

Effect of Applied Pressure Gradient on MHD flow between Parallel Plates under the Influence of Inclined Magnetic Field by Differential Transform Method

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Abstract - This paper studies the magnetohydrodynamic flow of a viscous incompressible flow between two parallel infinite plates under the influence of inclined magnetic field and applied pressure gradient. The upper plate is moving with constant velocity and the lower plate is held stationary and a pressure gradient is applied to the system which is under the influence of inclined magnetic field. The resulting governing partial differential equations are non dimensionalised and solved numerically by using the Differential Transform Method. The effect of negative and positive applied pressure gradient, Hartmann number, the angle of inclination of the magnetic field and the Reynolds Number on the flow field have been represented graphically. It is observed that an increase in the Hartmann number and the angle of inclination of magnetic field decreases the velocity profile and increasing the pressure gradient and Reynolds number effects the velocity profile.

Key Words: Magnetohydrodynamic, Reynolds number, Hartmann number, Pressure gradient, Differential Transform Method.

1. INTRODUCTION

Magnetohydrodynamic flow between two parallel plates under the influence of inclined magnetic field has many applications in different fields of Engineering and Technology. The interaction between the conducting fluid and the magnetic field modifies the fluid flow and effects the velocity profile depending on the orientation of the magnetic field. When a fluid moves through a magnetic field an electric field, and consequently a current may be induced. This current in turn interacts with the magnetic field to produce a force which effects the velocity of the fluid. In recent years MHD fluid flows has gained interest due to its widespread application in geothermal, oil reservoir engineering, separation of matter from fluids, MHD power generator, aerodynamics and nuclear engineering. The influence of magnetic field on a viscous incompressible fluid flow of electrically conducting fluid is of use in extrusion of plastics, manufacture of rayon, nylon.

Hannas Alvfan [20] a Swedish electrical engineer initiated the study of MHD. Shercliff [19] considered the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Drake (1965) considered the flow in channel due to periodic pressure gradient and solved by the method of separation of variables. Singh [10] studied steady MHD flow between two parallel plates. Manyogne, W A Kiema, Iyaya C W [7] studied the MHD Poiseuille flow between two infinite porous plates under the influence of magnetic field. Singh [14] investigated the hydromagnetic steady flow of viscous incompressible fluid between two parallel infinite plates under the influence of inclined magnetic field. Singh and Okwoyo [17] carried out a study of couette flow between two infinite parallel plates in the presence of transverse magnetic field. Agnes Mburu, Jackson Kwanza and Thomas Onyango [21] studied Magnetohydrodynamic fluid flow between two infinite parallel plates subjected to an inclined magnetic field under Pressure gradient. Despite the investigations done on MHD flow past parallel plates subjected to inclined magnetic field, the effect of negative and positive pressure gradient has received little attention. Here the main objective of the present investigation is to find the effect of applied pressure gradient and Reynolds number on MHD flow between two parallel infinite plates under the influence of magnetic field using Differential Transform Method(DTM)

2. MATHEMATICAL FORMULATION

We consider steady viscous flow along the X axis of an electrically conducting fluid between two horizontal parallel infinite plates located at $y=-h$ and $y=h$ and extending from $x=-\infty$ to $x=\infty$ and from $z=-\infty$ to $z=\infty$. The lower plate is stationary and upper plate is moving with constant velocity U.

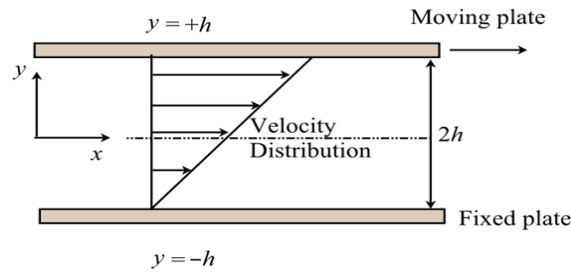


Fig 1. Geometrical Configuration of the flow.

3. GOVERNING EQUATIONS

A magnetic field of field strength represented by vector **B** is applied at right angle to the flow of an electrically conducting fluid moving with velocity **V**. Assuming that steady state conditions have been attained the interaction of the two fields gives rise to an electric field **E** at right angles to both **V** and **B** i.e $\mathbf{E} = \mathbf{V} \times \mathbf{B}$. Assuming the conducting fluid is isotropic, we denote its electrical conductivity by σ . The current density in conducting fluid $\mathbf{J} = \sigma \mathbf{E}$ by Ohm's law. Simultaneously with induced current occurs an induced electromotive force called the Lorentz force $\mathbf{F} = \mathbf{J} \times \mathbf{B}$. The equations that describe MHD flow are a combination of a continuity equation, Navier Stoke's equations of fluid dynamics and Maxwell's equation of electromagnetism.

The equation of continuity for the incompressible fluid flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

where u,v,w are the components of velocity of fluid in the x,y,z directions.

Momentum equations are given as follows:

x-momentum equation is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{F_x}{\rho} \tag{2}$$

y-momentum equation is

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \frac{F_y}{\rho} \tag{3}$$

z-momentum equation is

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \frac{F_z}{\rho} \tag{4}$$

where F_x, F_y, F_z are the components of $\mathbf{F} = \mathbf{J} \times \mathbf{B}$ in x, y and z directions respectively.

We are considering a two dimensional flow, therefore equation (1) reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

The plates are of infinite length, so we assume the flow is along x-axis only i.e velocity profile along the y and z axis $v=w=0$ and u depends on y only.

Therefore, $\frac{\partial u}{\partial x} = 0$ (6)

Since we have assumed steady flow, i.e flow variables are independent of time,

$$\frac{\partial u}{\partial t} = 0 \quad (7)$$

Therefore, equations (2), (3) and (4) can be written as

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{F_x}{\rho} \quad (8)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{F_y}{\rho} \quad (9)$$

Where, F_x, F_y and F_z are the components of the Lorentz force F , as the body force is neglected and replaced by Lorentz force.

Since there is no flow in the y-direction, using (5) and (6), we can rewrite the equations (8), (9) as

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{F_x}{\rho} \quad (10)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{F_y}{\rho} \quad (11)$$

There is no component of body force in y direction, and $F_x = \mathbf{J} \times \mathbf{B}$, $F_y = F_z = 0$ as $v=w=0$, the equation of motion becomes

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\mathbf{J} \times \mathbf{B}}{\rho} \quad (12)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (13)$$

Since we are considering the flow in x-direction then the flow will be affected by the magnetic flux which is perpendicular to the flow. Since we want to study the effect of different angles of inclination of the magnetic field then the velocity and magnetic flux profiles will be

$$\mathbf{V} = \mathbf{V}(u, 0, 0)$$

$$\mathbf{B} = \mathbf{B}(0, B \sin \theta, 0)$$

Where θ is the angle between \mathbf{V} and \mathbf{B}

Equation (13) implies p does not depend on y.

Since, $\mathbf{J} = \sigma \mathbf{E}$ and $\mathbf{E} = \mathbf{V} \times \mathbf{B}$, where \mathbf{V} is the fluid velocity along x-axis, the direction of fluid flow

$$\text{Now, } \mathbf{E} = \mathbf{V} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & 0 & 0 \\ 0 & B \sin \theta & 0 \end{vmatrix} = uB \sin \theta \hat{k} \quad (14)$$

$$\text{Now, } \mathbf{J} = \sigma \mathbf{E} = \sigma uB \sin \theta \hat{k}$$

$$\mathbf{J} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \sigma uB \sin \theta \\ 0 & B \sin \theta & 0 \end{vmatrix} = -\sigma uB^2 \sin^2 \theta \hat{i}$$

$$\text{Therefore, } F_x = -\sigma B^2 \sin^2 \theta u \quad (15)$$

Using (15), the equation of motion reduces to

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B^2 \sin^2 \theta}{\rho} u \quad (16)$$

4. NON -DIMENSIONALIZATION

To simplify (16) we non dimensionalize to reduce the parameters in the equation by using the following non dimensional quantities.

$$x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad p^* = \frac{p}{\rho U^2}, \quad u^* = \frac{u}{U}, \quad \text{Re} = \frac{U h}{\nu} \text{ where } \nu = \frac{\mu}{\rho}$$

$$\begin{aligned} \text{Now, } \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y} (U u^*) = \frac{\partial}{\partial y^*} (U u^*) \frac{\partial y^*}{\partial y} \\ &= U \frac{\partial u^*}{\partial y^*} \frac{1}{h} \end{aligned}$$

$$\therefore \frac{\partial u}{\partial y} = \frac{U}{h} \frac{\partial u^*}{\partial y^*} \quad (17)$$

$$\text{Now, } \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{U}{h} \frac{\partial u^*}{\partial y^*} \right)$$

$$\begin{aligned} \therefore \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial y^*} \left(\frac{U}{h} \frac{\partial u^*}{\partial y^*} \right) \frac{\partial y^*}{\partial y} \\ &= \frac{U}{h} \frac{\partial^2 u^*}{\partial y^{*2}} \left(\frac{1}{h} \right) \end{aligned} \quad (18)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{h^2} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (19)$$

$$\text{Next, } \frac{\partial p}{\partial x} = \frac{\partial}{\partial x} (\rho U^2 p^*)$$

$$\begin{aligned} &= \rho U^2 \left(\frac{\partial p^*}{\partial x} \right) \\ &= \rho U^2 \left(\frac{\partial p^*}{\partial x^*} \right) \left(\frac{\partial x^*}{\partial x} \right) \\ &= \rho U^2 \left(\frac{\partial p^*}{\partial x^*} \right) \left(\frac{1}{h} \right) \end{aligned}$$

$$\therefore \frac{\partial p}{\partial x} = \frac{\rho U^2}{h} \left(\frac{\partial p^*}{\partial x^*} \right) \quad (20)$$

$$\text{Similarly, } \frac{\partial p}{\partial y} = \frac{\rho U^2}{h} \left(\frac{\partial p^*}{\partial y^*} \right) \quad (21)$$

Substituting in equation (13) and (16) we get,

$$\left(\frac{\partial p^*}{\partial y^*} \right) = 0 \quad (22)$$

$$0 = -\frac{1}{\rho} \frac{\rho U^2}{h} \left(\frac{\partial p^*}{\partial x^*} \right) + \frac{\mu}{\rho} \frac{U}{h^2} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 \sin^2 \theta}{\rho} (U u^*) \quad (23)$$

$$0 = -\frac{U^2}{h} \left(\frac{\partial p^*}{\partial x^*} \right) + \frac{\mu}{\rho} \frac{U}{h^2} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 \sin^2 \theta}{\rho} (Uu^*) \quad (24)$$

Dividing throughout by $\frac{1}{\left(\frac{\mu U}{\rho h^2}\right)}$

$$0 = -\frac{\rho U h}{\mu} \left(\frac{\partial p^*}{\partial x^*} \right) + \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 h^2 \sin^2 \theta}{\mu U} (Uu^*) \quad (25)$$

Now writing $Re = \frac{\rho U h}{\mu}$ and $M^2 = \frac{\sigma B^2 h^2}{\mu}$ i.e $M = Bh \sqrt{\frac{\sigma}{\mu}}$

Dropping the * from equation (16) and (25) and simplifying we get

$$\left(\frac{\partial p}{\partial y} \right) = 0 \quad (26)$$

$$0 = -Re \left(\frac{\partial p}{\partial x} \right) + \frac{\partial^2 u}{\partial y^2} - M^2 \sin^2 \theta u \quad (27)$$

Where Re is called as the Reynold's Number and M is the Hartmann Number.

Differentiating equation (27) w.r.t x, We get

$$\frac{\partial^2 p}{\partial x^2} = 0 \quad (28)$$

Since P does not depend on y we express equation (28) as a total derivative

$$\frac{d^2 p}{dx^2} = 0 \quad (29)$$

From (29), we get

$$\frac{dp}{dx} = -P \quad (\text{a constant}) \quad (30)$$

Since u depends only on y equation (27) reduces to

$$\frac{d^2 u}{dy^2} - M^2 \sin^2 \theta u = -P Re \quad (31)$$

Equation (31) can be solved using boundary conditions

$u=0$, when $y=-h$

$u=U$, when $y=h$.

The non dimensional form of these boundary conditions are $u=0$, when $y=-1$

$u=1$, when $y=1$

The equation (31) is solved numerically by Differential Transform Method (DTM)

5. METHODOLOGY

The Differential Transform Method (DTM) is a numerical method for solving differential equations which was proposed by Zhou[1986]. The DTM gives exact values of the n^{th} order derivative of the analytic function at a point in terms of known and unknown boundary conditions in a fast manner, The DTM Technique uses the polynomials to approximate the exact solution. The differential transform of the k^{th} derivative of the function $u(x)$ is defined as follows

$$U(k) = \frac{1}{k!} \left[\frac{d^k u(x)}{dx^k} \right]_{x=x_0}$$

and the differential inverse transform of $U(k)$ is defined as

$$u(x) = \sum_{k=0}^{\infty} U(k)(x - x_0)^k$$

In real application, the function $u(x)$ is expressed as a finite series.

We apply DTM to equation (31) and express it in the form $u(x) = \sum_{k=0}^n U(k)(x - x_0)^k$

where $u(x) = \sum_{k=n+1}^{\infty} U(k)(x - x_0)^k$ is assumed to be negligible.

The following theorems can be deduced

Theorem 1. If $u(x) = y(x) \pm z(x)$ then

$$U(k) = Y(k) \pm Z(k)$$

Theorem 2. If $u(x) = ay(x)$ then

$U(k) = aY(k)$, where 'a' is a constant.

Theorem 3. If $u(x) = \frac{d^m y(x)}{dx^m}$, then

$$U(k) = \frac{(m+k)!}{k!} Y(k+m)$$

Theorem 4. If $u(x) = y(x)z(x)$, then

$$U(k) = \sum_{k_1=0}^k Y(k_1)Z(k - k_1)$$

Theorem 5. If $u(x) = x^n$, then

$$U(k) = \delta(k - n) = 1 \text{ for } k = n \\ = 0 \text{ for } k \neq n$$

6. SOLUTION

Applying DTM to equation (31) and using theorem 1 to 5, we get

$$(k+1)(k+2)U(k+2) - M^2 \sin^2 \theta U(k) = -P Re \quad (32)$$

Where $U(k) = \frac{1}{k!} \left[\frac{d^k u(y)}{dy^k} \right]_{y=0}$ and

$$u(y) = \sum_{k=0}^n U(k)(y)^k \quad (33)$$

and $u(x) = \sum_{k=n+1}^{\infty} U(k)(y)^k$ is assumed to be negligibly small.

From (32) we get the recurrence relation,

$$U(k+2) = \frac{-P Re + M^2 \sin^2 \theta U(k)}{(k+1)(k+2)} \quad (34)$$

For $k=0,1,\dots$ we find $U(2), U(3), U(4) \dots$ and so on in terms of $U(0)$ and $U(1)$.

Now from (33), we express $u(y)$ as a series, taking $n=11$

$$u(y) = U(0) + U(1)y + U(2)y^2 + U(3)y^3 + \dots + U(11)y^{11} \tag{35}$$

Substituting the boundary conditions $u(-1)=0$ and $u(1) = 1$ in above we get

$$U(0) - U(1) + U(2) - U(3) + \dots - U(11) = 0$$

$$U(0) + U(1) + U(2) + U(3) + \dots + U(11) = 1$$

Adding and subtracting the above two equations we get,

$$U(0) + U(2) + U(4) + \dots + U(10) = 0.5 \tag{36}$$

$$U(1) + U(3) + U(5) + \dots + U(11) = 0.5 \tag{37}$$

By assigning values to M, P, Re and θ , we obtain $U(2), U(3), U(4), U(5), \dots, U(11)$ in terms of $U(1)$ and $U(0)$ using equation (34)

These are further substituted in equation (36) and (37) to get $U(0)$ and $U(1)$.

Using $U(0)$ and $U(1)$ thus calculated we resubstitute to obtain $U(2), U(3), U(4), U(5), \dots, U(11)$

These known values are substituted in the series $u(y)$ in equation (33) and for different values of y the velocity profile is obtained.

7. TABLES AND GRAPHS

From the above methodology, using MATLAB the velocity profile has been calculated for different values of angle of inclination, Hartmann number, Pressure gradient P and Reynolds Number and the tables and graphs have been shown.

Table 1: Different Values of Theta

| y | theta=pi/6 | theta=pi/4 | theta=pi/3 | theta=pi/2 |
|------|------------|------------|------------|------------|
| -1 | 0 | 0 | 0 | 0 |
| -0.8 | 0.0822 | 0.0554 | 0.0402 | 0.0308 |
| -0.6 | 0.1596 | 0.1073 | 0.0773 | 0.0585 |
| -0.4 | 0.2354 | 0.1597 | 0.1156 | 0.0875 |
| -0.2 | 0.3127 | 0.217 | 0.1598 | 0.1227 |
| 0 | 0.3944 | 0.2836 | 0.2153 | 0.1696 |
| 0.2 | 0.484 | 0.3651 | 0.2889 | 0.2359 |
| 0.4 | 0.5849 | 0.4679 | 0.3894 | 0.3324 |
| 0.6 | 0.7013 | 0.6003 | 0.529 | 0.4747 |
| 0.8 | 0.8379 | 0.7731 | 0.7246 | 0.6858 |
| 1 | 1 | 1 | 1 | 1 |

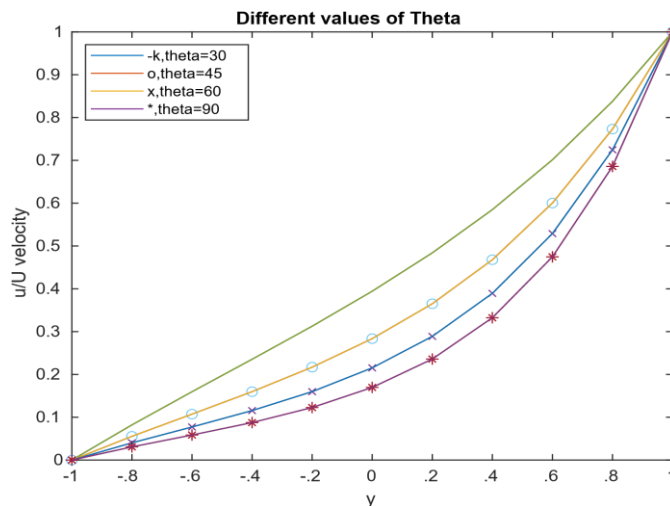


Fig 2. Velocity profile for Different Values of Theta, M=2, P=1, Re=.2

In Table 1. The velocity of fluid has been calculated for different angles of inclination of the magnetic field by using DTM method and it is observed that as angle of inclination of the magnetic field increases, the velocity profile decreases which can be observed in the graph shown in Fig 2.

Table 2: Different values of M

P=1, Theta=60, Re=.2

| y | M=.5 | M=1 | M=1.5 | M=2 |
|------|--------|--------|--------|--------|
| -1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| -0.8 | 0.1223 | 0.0920 | 0.0621 | 0.0402 |
| -0.6 | 0.2376 | 0.1788 | 0.1204 | 0.0773 |
| -0.4 | 0.3466 | 0.2630 | 0.1789 | 0.1156 |
| -0.2 | 0.4503 | 0.3470 | 0.2414 | 0.1598 |
| 0 | 0.5493 | 0.4334 | 0.3122 | 0.2153 |
| 0.2 | 0.6444 | 0.5249 | 0.3962 | 0.2889 |
| 0.4 | 0.7363 | 0.6241 | 0.4991 | 0.3894 |
| 0.6 | 0.8258 | 0.7341 | 0.6278 | 0.5290 |
| 0.8 | 0.9135 | 0.8582 | 0.7911 | 0.7246 |
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

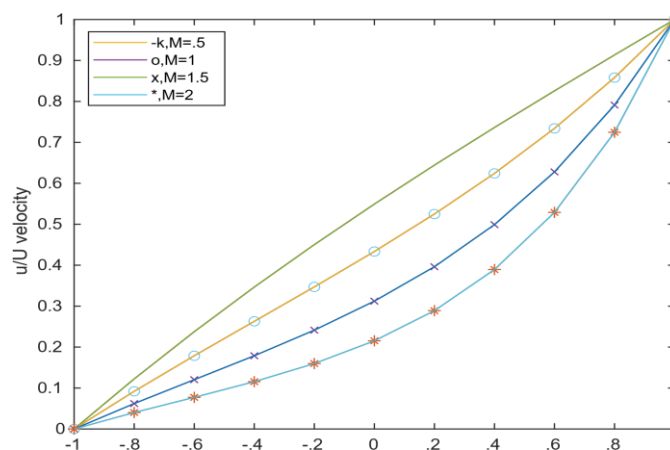


Fig 3. Velocity profile for Different Values of M

In Table 2, for different values of Hartmann number the velocity profile has been calculated and shown graphically. Fig 3 shows that as Hartmann number increases there is a decrease in the velocity. This is because Hartmann number is ratio of magnetic forces to viscous forces so larger the Hartmann number, stronger the magnetic forces which reduces the velocity due to action of Lorentz forces.

Table 3. Different values of Re (Reynolds number)

P=1, M=2, Theta= 60

| y | Re=.2 | Re=.4 | Re=.6 | Re=.8 |
|------|--------|--------|--------|--------|
| -1 | 0 | 0 | 0 | 0 |
| -0.8 | 0.0402 | 0.0583 | 0.0764 | 0.0945 |
| -0.6 | 0.0773 | 0.1075 | 0.1378 | 0.1681 |
| -0.4 | 0.1156 | 0.1537 | 0.1917 | 0.2298 |
| -0.2 | 0.1598 | 0.2022 | 0.2446 | 0.287 |
| 0 | 0.2153 | 0.2591 | 0.3029 | 0.3467 |
| 0.2 | 0.2889 | 0.3313 | 0.3737 | 0.4161 |
| 0.4 | 0.3894 | 0.4275 | 0.4655 | 0.5036 |
| 0.6 | 0.529 | 0.5593 | 0.5896 | 0.6199 |
| 0.8 | 0.7246 | 0.7427 | 0.7608 | 0.7789 |
| 1 | 1 | 1 | 1 | 1 |

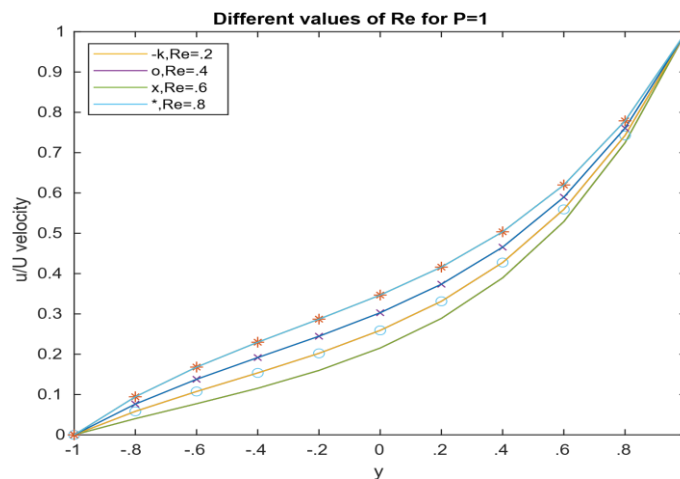


Fig 4. Velocity profile for Different Values of Re, P=1

In Table 3, for different values of Re but P positive the values of velocity profile have calculated by DTM method and shown graphically. Fig 4. Shows the effect of increase in Reynolds number when P is positive.

When P is positive, the pressure gradient is negative. This negative pressure gradient indicates the pressure is decreasing in the x direction along with decreasing viscous forces because of increased Reynolds number causes an increase in velocity profile.

Table 4. Different values of Re (Reynolds number)

P=-2, M=2, Theta= 60

| Y | Re=.2 | Re=.4 | Re=.6 | Re=.8 |
|------|---------|---------|---------|---------|
| -1 | 0 | 0 | 0 | 0 |
| -0.8 | -0.014 | -0.0502 | -0.0864 | -0.1226 |
| -0.6 | -0.0136 | -0.0742 | -0.1348 | -0.1953 |

| | | | | |
|------|--------|---------|---------|---------|
| -0.4 | 0.0013 | -0.0748 | -0.151 | -0.2271 |
| -0.2 | 0.0326 | -0.0522 | -0.137 | -0.2218 |
| 0 | 0.084 | -0.0036 | -0.0912 | -0.1788 |
| 0.2 | 0.1617 | 0.0769 | -0.008 | -0.0928 |
| 0.4 | 0.2751 | 0.199 | 0.1228 | 0.0467 |
| 0.6 | 0.4381 | 0.3775 | 0.317 | 0.2564 |
| 0.8 | 0.6704 | 0.6342 | 0.598 | 0.5618 |
| 1 | 1 | 1 | 1 | 1 |

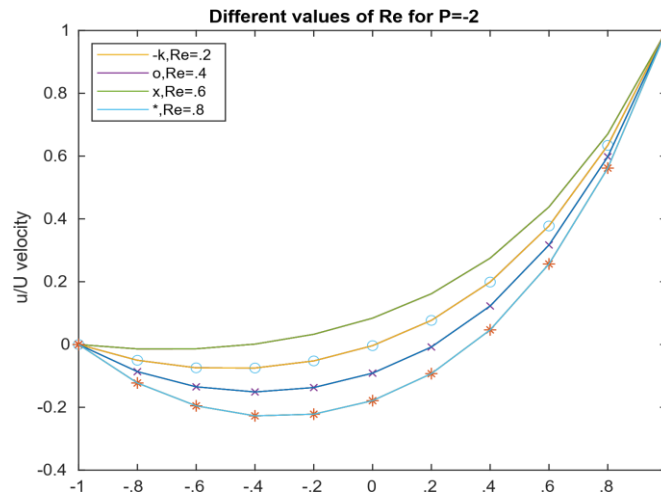


Fig 5. Velocity profile for Different Values of Re, P=-2

In Table 4, for different values of Re but P negative the values of velocity profile have been calculated by DTM method and shown graphically. Fig.5 shows the effect of increase in Reynolds number, when P is negative. When P is negative, the pressure gradient is positive. This positive pressure gradient called as adverse pressure gradient means the flow is moving into a region of increasing pressure which will cause a decrease in velocity as the fluid motion created by the plate is not strong enough to overcome this pressure gradient even as Reynolds number increases

Table 5: Different Values of P

| y | P=-3 | P= -2 | P=-1 | P=0 | P=1 | P=2 |
|------|---------|---------|--------|--------|--------|--------|
| -1 | 0 | 0.0000 | 0 | 0 | 0 | 0 |
| -0.8 | -0.0321 | -0.014 | 0.0041 | 0.0221 | 0.0402 | 0.0583 |
| -0.6 | -0.0439 | -0.0136 | 0.0167 | 0.047 | 0.0773 | 0.1075 |
| -0.4 | -0.0367 | 0.0013 | 0.0394 | 0.0775 | 0.1156 | 0.1537 |
| -0.2 | -0.0098 | 0.0326 | 0.075 | 0.1174 | 0.1598 | 0.2022 |
| 0 | 0.0402 | 0.084 | 0.1278 | 0.1716 | 0.2153 | 0.2591 |
| 0.2 | 0.1193 | 0.1617 | 0.2041 | 0.2465 | 0.2889 | 0.3313 |
| 0.4 | 0.2371 | 0.2751 | 0.3132 | 0.3513 | 0.3894 | 0.4275 |
| 0.6 | 0.4078 | 0.4381 | 0.4684 | 0.4987 | 0.529 | 0.5593 |
| 0.8 | 0.6523 | 0.6704 | 0.6884 | 0.7065 | 0.7246 | 0.7427 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

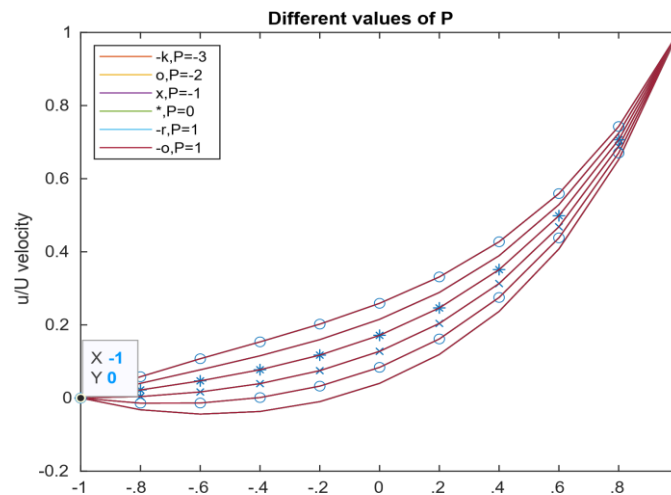


Fig 6. Velocity profile for Different Values of P for M=2, theta=60, Re=.2

In Table 5. The velocity profile has been calculated for different values of P using DTM method and shown graphically. Fig. 6 shows the effect of pressure gradient on the fluid flow. It is observed that when P is positive (P=1, 2), the pressure gradient $\frac{dp}{dx} = -P < 0$. That is we have a negative pressure gradient which favors the flow and overcomes the shear stress and the velocity increases across the plate.

When P is negative (P=-3,-2,-1), the pressure gradient $\frac{dp}{dx} = -P > 0$. That is we have a positive pressure gradient or adverse pressure gradient which fluids find difficult to negotiate therefore the velocity becomes negative in the lower half region and then increases slowly across the plate. As P increases from negative to positive it is observed that the velocity of fluid increases as the pressure gradient is increasing from positive to negative.

8. VALIDATIONS OF RESULTS:

The results obtained by using DTM were compared with Singh (2014) who used an analytic method and were found to in total agreement. However he had not considered the effect of negative and positive pressure gradient and Reynolds number on fluid flow.

9. RESULTS AND DISCUSSION

The problem of the effect of inclined magnetic field, pressure gradient on MHD flow between two infinite parallel plates with upper plate moving with constant velocity and lower plate stationary along with applied pressure gradient has been investigated. From the analysis the following results were observed

- 9.1 As the angle of inclination of magnetic field increases it is found that there is a decrease in the velocity profile.
- 9.2 As the strength of the magnetic field increases, there is a decrease in the velocity profile.
- 9.3. Negative and positive pressure gradient increase or decrease the velocity
- 9.4 Increase in Reynolds number effects the velocity of flow.

10. CONCLUSION

The study concerns the effect of inclination of magnetic field, Pressure gradient, Reynolds number on MHD flow between parallel plates. The results obtained using DTM are in agreement with the analytic method and show that for an increased angle of inclination of magnetic field the velocity decreases. An increase in the magnetic field strength M also results in decrease in the velocity. Further, the effect of pressure gradient on the fluid flow has been observed, where negative and positive pressure gradient increases and decreases the flow rate respectively. Also it is observed that increase in Reynolds number results in increase or decrease of velocity depending on whether a negative or positive pressure gradient is applied, which can be used in

controlling the flow in some engineering problems. Pressure gradient is one of the factors that influence a fluid flow immensely. Differential Transform method is found to be fast, accurate and converges very fast to the exact solution.

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