The Effect of Tooth Thickness on Root Stress of Internal Spur Gear Mechanism

Tufan Gürkan Yılmaz[1], Fatih Karpat[2]

[1]*Department of Mechanical Engineering, Bursa Uludag University & Bursa, Turkey*
[2]*Department of Mechanical Engineering, Bursa Uludag University & Bursa, Turkey*

---***---

Abstract - *This study aims to enhance the load carrying capacity of the internal spur gear mechanism by making stress equal between pinion and internal gear by shifting standard thickness on pitch circle. As standard, this thickness value is half of pitch. Actually, it can be shifted if the total pitch value is equal to πm. To understand the effect of tooth thickness variation on stress, finite element analyses are implemented. When the thickness is on pitch circle, the root stress decreases. Specific thickness values which make maximum stress equal is determined. Besides, the total mechanism weight is decreased with this method.*

*Key Words***:** Internal spur gear, tooth thickness, finite element analysis, bending stress, lightweighting

1. INTRODUCTION

The internal spur gear mechanism is composed of internal and pinion gear. Internal spur gear tooth has bigger thickness on root zone that it is subjected to lower stress than pinion. This difference can be removed with decreasing thickness on pitch circle of internal gear and increasing thickness on pitch circle of pinion. The pinion shaper is the common generation tool of internal and pinion gear. The geometry of this cutter is similar to pinion gear just tip of cutter is sharpened to cut teeth.

In literature, there are various publications about different kinds of tooth designs to decrease root stress. One of these methods is using asymmetric gear that it has different pressure angle on drive and coast sides. With increasing pressure angle, the stress levels decrease [1-2]. Yılmaz et al. investigated effect of changing cutter tip radius on external and internal spur gears. When tip radius is increased, the stress values decrease [3-4]. Another method is to use artificial curves. Spitas et al. offered circular curve to reduce stress. However it is just effective when tooth number is under the 17 [5]. Zou et al. conducted an optimization study to reach an optimum curve. With specified curve, the stress values decrease approximately 20% [6]. Using different tooth thicknesses from standard value is novel approach. Sekar and Muthuveerappan conducted finite element analysis to obtain the effect of tooth thickness on stress of external spur gear mechanisms. Balanced stress values are determined for given teeth number, pressure angle and profile shifting coefficient [7].

In the current study, influence of tooth thickness on stress of internal spur gear mechanism was searched by finite element analyses. For this, the mathematical equations of pinion cutter were set. A MATLAB code was generated for geometrical design of external and internal gear. The output file which contains coordinates of gear tooth was exported to CATIA for generating three teeth FEA model. Case studies were conducted for determined gear parameters. As a result of analyses, the specific tooth thickness value was obtained. The weight reduction of mechanism was also reported.

2. MATHEMATICAL EQUATIONS OF PINION CUTTER

Pinion (external) and internal spur gears are generated pinion type cutters commonly. The geometry of pinion type cutters determines the stress behavior of gear under loading. For this reason, to design a gear first the cutters equations have to be set properly. In Fig.1, the regions of pinion cutter are illustrated.
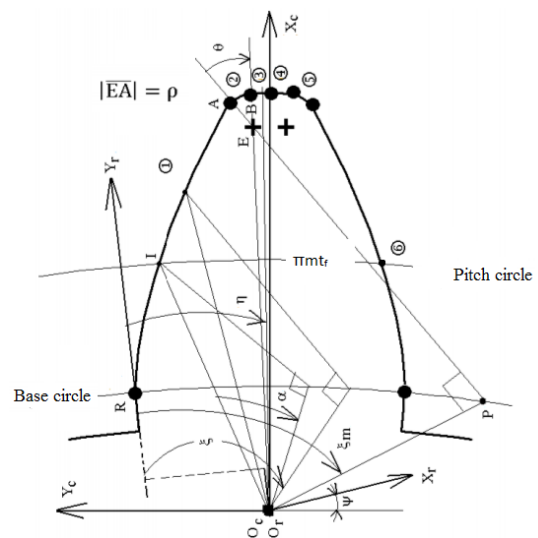

Fig -1: The geometry of pinion type cutter tooth

First and sixth regions of the tooth are generated involute section, second and fifth sections are generated trochoid section, and finally third and fourth sections of tooth are generated bottom land of gear. $S_R(X_R,Y_R)$ is reference coordinate system and $S_c(X_c,Y_c)$ is coordinate system of cutter. Where α is pressure angle, ψ is rotating angle, ξ is the involute angle and design parameter of first and sixth regions

changes $0 \leq \xi \leq \xi_m$, ξ_m is the maximum involute angle and t_f is tooth thickness coefficient.

The equations of regions are presented successively.

$$R_c^1 = \begin{bmatrix} r_b \cos(\xi - \psi) + r_b\,\xi \sin(\xi - \psi) \\ -r_b \sin(\xi - \psi) + r_b\,\xi \cos(\xi - \psi) \\ 1 \end{bmatrix} \qquad (1)$$

$$R_c^6 = \begin{bmatrix} r_b \cos(\xi - \psi) + r_b\,\xi \sin(\xi - \psi) \\ r_b \sin(\xi - \psi) - r_b\,\xi \cos(\xi - \psi) \\ 1 \end{bmatrix} \qquad (2)$$

$$\psi = \frac{\pi t_f}{z_c} + tan(\alpha) - (\alpha) \qquad (3)$$

Where r_b is the radius of the base circle and z_c is teeth number of pinion cutter.

$$R_c^2 = \begin{bmatrix} (r_b \cos(\xi_m - \psi) + r_b \xi_m \sin(\xi_m - \psi) \\ -\rho_1 \sin(\xi_m - \psi) + \rho_1 \cos(\theta_1 + \xi_m - \psi)) \\ (-r_b \sin(\xi_m - \psi) + r_b \xi_m \sin(\xi_m - \psi) \\ -\rho_1 \sin(\xi_m - \psi) + \rho_1 \cos(\theta_1 + \xi_m - \psi)) \\ 1 \end{bmatrix} \qquad (4)$$

$$R_c^5 = \begin{bmatrix} (r_b \cos(\xi_m - \psi) + r_b \xi_m \sin(\xi_m - \psi) \\ -\rho_2 \sin(\xi_m - \psi) + \rho_2 \cos(\theta_2 + \xi_m - \psi)) \\ (r_b \sin(\xi_m - \psi) - r_b \xi_m \sin(\xi_m - \psi) \\ \rho_2 \sin(\xi_m - \psi) - \rho_2 \cos(\theta_2 + \xi_m - \psi)) \\ 1 \end{bmatrix} \qquad (5)$$

Where θ is design parameter of region 2-5 and changes

$0 \leq \theta_{1,2} \leq (\pi/2) - \arctan(\xi_m - (\rho_{1,2}/r_b))$.

The mathematical equation of region 3-4 is presented;

$$R_c^3 = \begin{bmatrix} r_a \cos(\eta - \psi) \\ r_a \sin(\eta - \psi) \\ 1 \end{bmatrix} \qquad (6)$$

$$R_c^4 = \begin{bmatrix} r_a \cos(\eta - \psi) \\ -r_a \sin(\eta - \psi) \\ 1 \end{bmatrix} \qquad (7)$$

$$r_a = \sqrt{r_b + (r_b \xi_m - \rho)^2} + \rho \qquad (8)$$

Where η is designed parameter of region 3-4 and changes $\xi_m + \delta - (\pi/2) \leq \eta \leq (\pi t_f/z_c) + \tan(\alpha) - (\alpha)$.

$$\delta = \frac{\pi}{2} - \arctan(\xi_m - (\rho_{1,2} - r_b)) \qquad (9)$$

Normal vector of cutter regions could be determined by differential geometry with the following equation;

$$n_c^i = \frac{\dfrac{dR_c^i}{dv_j} x k_c}{\left| \dfrac{dR_c^i}{dv_j} x k_c \right|} \quad for\ i = 1 - 6;\ \ v_j = \xi, \theta, \eta \qquad (10)$$

During pinion and internal gear machining, there is a movement relation between the cutter and gear blank. This is a connected system which means when the coordinates of a point on cutter are known, then its corresponding point on gear is apparent. In following sections, with using coordinate transformation matrix, differential geometry and gearing theory; the equations of pinion and internal gear tooth were obtained. The coordinate system S_f (X_f, Y_f) is the fixed coordinate system, $S_g(X_g, Y_g)$ defines the coordinate system of gear.

Equations of internal and pinion gear

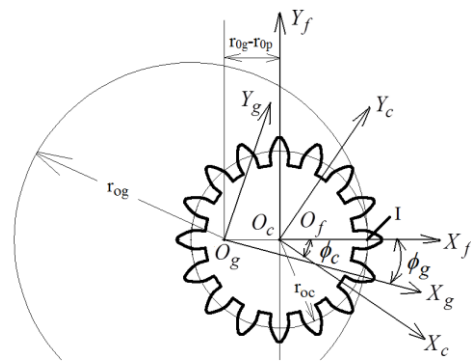The movement relation between generated gear and pinion cutter is illustrated in Fig.2 and Fig.3.

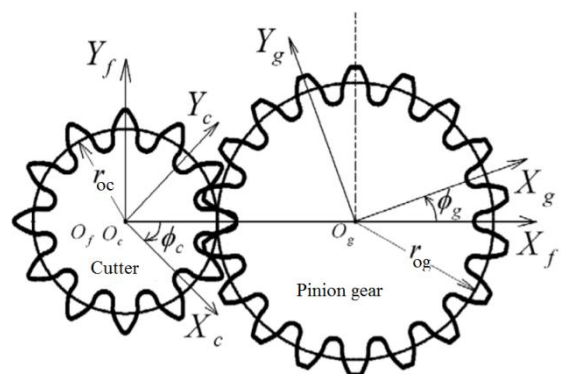**Fig -2**: Movement relation between internal gear blank and cutter

Fig -3: Movement relation between pinion gear blank and cutter

Coordinate transformation matrix could be derived from the coordinate axis of gear blank and cutter for internal gear.

$$M_{gc} = \begin{bmatrix} \cos(\phi_g - \phi_c) & -\sin(\phi_g - \phi_c) & (r_{0g} - r_{0c})\cos(\phi_g) \\ \sin(\phi_g - \phi_c) & \cos(\phi_g - \phi_c) & (r_{0g} - r_{0c})\sin(\phi_g) \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

For pinion gear is,

$$M_{gc} = \begin{bmatrix} \cos(\phi_g + \phi_c) & \sin(\phi_g + \phi_c) & (-r_{0g} - r_{0c})\cos(\phi_g) \\ -\sin(\phi_g + \phi_c) & \cos(\phi_g + \phi_c) & (r_{0g} + r_{0c})\sin(\phi_g) \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

Where $\emptyset_g$ and $\emptyset_c$ are rotating angle, roc is the radius of the pitch circle of the cutter and rog is the radius of the pitch circle of the internal gear. The angle equation between the pinion gear and cutter is presented in the following equation.

$$\phi_g = (\frac{r_{0c}}{r_{0g}})\phi_c \quad (13)$$

The mathematical equation of pinion and internal gear is obtained by the following equation.

$$R_g^i = [M_{gc}]R_c^i \quad (14)$$

According to gearing theory, the common normal vector of cutter and blank passes on the instantaneous rotating center (I). The mathematical expression of this fact is presented in the following equation;

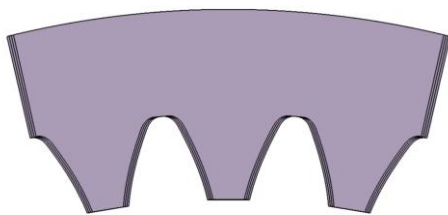$$\frac{X_c - x_c^i}{n_{cx}^i} = \frac{Y_c - y_c^i}{n_{cy}^i} \quad for\ i = 1 - 6 \quad (15)$$

Fig -4: 2D design of internal gear with the different tooth thickness
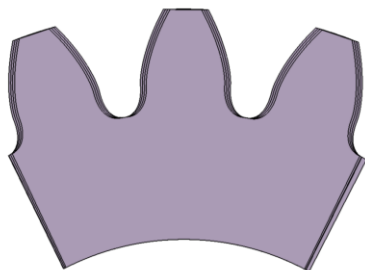

Fig -5: 2D design of pinion gear with the different tooth thickness

3. FINITE ELEMENT ANALYSIS

To understand changing of bending stress for internal gear and pinion gear of internal spur gear mechanism with unequal tooth thickness FEA was conducted for defined gear parameters. Datasheet of the case is given in Table 1.

Table -1: Datasheet of Internal Spur Gear Mechanism

Gear parameter	Internal	Pinion
Module (m)	4 mm	4 mm
Number of teeth (z)	40	20
Pressure Angle (α)	20º	20º
Addendum coefficient (h_a)	0.8	1
Dedendum coefficient (h_f)	1.25	1.25
Tip radius coefficient of cutter (ρ)	0.3	0.3
Tooth thickness coefficient at pitch circle (t_f)	0.5-0.47 0.44-0.41	0.5-0.53 0.56-0.59
Profile shifting (x)	0	0

100 N normal force is applied on its highest point single tooth contact for internal and pinion gear separately. The load and boundary condition are illustrated in Fig.6.
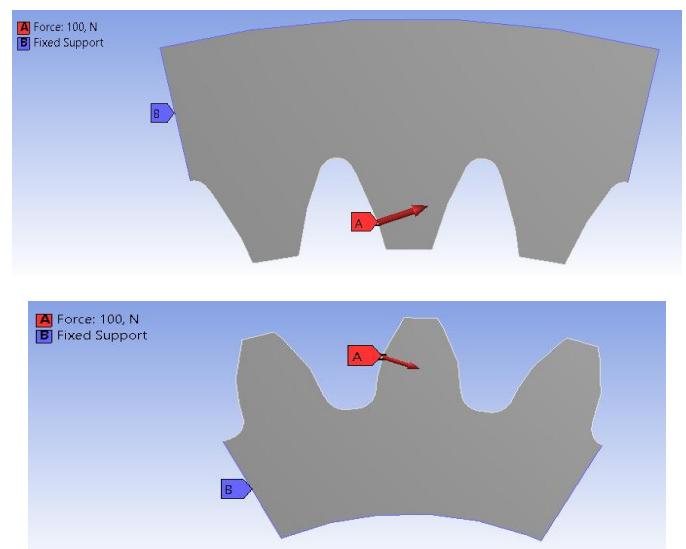

Fig -6: Load and boundary conditions for internal and pinion gear

Fixed Support was given both lateral sides and rim side in analyses. 0.2 element size and quadrilateral element type were used for meshing. Plane strain condition was assumed for solving.

In Fig.7 and Fig.8, the bending stress values of pinion gear with 0.5×πm and 0.59× πm are presented as the sample.
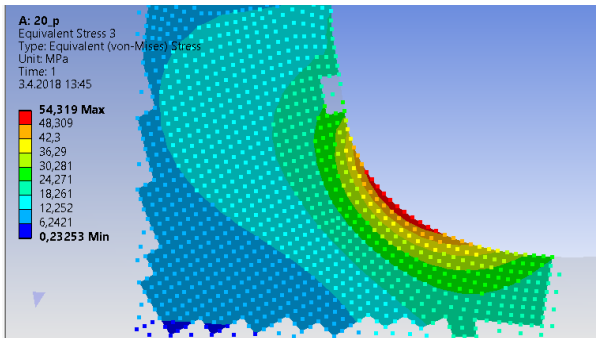
Fig -7: Pinion gear with 0.5× πm tooth thickness on the pitch circle

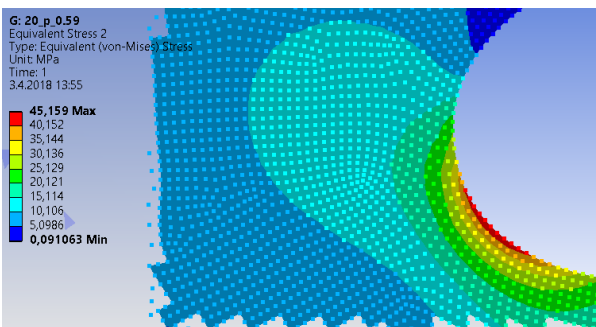**Fig -8**: Pinion gear with 0.59× πm tooth thickness on the pitch circle

In Fig.9 and Fig.10, the bending stress values of internal gear with 0.5×πm and 0.41× πm are presented as the sample.
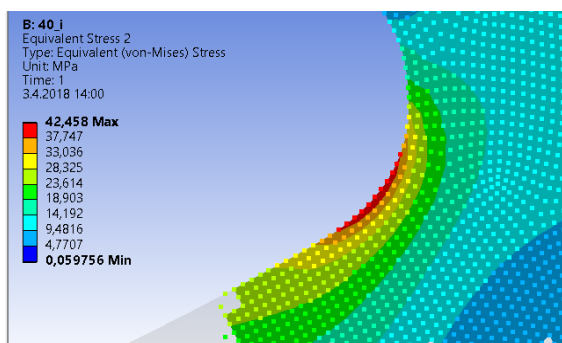
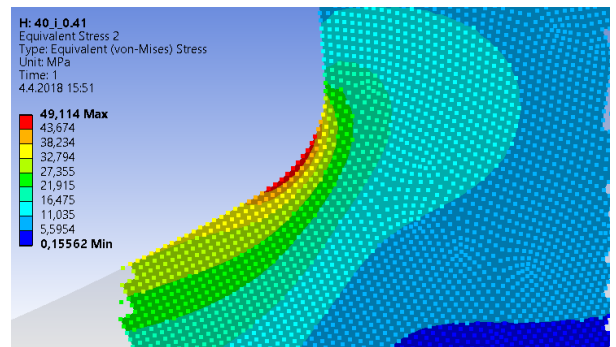**Fig -9**: Internal gear with 0.5× πm tooth thickness on the pitch circle

Fig -10: Internal gear with 0.41× πm tooth thickness on the pitch circle

To determine tooth thicknesses that balance bending stress; stress with changing thickness graphic for pinion and internal gear is presented in Chart 1 and 2 successively.
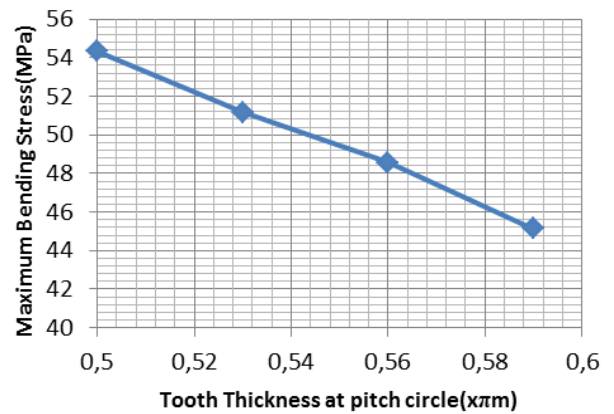

Chart -1: Relation between bending stress and tooth thickness for pinion
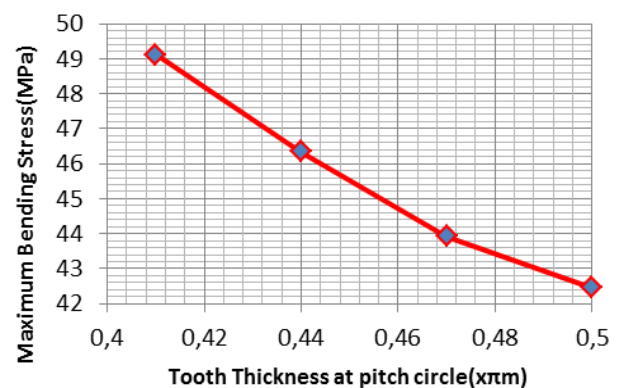

Chart -2: Relation between bending stress and tooth thickness for internal gear

According to results, when tooth thickness value is 0,572×πm for pinion and 0,428×πm for internal gear, the stress is getting balanced (approximately 47,3 MPa). Please note that the total thickness value has to be 1×πm.

When tooth thickness is increased from 0,5×πm to 0,572×πm the stress level decreases 11% nearly for the pinion. On the other hand, when the thickness is decreased from 0,5×πm to 0,428×πm for internal gear, the stress increases 11%.

After specifying tooth thickness values, the weight reduction of the mechanism could be obtained. In Table 2, the weight change of pinion and internal gear was presented. Facewidth is assumed 20 mm.

Table -2: Weight Reduction of Internal Spur Gear Mechanism

Tooth thickness (t_f)	Pinion weight(gr)	Internal gear weight(gr)
0,5×πm	492,53	1262,13
0,572× πm	517,5	-
0,428× πm	-	1219,3
Difference	24,97	-42,83
Total	-17,86	

3. CONCLUSION

In this study, the effects of tooth thickness at pitch circle on bending stress of internal and pinion gear of internal spur gear mechanisms. The balance stress value was tried out to find. For this, first the mathematical equation of pinion cutter was defined then equations of internal and pinion gear were obtained. 2D FEA model was created in the CAD program. Finite element analyses were conducted for a case study with different tooth thickness. According to results, for standard cutter the stress difference between pinion and internal gear was nearly 20%. To diminish this difference, the thickness was increased for pinion and decreased for internal gear. When tooth thickness value is 0.572×πm for pinion and 0.428×πm for internal gear; the stress values become equal. With this design change total weight was decreased and load carrying capacity of internal gear mechanism was increased.

REFERENCES

[1] A. Kapelevich, "Geometry and design of involute spur gears with asymmetric teeth" Mech. Mach. Theory, vol. 35, January 2000, pp. 117-130, 2000. doi: 10.1016/S0094-114X(99)00002-6.

[2] T. Masuyama and N. Miyazaki, "Evaluation of load capacity of gears with an asymmetric tooth profile," International Journal of Mechanical and Materials Engineering, vol.11, November 2016, doi: 10.1186/s40712-016-0064-0.

[3] T.G. Yılmaz, O. Dogan, C. Yuce, F. Karpat, "Improvement of the loading capacity of internal spur gears with using asymmetric trochoid profile," Proc. ASME International Mechanical Engineering Conference&Exhibtion, Nov. 2017, pp. V011T15A018, doi: 10.1115/IMECE2017-71009.

[4] T.G. Yılmaz, O. Dogan, F. Karpat, "Stress analysis of thin rimmed spur gears with asymmetric trochoid," in Proc. International Conference on Mechanics and Industrial Engineering, June, 2017. doi: 10.11159/icmie17.132.

[5] C. Spitas and V. Spitas, "A FEM study of the bending strength of circular fillet gear teeth compared to trochoidal fillets produced with enlarged cutter tip radius," Mech. Based Des. Struc., vol. 35, Feb. 2007, pp. 59-73, doi: 10.1080/15397730601182802.

[6] T. Zou, M. Shaker, J. Angeles, A. Morozov, "An innovative tooth root profile for spur gears and its effect on service life" Meccanica, vol. 52, Sep. 2016, pp. 1825-1841, doi: 10.1007/s11012-016-0519-7.

[7] P. Sekar, P., and G. Muthuveerappan, G, "A Balanced Maximum Fillet Stresses on Normal Contact Ratio Spur Gears to Improve Load Carrying Capacity Through Nonstandard Gears", Mech. Based Des. Struc, vol.43, April 2015, pp. 150-163, doi: 10.1080/15397734.2014.934833.