EFFICIENT BONDAGE NUMBER OF A JUMP GRAPH

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ABSTRACT:- A set S of vertices in jump graph J(G) is an efficient domination set, If every vertex in V-S is adjacent exactly one vertex in S. The efficient domination number γ_e(J(G)) of J(G) is minimum number of vertices is an efficient dominating set of J(G). In general γ_e(J(G)) can be made to increase by removal of edges from J(G). Our main objective is to Study this phenomenon. Let E be set of edges of J(G) such that γ_e(J(G)- E) > γ_e(J(G)). Then we define the efficient bondage number b_e(J(G)) of J(G) to be the minimum number of edges in E. In this communication an upper bound for b_e(J(G)) has been established and its exact values for some classes of graph have been found. In addition Nordhaus-Gaddum type results are established.

Key words: dominating set, bondage number.

Mathematical classification: b05C56.

1. INTRODUCTION: Dominating sets were studied by Berge.C[1] and ore[2] Domination alteration sets in gaphs were studied by Bauer et.al[3] . A similar5concept named as the bondage number of a graph was studied by Fink et.al.,[4]. The efficient domination number was introduced by Cockayayne et.al., [5]. In this communication we study stability of γ_e(J(G)) by defining the efficient bondage number b_e(J(G)) of a jump gaph J(G), The graphs considered in this communication are finite undirected, without loops, multiple edges and isolated vertices. Any undefined terms here may be found in Harary [6]. A set X of vertices is a dominating set of J(G)(if every vertex in X is adjacent at least one vertex in X. The domination number γ (J(G) of J(G) is the minimum number of vertices in a dominating set of J(G). Let E be a set of edges such that γ(J(G)- E) > γ(J(G)). Then the bondage number b(J(G)) of J(G) is the minimum number of edges in E. A set S of vertices in J(G) is an efficient dominating set if every vertx u in V-S is adjacent to exactly one vertex in S. The efficient domination number γ_e(J(G)) of J(G) is the minimum number of vertices is an efficient dominating set of J(G).

Let E be a set of edges such that γ_e(J(G) _ E) > γ_e(J(G)).Then we can define the efficient bondage number b_e(J(G)) is the minimum number of edges in E. Here we note that if

γ_e(J(G)) = p then b_e(J(G)) does not exists.

2. Results:

The following results is straight fowar hence we omit the proof.

Theorem1: for any graph J(G) with p vertices

γ_e(J(G)) = 1 if and only if Δ(J(G))=p-1.

Theorem 2: For any path P_p with p=2 vertices

$\gamma_e(J(P_p)) = \lceil p/3 \rceil$

Theorem 3: For any cycle C_p with p ≥ 3vertices

$\gamma_e(C_p) = \lceil \frac{p}{3} \rceil$ if p ≡0,1 (mod 3)

$= \lceil \frac{p}{3} \rceil = 1$ if p ≡ 2(mod 3)

Hence ⌈x⌉ denotes the least integer greater than or equal to x,

3. Main Results

Theorem 3.1: Let J(G) be a graph Δ(J(G)) = p – 1 Then b_e(J(G))= ┌ $\frac{n}{2}$ ┐ where n is the number of vertices of degree p - 1.

Proof: Let u_1, u_2, u_3………u_n be the n vertices of degree p – 1 then clearly removal of fewer then ┌ $\frac{n}{2}$ ┐ edges results into a graph J(G') having maximum degree Δ (G')= p – 1.

Hence b_e(J(G)) ≥ ┌ $\frac{n}{2}$ ┐

Now we consider the following cases.

Case (i): If n is even then the removal of $\frac{n}{2}$ independent edges u_1u_2, u_3u_4,……..$u_{n-1}u_n$ results into a graph J(H) having Δ (J(G)) = p - 2 Hence b_e(J(G)) = $\frac{p}{2}$.

Case (ii) : If n is odd then the removal of $\frac{n-1}{2}$ independent edges u_1u_2, u_3u_4,…………$u_{n-2}u_{n-1}$ yields a graph J(H') containing exactly one vertex u_n of degree p-1. Thus by removing an edge incident with u_n we obtain a graph J(H'') with Δ(J(H'')) = p – 2 γ_e(J(G')) ≥ 2.

Hence from case (i) and (ii) it follows that

B_e(J(G)) = $\frac{n}{2}$ if p is even

= $\frac{n-1}{2}$ + 1 if p is odd

B_e(J(G)) = ┌ $\frac{n}{2}$ ┐

Hence the proof.

The following result directly from Theorem 3.1

Proposition 3.2: For any complete graph K_p with p≥ 2 vertices $b_e(J(K_p))$= ┌ $\frac{p}{2}$ ┐

Proof: By theorem 3.1 $b_e(K_p)$ = $\frac{n}{2}$ since n=p

Proposition 3.3: For any wheel W_p with p ≥ 5 vertices $b_e(J(W_p))$ = 1

Proof: Since W_p contains exactly one vertex of degree p – 1 Hence

$B_e(J(W_p))$ = ┌ $\frac{1}{2}$ ┐ = 1

Theorem3.4L Let $K_{m,n}$ be a complete bipartite graph other then C_3 with 1 ≤ m ≤ n then

$B_e(J(K_{m,n}))$ = m

Proof: Let $v=v_1 \cup v_2$ be the vertex sets of $K_{m,n}$ where | v_1 |=m and | v_2 |=n let v ∈ v_2 then by removing all edges incident with v we obtain a graph J(G') containing two components

$_{K1}$ and $K_{m,n-1}$

Hence γ_e(J(G')) = $\gamma_e(J(K_1))$ + γ_e $(J(K_{m,n-1}))$

= 1 + $\gamma_e(JK_{m,n})$ ≥ γ_e $(J(K_{m,n}))$

Thus $b_e(J(K_{m,n}))$ = deg v = $|v_1|$=m

Proposition 3.5: For any cycle C_p with $p \geq 3$ vertices

$b_e(J(C_p)) = 2$ if $p \equiv 0 \pmod 3$

$=3$ if $p \equiv 1 \pmod 3$

$= 4$ if $p \equiv 2 \pmod 3$

Proof: Let C_p be a cycle with $p \geq 3$ vertices . Then we consider the following cases,

Case 1: If $p \equiv 0 \pmod 3$ let J(H) be a graph OBTAINED BY REMOVING TWO ADJACENT EDGES FROM c_P. Then clearly J(H) consists of an isolated vertex and a path of order p – 1.

Thus $\gamma_e(J(H)) = 1 + \gamma_e(J(P_{p-1})) = 1 + \lceil \frac{p-1}{3} \rceil$

Since $p \equiv 0 \pmod 3$ $\lceil \frac{p-1}{3} \rceil = \lceil \frac{p}{3} \rceil$

Therefore $\gamma_e(J(H)) = 1 + \lceil \frac{p}{3} \rceil$

$= 1 + \gamma_e(J(C_p)) > \gamma_e(J(C_p))$

Hence $b_e(J(C_p))=2$

Case 2: if $p \equiv 1 \pmod 3$ then the removal of three consecutive edgs from $J(C_p)$ results in a graph J(H) consisting of two isolated vertices and a path of order p – 2 Hence,

$\gamma_e(J(H)) = 2 + \gamma_e(J(P_{p-2}))$

$= 2 + \lceil \frac{p-2}{3} \rceil$

$= 1 + \lceil \frac{p}{3} \rceil$

$= 1 + \gamma_e(J(C_p)) > \gamma_e(J(C_p))$

Thus $b_e(J(C_p)) = 3$.

Case 3: if $p \equiv 2 \pmod 3$ then by removing four consecutive edges from C_p we obtain a graph J(H) containing three isolated vertices and a path of order p – 3 then

$\gamma_e(J(H)) = 3 + \gamma_e(J(P_{p-3})) = 3 + \lceil \frac{p-3}{3} \rceil$

$= 2 + \lceil \frac{p}{3} \rceil > \gamma_e(J(C_p))$

Hence $b_e(JC_p)) = 4$

Hence the proof.

Proposition 3.6: For any path P_p with $p \geq 2$ vertices then

$b_e(J(P_p)) = 2$ if $p \equiv 1 \pmod 3$

$= 1$ otherwise.

Proof: Let P_p be a path with p > 2 then we consider the following cases,

Case 1: if $p \equiv 1 \pmod 3$ then the removal of two end edges results a graph J(G') containing two isolated vertices and path of order p – 2. Hence.

$$\gamma_e(J(G')) = 2 + \gamma_e(J(P_{p-2}))$$

$$= 2 + \lceil \frac{p-2}{3} \rceil$$

$$= \lceil \frac{p}{3} \rceil + 1 > \lceil \frac{p}{3} \rceil = \gamma_e(J(P_p))$$

Thus $b_e(J(P_p)) = 2$

Case 2: If $p \not\equiv 1 \pmod 3$ then the removed of an edge from $J(P_p)$ results a graph J(H) containing an isolated vertex and a path of oder p – 1.

$$\therefore \gamma_e(J(H)) = 1 + \gamma_e(J(P_{p-1}))$$

$$= 1 + \lceil \frac{p-1}{3} \rceil$$

$$= \lceil \frac{p}{3} \rceil = \gamma_e(J(P_p))$$

Hence $b_e(J(P_p)) = 1$

Theorem3.7: For any connected graph J(G) with $p \geq 2$ vertices $b_e(J(G)) \leq p - 1$.

Further the bound is attained if $G = C_p$ with $3 \leq p \leq 5$

Proof: On the contrary suppose $b_e(J(G)) \geq p$ let E_u denote the set of edges incident with a vertex u. Then clearly

$$\gamma_e(J(G- E_u)) \geq \gamma_e(J(G))$$

Which is a contradiction and $| E_u | \leq p - 1$

Hence $b_e(J(G)) \leq p - 1$

Further for $J(G) = J(C_p)$ with $3 \leq p \leq 5$ it is easy to see that

$$b_e(J(G)) = p - 1$$

Theorem 3.8: Let u and v be distinct adjacent vertices in a non trivial graph J(G) then

$$B_e(J(G)) = \min \{ \deg u + \deg v \}.$$

Proof: Let u and v be two distinct adjacent vertices of J(G) such that eg u + deg v is minimum. Suppose $b_e(J(G))$ = deg u + deg v. Let E_u denote the set of edgs that are incident with u and v Then clearly | E |= deg u +deg v – 1 and hence $\gamma_e(J(G) - E) = \gamma_e(J(G))$. Since u and v are isolated vertices in J(G) – E, $\gamma_e(J(G) - u - V) = \gamma_e(J(G)) - 2$. Thus for any minimum efficient dominating set S of J(G) – u – v, S ∪ | u | is an efficient dominating set of J(G) with cardinality $\gamma_e(J(G)) - 1$, a contradiction. Hence $b_e(J(G)) \leq \min \{ \deg u + \deg v \}$.

Corollary 3.8.1: For any nontrivial graph J(G)

$$b_e(J(G)) \leq \delta(J(G)) + \Delta(J(G))$$

Proof: This follows from the Theorem 3.8

Now we obtain a Nordhaus-Gaddum type result.

Theorem 3.9 For any graph J(G)

(i) $b_e(J(G)) + b_e(J(\bar{G})) \leq 2 (p - 1)$ and
(ii) $b_e(J(G)) . b_e(J(\bar{G})) \leq 2p (\delta(J(G)) + \Delta (J(G)))$

Proof: By corollary 3.8.1 we have

$$b_e(J(G)) \leq \delta(J(G)) + \Delta (J(G))$$

and $b_e(J(\bar{G})) \leq \delta(J(\bar{G})) + \Delta (J(\bar{G}))$

Hence $b_e(JG)) + b_eJ(\bar{G}) = \delta(J(G)) + \Delta (J(G)) + \delta(J(\bar{G})) + \Delta (J(\bar{G}))$

$$= \delta(J(G)) + \Delta (J(G)) + p -1 - \Delta (J(G)) + p - 1 - \delta(J(G))$$

$$=2(p - 1)$$

Also $b_e(JG)) . b_eJ(\bar{G}) = (\delta(J(G)) + \Delta (J(G))) . (\delta(J(\bar{G})) + \Delta (J(\bar{G})))$

$$= (\delta(J(G)) + \Delta (J(G))) (p -1 - \Delta (J(G)) + p - 1 - \delta(J(G)))$$

$$= (\delta(J(G)) + \Delta (J(G))) (2(p-1) - \Delta(J(G)) - \delta (J(G)))$$

$$= 2p ((\delta(J(G)) + \Delta (J(G))))$$

Thus $b_e(J(G)) + b_e(J(\bar{G})) \leq 2 (p - 1)$ and
$b_e(J(G)) . b_e(J(\bar{G})) \leq 2p (\delta(J(G)) + \Delta (J(G)))$.

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