$. Let $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U,called as the Nano topology with respect to X. Elements of the Nano topology are known as the Nano open sets in U and $(U, \tau_R(X))$ is called the Nano topological space.Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

Let U be the universe, R be an equivalence relation on U and$\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$where$X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

$(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence Relation on U and U/R denotes the family of Equivalence class of U by R.

Definition 2.4[2] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by $Nint(A)$.That is $Nint(A)$ is the largest Nano open subset of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing A and is denoted by $NCl(A)$.That is $NCl(A)$ is the smallest Nano closed set containing A.

Definition2.5 Let $(U, \tau_R(X))$ be a Nano topological space with respect to X where $X \subseteq U$.Then P is said to be

(i) Nano semiopen [3] if $P \subseteq NCl(Nint(P))$

(ii) Nano regular open [3] if $P = Nint(NCl(P))$

(iii) Nano α open [3] if $P \subseteq Nint(NCl(Nint(P)))$

Definition2.6[3]The Nano delta interior of a subset A of U is the union of all Nano regular open sets of U contained in A and is denoted by $N\delta int(A)$ or a subset A is called Nano δ_open if $A = \delta int(A)$.

3. NANO GENERALIZED δSEMI CLOSED SET

Definition3.1 A subset P of $(U, \tau_R(X))$ is called Nano generalized δ_semiclosed set (briefly $Ng\delta s\_closed$) if $NsCl(P) \subseteq Q$, whenever $P \subseteq Q$ and Q is $N\delta\_open$ set in U.

Example3.2 U={a,b,c,d} with U/R={{a},{c},{b,d}} and X={a,b}.$\tau_R(X) = \{U, \emptyset, \{a\}, \{a,b,d\}, \{b,d\}\}$,then $Ng\delta s\_closed$={U, ∅ , {a}, {b}, {c}, {d}, {a,c}, {b,c}, {b,d} ,{c,d}, {a,b,c}, {a,c,d}, {b,c,d}}

Remark3.3 Intersection of two $Ng\delta s\_closed$ set is again $Ng\delta s\_closed$. But the union of two $Ng\delta s\_closed$ sets need not be $Ng\delta s\_closed$.

Example 3.4 U= {a,b,c,d} with U/R={{a},{c},{b,d}} and X={a,b},$\tau_R(X) = \{U, \emptyset, \{a\}, \{a,b,d\}, \{b,d\}\}$then Ng$\delta$s_closed={U, ∅ , {a}, {b}, {c}, {d}, {a,c}, {b,c}, {b,d} ,{c,d}, {a,b,c}, {a,c,d}, {b,c,d}} . Here {a} and {b} are $Ng\delta s_closed$sets but{a,b} is not $Ng\delta s\_closed$ set.

Theorem3.5A subset P of (U,$\tau_R(X)$) is $Ng\delta s\_closed$ set if $NsCl(P) - P$ does not contain any non empty $N\delta\_closed$ set.

Proof: Suppose P is $Ng\delta s_closed$set and Q be a $N\delta\_closed$ set in U such that $Q \subseteq NsCl(P) - P$.This implies $Q \subseteq NsCl(P)$and $Q \subseteq U - P$i.e $P \subseteq U - Q$.This implies U-Q *is* $N\delta\_open$ set containing a $Ng\delta s\_closed$ set P.$NsCl(P) \subseteq U - Q \Longrightarrow Q \subseteq U - NsCl(P)$.Thus $Q \subseteq NsCl\,(P) \cap (U - NsCl\,(P)) = \emptyset$. This shows $Q = \emptyset$.

Remark 3.6 The converse of the above theorem need not be true

Example:**3.7** Let U={a,b,c,d} with U/R={{a}, {c}, {b,d}}, X={a,b} then$\tau_R(X) = \{U, \emptyset, \{a\}, \{a,b,d\}, \{b,d\}\}$ Let P={a,b,d} ⊆ U , $NsCl(P) - P$=U-{a,b,d} ={c} which does not contain any non empty $N\delta\_closed$ set. Therefore P is not $Ng\delta s\_closed$.

Theorem3.8 A $Ng\delta s\_closed$ set P is Nano semiclosed if and only if $NsCl(P) - P$ is $N\delta\_closed$.

Proof: Let P be a $Ng\delta s\_closed$ set and also Nano semiclosed in U then $NsCl(P) = P$implies $NsCl(P) - P = \emptyset$,which is $N\delta\_closed$ set.

Conversely $NsCl(P) - P$ is $N\delta\_closedset$ and P is $Ng\delta s_closed$set,$NsCl(P) - P$ is $N\delta\_closed$ subset of itself and by theorem 3.5$NsCl(P) - P = \emptyset$.i.e$NsCl\,(P) = P$ this gives P is Nanosemiclosed.

Theorem3.9 If P is a $Ng\delta s\_closed$ set and $P \subseteq Q \subseteq NsCl\,(P)$, then Q is $Ng\delta s\_closed$ set.

Proof: Let $Q \subseteq O$ and O be $N\delta\_open$ in $(U, \tau_R(X))$ since $P \subseteq Q \Longrightarrow P \subseteq O$ and P is $Ng\delta s\_closed$ set which implies $NsCl\,(P) \subseteq O$. By hypothesis $Q \subseteq NsCl\,(Q) \subseteq O$, which implies $NsCl(Q) \subseteq NsCl(P) \subseteq O$ this implies $NsCl(Q) \subseteq O$ .Therefore Q is $Ng\delta s\_closed$ set.

Definition 3.10 A set which is both Nano semiopen and Nano semiclosed is Nano semi regular.

Theorem 3.11 If P is both $N\delta\_open$ and $Ng\delta$s_closed, then P is Nano semiclosed and hence Nano semi regular open .

Proof: Suppose P is both $N\delta\_open$ and $Ng\delta s_closed$since $P \subseteq P \Longrightarrow NsCl\,(P) \subseteq P$.But$P \subseteq NsCl\,(P)$ is always true.So $NsCl(P) = P$ this tells P is Nano semiclosed. Since P is $N\delta\_open$ and every $N\delta\_open$ is Nano semiopen. Therefore P is both Nano semiopen and Nano semiclosed, hence P is Nano semi regular.

Theorem 3.12 For a space U the following are equivalent

(i) Every $N\delta\_open$ set of U is Nano semiclosed.

(ii) Every subset of U is $Ng\delta s\_closed$.

Proof: (i)⟹ (ii) Suppose (i) holds .Let P be any subset of U and Q be a $N\delta\_open$ set such that P⊆Q this implies $NsCl(P) \subseteq NsCl\,(Q)$.By hypothesis Q is Nano semiclosed, this gives $NsCl(Q) = Q$. Hence $NsCl(P) \subseteq Q$ therefore P is $Ng\delta\_closed$ set in U.

(ii)⟹(i) suppose (ii) holds and $Q \subseteq U$ is $N\delta\_open$ set by (ii) Q is Ngs_closed.Therefore$NsCl(Q) \subseteq Q$But $Q \subseteq NsCl\,(Q)$ is always true. Therefore$NsCl\,(Q) = Q$. This shows that, Q is Nano semiclosed.

Theorem3.13 For any$x \in U$, the set U-{x} is $Ng\delta s\_closed$ set or$N\delta_open$.

Proof: Suppose for any x∈U, U-{x} is not $N\delta\_open$ .Then U is the only $N\delta$_openset containing U-{x}. Therefore, $NsCl\ (U-\{x\}) \subseteq U$.Hence U-{x} is$Ng\delta s_ closed$set.

Definition 3.14A set A of U is called Nano generalized δ_semiopen (briefly$Ng\delta s_open$) set if its complement U-A or$A^c$is $Ng\delta s\_closed$ in U.

Theorem3.15 A set P is $Ng\delta s\_open$ if and only if $Q \subseteq Nsint(P)$, whenever Q is $N\delta\_closed$ and $Q \subseteq P$.

Proof: Let P be a $Ng\delta s\_open$ set in U. Suppose$Q \subseteq P$, where Q is $N\delta\_closed$ then U-P is $Ng\delta s_closed$set contained in a $N\delta\_open$ set U-Q.This implies$NsCl(U-P) \subseteq U-P$. Therefore,$U-Nsint(P) \subseteq U-P$,which implies $Q \subseteq Nsint(P)$.

Conversely, suppose $Q \subseteq Nsint(P)$,whenever $Q \subseteq P$ and Q is $N\delta\_closed$. Then $U-Nsint(P) \subseteq U-Q$ whenever $U-P \subseteq U-Q$ and $U-Q$ is $N\delta\_open$.This implies $NsCl(U-P) \subseteq U-Q$ whenever $U-P \subseteq U-Q$ and $U-Q$ is $N\delta\_open$ .This shows that U-P is $Ng\delta s\_closed$ in U, hence P is $Ng\delta s\_open$ set in U.

Theorem3.16 If P is $Ng\delta s\_open$ set of space U, then Q=U whenever Q is $N\delta_open$and $Nsint(P)\cup(U-P) \subseteq Q$.

Proof: Let P be a$Ng\delta s_open$ set and Q be a $N\delta\_open$ set in U such that$Nsint(P)\cup(U-P) \subseteq Q$ .Then $U-Q \subseteq U-\big(Nsint(P)\cup(U-P)\big) \subseteq (U-Nsint(P))\cap P$.Thatis $U-Q \subseteq NsCl(U-P)-(U-P)$.Since $U-P$ is $Ng\delta s\_closed$ set and by theorem 3.5,$NsCl(U-P)-(U-P)$ does not contain any non empty $N\delta\_closed$ set which implies $U-Q=\emptyset$.Hence U=Q.

Theorem3.17 If $Nsint(P) \subseteq Q \subseteq P$ and P is $Ng\delta s_open$set, then Q is $Ng\delta s_open$set .

Proof: Let P be a $Ng\delta s\_open$ set and $Nsint(P) \subseteq Q \subseteq P$,implies$U-P \subseteq U-Q \subseteq U-Nsint(P)$That is $U-P \subseteq U-Q \subseteq NsCl(U-P)$.Now $U-P$ is $Ng\delta s\_closed$ set and by theorem 3.9, $U-Q$is $Ng\delta s\_closed$ set in U. This shows that Q is $Ng\delta s\_open$ set.

Definition 3.18 A Space $(U,\tau_R(X))$ is called $Ng\delta sT_{1/2}$ space if every $Ng\delta s_closed$set in it is Nano semiclosed.

Theorem3.19 For a Nano topological space $(U,\tau_R(X))$ the following are equivalent.

(i) U is $Ng\delta sT_{1/2}$ space

(ii) Every singleton set of U is either $N\delta\_closed$ or Nano semiopen.

Proof: (i)⟹(ii)

If {x} is not $N\delta\_closed$ then U-{x} is not $N\delta\_open$ then the only$N\delta_open$ set containing U-{x} is U. Therefore U-{x} is $Ng\delta s\_closed$ set in U.By (i)U-{x} is Nano semiclosed ,which implies {x} is Nano semiopen.

(ii)⟹(i)

Let $P \subseteq U$ be $Ng\delta s\_closed$ set and $x \in NsCl(P)$ then consider the following cases

Case (i) Let {x} be $N\delta\_open$ since $x \in NsCl\ (P)$ then $\{x\} \cap NsCl\ (P) \neq \emptyset$ this implies $x \in P$

Case (ii) Let {x} be$N\delta_closed$. Assume that $x \notin P$ then$x \in NsCl\ (P)-P$, which implies $\{x\} \subseteq NsCl\ (P)-P$ this is not possible according to theorem 3.5 this shows that$x \in P$.

So in both cases $NsCl(P) \subseteq P$ .Since the reverse inclusion is trivial, implies$NsCl(P)=P$ therefore P is Nano semiclosed.

Theorem3.20 (i) Every $N\delta_closed$is $Ng\delta s\_closed$.

(ii) Every Nano closed is $Ng\delta s\_closed$

(iii) Every Nano semiclosed is $Ng\delta s\_closed$

(iv) Every $N\alpha closed$ is $Ng\delta s\_closed$

Remark 3.21 From following example it is clear that converse of the above theorem need not be true

Example 3.22 Let U={a,b,c,d} with U/R={{a}, {c}}, {b,d}}, X={a,b} $\tau_R(X)$={U,∅,{a},{a,b,d},{b,d}}

Nano δ_ closed = {U, ∅, {b,c,d},{c},{a,c}}

Nano closed sets ={ U,∅, {b,c,d},{c}, {a,c}}

Nano semiclosed={U,∅, {a},{c}, {a,c}, {b,d},{b,c,d}}

Nano α closed = {U,∅,{a,c},{b,c,d},{c}}

$Ng\delta s\_closed$ ={U, ∅ , {a}, {b}, {c}, {d}, {a,c}, {b,c}, {b,d} ,{c,d}, {a,b,c}, {a, c, d}, {b, c, d}}

Clearly {b} is $Ng\delta s\_closed$, but it is not $N\delta\_closed$, Nano closed, Nano semiclosed and $N\alpha closed$

REFERENCES

1. Bhuvaneswari. K and K. MythiliGnanapriya, on Nano Generalized Closed Sets in Nano Topological Space International Journal of scientific and Research Publications,Volume 4,2014
2. LellisThivagar .M and Carmel Richard, Note on Nano topological spaces, Communicated
3. LellisThivagar .M and Carmel Richard, On Nano forms of weakely open sets. International Journal of Math.Stat. Innovation, vol 1, (2013), 31-37

4. Levine.NGeneralized Closed sets in Topology,Rend Cire.Math.Palermo,19(2),89-96
5. S.S. Benchalli and Umadevi I. Neeli International Journal of Applied Mathematics Volume 24 ,2011,21-38