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Differential Transform Method for the Vaccination Model of the Cholera Carrier

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Abstract - *Approximately 1.3 billion people are at risk of cholera worldwide. Cholera is a rapidly dehydrating diarrhoeal disease and is spread through faecal contamination of water and food. This study aims to present solution of the deterministic mathematical model of the vaccination model of cholera carrier using differential transform method (DTM). The recurrence DTM relation of the model is generated and plotted. The comparison between differential transform method (DTM) solution and Runge-Kutta (RK4) was performed and tested. The method gives accuracy and the results have shown remarkable performance in the convergence rate and analysis of the model.*

Key Words: Cholera carrier, Convergence rate, Differential transform, Epidemic model, Runge-Kutta, Vaccination

1.0 INTRODUCTION

A preventable devastating disease cholera remains a global health challenge for past 200years. Cholera is a gastrointestinal virulent diarrhoel illness whose ethological agents is the bacterium vibrio cholerae. The bacteria that cause cholera infection is usually transmitted through ingesting water contaminated with human faeces or vomitus [15]. The disease also is transmitted through ingesting contaminated foods (raw, shellfish, vegetables and sea foods). The annual burden of cholera has been estimated at 1.3 to 4.0 million cases and 21 000 to 143 000 deaths worldwide [22].

Currently there are three WHO pre-qualified oral cholera vaccines (OCV): Dukoral, Shanchol, and Euvichol [22]. The vaccines should always be used in conjunction with other cholera prevention and control strategies as it complements the other control measures, and therefore it should be implemented in relevant settings as part of comprehensive cholera control strategies or while the other activities are being developed [23].

According to [6], the differential transformation method (DTM) of Pukhov and Zhou is frequently used as a new method for solving differential equations, in both linear and nonlinear systems. Zhou [24] has proposed differential transform method to study the initial value problems in the electrical circuit for a computation in linear and nonlinear systems. Since that time, the technique has been used to computes equation such as differential algebraic equations, fractional differential equation, Lane-Emden type equations and Schrodinger equations [2,3,8,4,14,16]. Furthermore, the application of DTM were extended to mathematical modeling and simulations [1,10,19,20], in order to observe the behavior of the models convergence.

The related recent work on application of DTM to the study in epidemiology are that of Ibrahim and Ismail [11] proposed a new modification of the DTM to study a SIRC influenza model in which new initial conditions and parameter values were used. Also [4] study the application of differential transform method and variational iteration method in the numerical solution of SIR model, their result shows that both methods are accurate and efficient for computation of ODEs. A new method for computation of epidemics model was discussed by [5] their results are compared with the results obtained from the different numerical method. Benhammouda [7] study the application of the DTM to used 4-5th order Runge Kutta method with degree four interpolant (RKF45) and they obtained the high accuracy results in the numerical solution of the lake system problem. Moon [17] apply DTM to the demonstrated solution of some nonlinear differential equations. Differential Transformation Method (DTM) for Solving SIS and SI Epidemic Models were developed by [8]. There are scarce literatures to study cholera using DTM and the most recent one was found in the work of [12] in which they obtained solution of cholera carrier epidemic model using DTM.

This study is the motivation of the work by [12], their work was extended by incorporating the exposed and vaccinated individuals in the populations, that leads to the development of the new model. However, this paper aimed to study the differential transform method for the solution of the vaccination model of the cholera carrier.

This work was organized into four main sections. Section one briefly provide the general introduction, model formulation, concepts of differential method. Section two provide Solution of the Vaccination Model of a Cholera Carrier, Vaccination Model Solution Using DTM, Numerical Result and Discussion. Section three provide conclusions, recommendations, acknowledgments, references and biography.

1.1 Model Formulation

The following assumptions are considered to design the vaccination model of cholera carrier:

i. Population have equal access to vaccination.

ii. Using combine incidence rates of the form $\beta_1 IS + \beta_1 \eta VS + \frac{\beta_2 B}{1+\tau B}$

iii. re-infection of vaccinated individual may occur.

iv. modification parameter to reduce the shedding rate of vaccinated individuals.

Using mentioned assumptions, the below schematic diagram establishes the interaction between different populations:

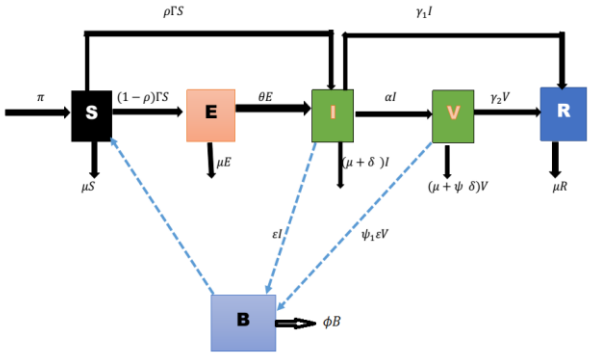

Figure 1: Vaccination Model Diagram

Thus, the model formulation is governed by the following system of nonlinear differential equations:

$$\frac{dS}{dt} = \pi - \left[\beta_1 (I + \eta V) + \frac{\beta_2 B}{1+\tau B}\right] S(t) - \mu S(t) \quad (1)$$

$$\frac{dE}{dt} = (1-\rho)\left[\beta_1 (I + \eta V) + \frac{\beta_2 B}{1+\tau B}\right] S(t) - (\mu + \theta) E(t) \quad (2)$$

$$\frac{dI}{dt} = \rho\left[\beta_1 (I + \eta V) + \frac{\beta_2 B}{1+\tau B}\right] S(t) + \theta E(t) - (\mu + \delta + \alpha + \gamma_1) I(t) \quad (3)$$

$$\frac{dV}{dt} = \alpha I(t) - (\mu + \psi\delta + \gamma_2) V(t) \quad (4)$$

$$\frac{dR}{dt} = \gamma_1 I(t) + \gamma_2 V(t) - \mu R(t) \quad (5)$$

$$\frac{dB}{dt} = \varepsilon I(t) + \varepsilon\psi_1 V(t) - \phi B(t) \quad (6)$$

Subject to the initial conditions

$$\left.\begin{aligned} S(0) = S_0,\ E(0) = E_0,\ I(0) = I_0 \\ V(0) = V_0,\ R(0) = R_0,\ B(0) = B_0 \end{aligned}\right\} \quad (7)$$

Table -1: Description of Variables

Variable	Description
$S(t)$	Susceptible Individuals
$E(t)$	Exposed Individuals
$I(t)$	Infective Individuals
$V(t)$	Vaccinated Individuals
$R(t)$	Removed Individuals
$B(t)$	Pathogen Individuals

Table -2: Description of Vaccination Model

Parameter	Description
π	Recruitment rate
ρ	Progression to infective class
μ	Natural death
β_1	Transmission coefficient associated to infective class
β_2	Transmission coefficient associated to vibrio cholera
ψ	Modification parameter associated with reduced mortality of vaccinated individuals
ψ_1	Modification parameter associated with vaccine failure to reduced shielding rate of vaccinated individuals
τ	Saturated constant
θ	Progression rate from $E(t)$ to $I(t)$ compartment
α	vaccination rate
γ_1	Recovery rate for infectious individuals
γ_2	vaccine recovery rate
ε	Shedding rate for infectious individuals Saturation constant
δ	Cholera induced death rate
ϕ	Decay rate of vibrio cholerae.

1.2 Differential Transform Method (DTM)

Concept of the Differential Transforms Method

With reference to [12].

Given that the function $b(t)$ in Taylor series about a point $t=0$, such that $b(t)=\sum_{i=0}^{k}\frac{t^k}{k!}\left[\frac{d^k b}{dt^k}\right]_{t=0}$, then the differential transform of $b(t)$ is given as, $B(k)=\frac{1}{k!}\left[\frac{d^k b}{dt^k}\right]_{t=0}$ and the inverse differential transform is $b(k)=\sum_{k=0}^{\infty}t^k B(k)$.

Some operational properties of Differential Transform Method

Given two uncorrelated functions $u(t)$ and $v(t)$ with time t, then $U(k)$ and $V(k)$ are the transformed functions corresponding to $u(t)$ and $v(t)$, respectively. Then, the following properties hold:

1 If $b(t)=u(t)\pm v(t)$, *Then* $B(k)=U(k)\pm V(k)$

2 If $b(t)=\alpha u(t)$, *Then* $B(k)=\alpha U(k)$

3 If $b(t)=\frac{du(t)}{dt}$, *Then* $B(k)=(k+1)U(k+1)$

4 If $b(t)=\frac{d^2u(t)}{dt^2}$, *Then* $B(k)=(k+1)(k+2)U(k+2)$

5 If $b(t)=\frac{d^m u(t)}{dt^m}$, *Then* $B(k)=(k+1)(k+2)...(k+m)U(k+m)$

6 If $b(t)=u(t)v(t)$, *Then* $B(k)=\sum_{l=0}^{k}V(l)U(k-l)$

7 If $b(t)=u(t)v(t)$, *Then* $B(k)=\sum_{l=0}^{k}V(l)U(k-l)$

8 If $b(t)=t$, *Then* $B(k)=\delta(k-1)$

9 If $b(t)=t^m$, *Then* $B(k)=\delta(k-m)$, $\delta(k-m)=\begin{cases}1 & if\ k=m\\ 0 & if\ k\neq m\end{cases}$

10 If $b(t)=\exp(\lambda t)$, *Then* $B(k)=\frac{\lambda^k}{k!}$

11 If $b(t)=(1+t)^m$, *Then* $B(k)=\frac{m(m-1)...(m-k+1)}{k!}$

12 If $b(t)=\sin(wt+\alpha)$, *Then* $B(k)=\frac{w^k}{k!}\sin\left(\frac{nk}{2}+\alpha\right)$

2. SOLUTION OF THE VACCINATION MODEL OF A CHOLERA CARRIER

2.1 Vaccination Model Solution using DTM

Using operational properties (1), (2), (3), (6) and (7) of DTM in subsection (1.2) and applying it into the model equation (1-6), the first equation is transformed into

$$(k+1)S(k+1)=\pi\cdot\delta(k,0)-\begin{bmatrix}\beta_1[\sum_{l=0}^{k}[S(l)I(k-l)]]\\ +\eta.\sum_{l=0}^{k}[S(l)V(k-l)]\\ +\beta_2\sum_{l=0}^{k}[S(l)P(k-l)]\end{bmatrix}-\mu S(k)$$

Dividing through by (k+1) so as to make S(k+1) the subject of the expression, then

$$S(k+1)=\frac{1}{k+1}\left[\pi\cdot\delta(k,0)-\begin{bmatrix}\beta_1[\sum_{l=0}^{k}[S(l)I(k-l)]]+\\ \eta.\sum_{l=0}^{k}[S(l)V(k-l)]\\ +\beta_2\sum_{l=0}^{k}[S(l)P(k-l)]\end{bmatrix}-\mu S(k)\right] \quad (8)$$

Second equation of model system (2) is transformed into

$$(k+1)E(k+1)=\begin{bmatrix}\beta_1[\sum_{l=0}^{k}[S(l)I(k-l)]]\\ +\eta\sum_{l=0}^{k}[S(l)V(k-l)]\\ +\beta_2\sum_{l=0}^{k}[S(l)P(k-l)]\end{bmatrix}-Q_1E(k)$$

This implies

$$E(k+1)=\frac{1}{k+1}\left[\beta_1\begin{bmatrix}\sum_{l=0}^{k}[S(l)I(k-l)]\\ +\eta\sum_{l=0}^{k}[S(l)V(k-l)]\\ +\beta_2\sum_{l=0}^{k}[S(l)P(k-l)]\end{bmatrix}-Q_1E(k)\right] \quad (9)$$

In the same vein, equation (3) transformed into

$$(k+1)I(k+1)=\left[\theta E(k)-Q_2I(k)\right]$$

$$\Rightarrow I(k+1)=\frac{1}{k+1}\left[\theta E(k)-Q_2I(k)\right] \quad (10)$$

Following the same analysis, equation (4), (5) and (6) respectively transformed into

$$V(k+1)=\frac{1}{k+1}\left[\alpha I(k)-Q_3I(k)\right] \quad (11)$$

$$R(k+1)=\frac{1}{k+1}\left[\gamma_1I(k)-\gamma_2I(k)-\mu R(k)\right] \quad (12)$$

$$B(k+1)=\frac{1}{k+1}\left[V\varepsilon I(k)-\varepsilon I(k)-\phi P(k)\right] \quad (13)$$

Thus, the differential transform of system of equations (1-6) is given as:

$$S(k+1)=\frac{1}{k+1}\left[\pi\cdot\delta(k,0)-\left[\begin{array}{l}\beta_1[\sum_{l=0}^{k}[S(l)I(k-l)]]\\ +\eta.\sum_{l=0}^{k}[S(l)V(k-l)]\\ +\beta_2\sum_{l=0}^{k}[S(l)P(k-l)]\end{array}\right]-\mu S(k)\right]$$

$$E(k+1)=\frac{1}{k+1}\left[\beta_1\left[\begin{array}{l}\sum_{l=0}^{k}[S(l)I(k-l)]\\ +\eta\sum_{l=0}^{k}[S(l)V(k-l)]\\ +\beta_2\sum_{l=0}^{k}[S(l)P(k-l)]\end{array}\right]-Q_1E(k)\right]$$

$$I(k+1)=\frac{1}{k+1}\left[\theta E(k)-Q_2I(k)\right]$$

$$V(k+1)=\frac{1}{k+1}\left[\alpha I(k)-Q_3I(k)\right]$$

$$R(k+1)=\frac{1}{k+1}\left[\gamma_1 I(k)-\gamma_2 I(k)-\mu R(k)\right]$$

$$B(k+1)=\frac{1}{k+1}\left[V\varepsilon I(k)-\varepsilon I(k)-\phi P(k)\right]$$

Subject to the initial conditions

$$\begin{aligned}&S(0)=1.4, E(0)=0.55,\ I(0)=0.46,\\ &V(0)=0.3, R(0)=0.29,\ B(0)=10.\end{aligned} \quad (14)$$

The iteration of the resulting differential transform of vaccinated model of cholera carrier using Maple 18 is obtained by using the initial conditions and the parameter values given by Table 3 as follows

Table -3: Parameter of Vaccination Model

Parameter	Description	Value	Source
η	Modification Parameter	$0-1$	Varied
μ	Natural death	0.2	Estimate
β_1	Force of infection in human susceptible	0.05	Estimate
β_2	Force of infection in pathogen	0.05	[18]
ψ_1	Vaccinated reduced failure	0.001	Varied
ρ	Fast progression rate	$0-1$	Varied
ε	Shedding rate	0.01	[18]
α	vaccine rate	$0-1$	Varied
γ_1	Natural recovery rate	0.2	[9]
γ_2	Vaccine recovery rate	$0-1$	Varied
ψ	Modification reduced mortality	0.05	Estimate
ϕ	Decay of vibrio	0.02	Estimate
τ	Saturation constant	$0-0.02$	Varied
θ	Progression rate E to I	0.3	Estimate
δ	Induced death	0.015	[25]
π	Recruitment	0.6	Estimate

$$\begin{aligned}&S(1)=-0.4122210, E(1)=0,6167210,\\ &I(1)=-0.10260,\ V(1)=0.157775,\\ &R(1)=0.1012, B(1)=-0.15397\end{aligned} \quad (15)$$

$$S(2) = 0.1579954110,\ E(2) = -0.1815290160,$$
$$I(2) = 0.0153910500,\ V(2) = -0.09632941560,\quad (16)$$
$$R(2) = 0.02829775000,\ B(2) = -0.003582411250$$

$$S(3) = -0.04010673447,\ E(3) = 0.04228073820,$$
$$I(3) = -0.001790993278,\ V(3) = 0.02653593841,\quad (17)$$
$$R(3) = -0.01783899190,\ B(3) = 0.0005253088943$$

$$S(4) = 0.00786579768,\ E(4) = -0.008050981800,$$
$$I(4) = 0.000219227006,\ V(4) = -0.005006963365,\quad (18)$$
$$R(4) = 0.004199986928,\ B(4) = -0.00004673797798$$

Using the iterated values obtained, the series solution of the first five (5) approximations of each compartment $S(t), E(t), I(t), V(t), R(t), and\ B(t)$ are computed and presented below:

$$S(t) = 1.4 - 0.4122210t + 0.1578854110t^2 - 0.04010673447t^3 + 0.007836579768t^4 + \square\square\square\quad (19)$$

$$E(t) = 0.55 + 0.6167210t - 0.1815290160t^2 + 0.04228073820t^3 - 0.008050981800t^4 + \square\square\square\quad (20)$$

$$I(t) = 0.4 - 0.10260t + 0.01513910500t^2 - 0.001790993278t^3 + 0.0002109227006t^4 + \square\square\square\quad (21)$$

$$V(t) = 0.3 + 0.157775t - 0.09632041560t^2 + 0.02653593841t^3 - 0.005006963365t^4 + \square\square\square\quad (22)$$

$$R(t) = 0.29 - 0.1012t + 0.02829775000t^2 - 0.01783899190t^3 + 0.004199986928t^4 + \square\square\square\quad (23)$$

$$B(t) = 10 + 0.15397t - 0.003582411250t^2 + 0.0005253088943t^3 - 0.00004673797798t^4 + \square\square\square\quad (24)$$

The graphs of each compartment with time was plotted and displayed as follow

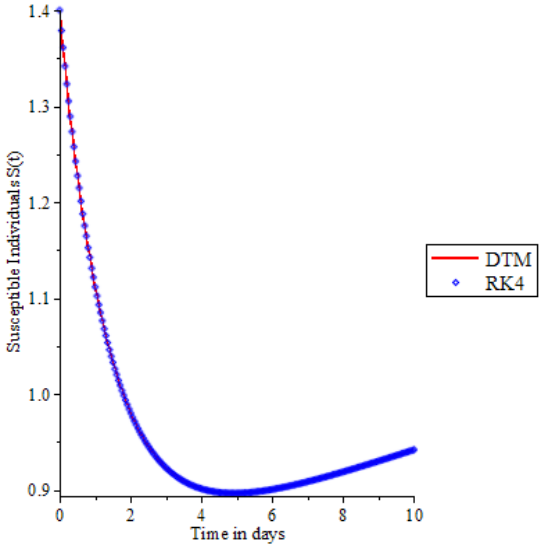

Figure -2: Solution of Susceptible Individuals by DTM and RK4

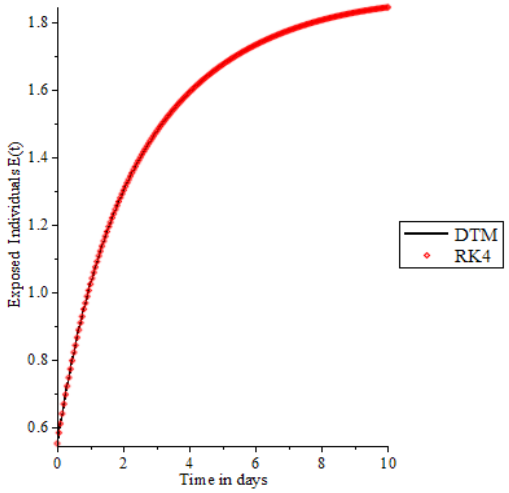

Figure -3: Solution of Exposed Individuals by DTM and RK4

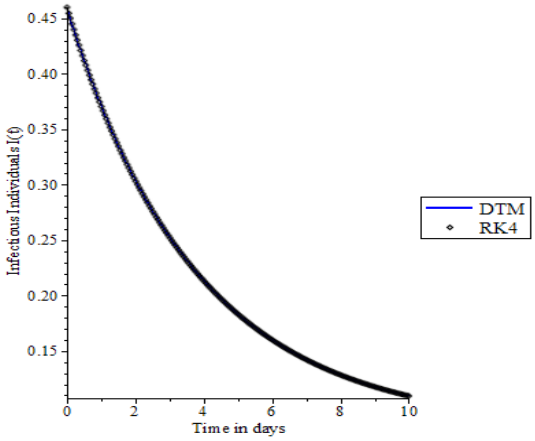

Figure -4: Solution of Infected Individuals by DTM and RK4

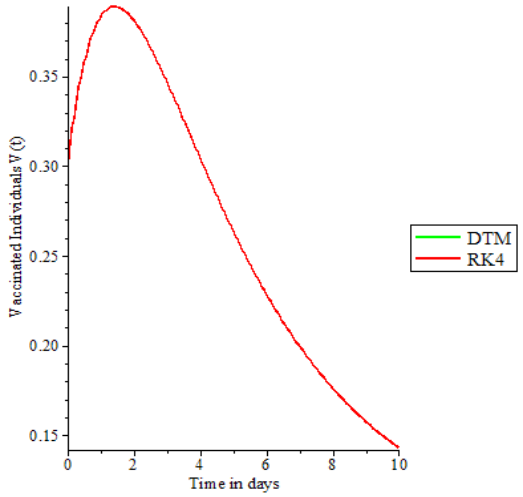

Figure -5: Solution of Vaccinated Individuals by DTM and RK4

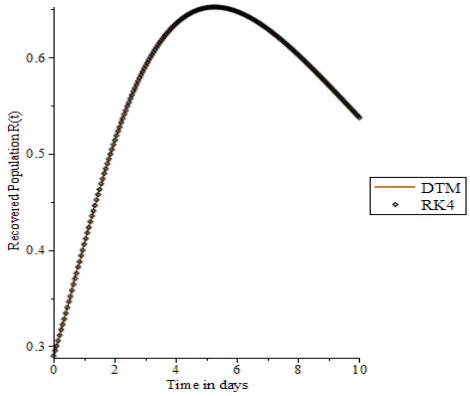

Figure -6: Solution of Recovered Individuals by DTM and RK4

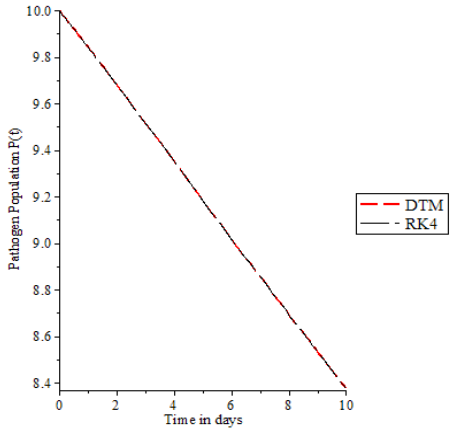

Figure -7: Solution of Pathogen Population by DTM and RK4

Figure Simulations (2-7) depicts the comparison between DTM and RK-4 solutions of the model system (2-7) populations against Time (in days).

The Differential Transform Method (DTM) is demonstrated against maple built-in fourth order Runge-Kutta procedure for the solutions of vaccination model of cholera individuals. Fig 2 shows population of susceptible drastically suffered from the early breakout of the cholera. Fig 3 shows that activities are regaining as time goes on. Fig 4 indicates that the infected individual's population would vanish as a result of measures to eradicate the cholera disease in the population over time. Fig 5 shows that the vaccine administered decreases in efficacy overtime. Fig 6 shows recovered population rises and going back to susceptible overtime. Fig 7 following environmental sanitation and taking other control measures the pathogen population would wipe out completely in the population over time.

3. CONCLUSIONS

In this paper, solution of the vaccination model of cholera carrier epidemic model using differential transform method were successfully presented and analyzed rigorously. By using DTM operational properties, the vaccination model of a cholera carrier was transformed and the power series solutions were generated. The accuracy of DTM was demonstrated and comparison was made against the maple built-in fourth order Runge-Kutta method. Plotted DTM solutions were found to be in good agreement with the popular Runge-Kutta solution. The results revealed that the method has high accuracy and required less computational work compared to the other methods.

In view to that it is recommended that mathematical modeling problems can be solved using DTM because of it is efficiency, reliability, high accuracy, and fast convergence rate.

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