THE STRONG NON-SPLIT DOMINATION NUMBER OF A JUMP GRAPH

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ABSTRACT - *A dominating set D of a graph J(G)=(V,E) is a strong non-split dominating set if the induced sub graph <V-D> is complete. The non split domination number γ_{ns}(J(G)) of J(G) is minimum cardinality of a strong non split dominating set. In this paper we relate this parameter to other parameters of jump graph J(G) and obtain its exact values for some standard graphs.*

INTRODUCTION:

All the graphs considered here are assumed to be finite undirected, nontrivial, and connected without loops or multiple edges. Any undefined tem in this paper may be found in Hynes et.al.,[2].

Let J(G) be a jump graph. A set D⊆ V is a dominating set of J(G) if every vertex in V-D is adjacent to some vertex in D. The domination number γ (J(G)) of J(G) is the minimum cardinality of a dominating set.

N.Pratap Babu Rao and Sweta .N introduced the concept of non split domination in jump graph.

A dominating set D of a graph S_1={ 1} is a non split dominating set if the induced sub graph <V-D> is connected. The non split dominating number γ_{ns}(J(G)) of J(G) is the minimum cardinality of a non split dominating set.

Dominating sets whose induce a complete sub graph have a great diversity of applications one such application is the following.

In setting up the communication limits in a network one might want a strong core group there can communicate with each other member of the core group and so that everyone in the group receives the message from someone outside the group and communicate it to every other in the group This suggests the following definition.

Definition 1: A dominating a set of vertices a γ -set if it is a dominating set with cardinality γ (J(G)). Similarly, a γ_{ns}-set an a γ_{sns}-set one defined unless and otherwise stated, the graphs has p vertices and q edges.

2. RESULTS

Theorem:1 For any graph G

γ (J(G)) ≤ γ_{ns}(J(G)) ≤ γ_{sns}(J(G)

Proof: This follows from the fact that every strong non split dominating set of J(G)is a non split dominating set and every non split dominating set is a dominating set.

The following characterization is easy to see hence we omit its proof

Theorem 2 : A strong non split dominating set D of J(G) is minimal if and only if for each vertex v∈ D, one of the following conditions is satisfied.

- **i)** There exists a vertex u ∈ V-D suchthat N(u)⋂ D –{v}
- **ii)** V is an isolated vertex in <D>
- **iii)** There exists a vertex w∈ V-D such that w is not adjacent to v Now we obtain a relationship between γ_{sns}(J(G)) and γ_{sns}(J(H)) where J(H) ia an spanning sub graph of J(G).

Theorem 3: For any spanning sub graph J(H) of J(G)

$\gamma_{sns}(J(G)) \leq \gamma_{sns}(J(H))$

Proof: Since every non split dominating set of J(H) is a strong non split dominating set of J(G).

Above inequality holds.

Next we obtain a lower bound on $\gamma_{sns}(J(G))$

Theorem4: For any jump graph J(G)

$\beta_0(J(G) \leq \gamma_{sns}(J(G))$

where $\beta_0(J(G))$ is the independent number of J(G)

Proof : Let D be a $\gamma_{sns}(J(G))$-set of J(G) and S be an independent set of vertices in J(G). Then either $S \subseteq D$ or S contains at most one vertex from V-D and at most |D|-1 vertices from D

This implies $\beta_0(J(G) \leq \gamma_{sns}(J(G))$.

The bound given in above inequality is sharp.

For example for a complete jump graph $J(K_p)$ $\gamma_{sns}(J(K_p)) = 1 = \beta_0(J(K_p))$

Now we prove the major result of this paper from which we can deduce the exact values of $\gamma_{sns}(J(G))$ for some standard graphs

Theorem 5: For any graph J(G),

$p - w(J(G)) \leq \gamma_{sns}(J(G))$, $\gamma_{sns}(J(G)) \leq p - w(J(G)) + 1$

where w(J(G)) is a clique number of J(G) be a complete

Proof : Let D be a γ_{sns} – set of J(G). Since (V-D) is complete

$W(J(G)) \geq |V-D|$………….(A)

Let $\langle S \rangle$ be a complete graph with $|S| = w(J(G))$. Then for any vertex $u \in S$,

$\langle V-D \rangle \cup |u|$ is a strong nonsplit dominating set of J(G) and

$\gamma_{sns}(J(G)) \leq p - w(J(G) + 1$……………(B)

from (A) and (B) $p - w(J(G)) \leq \gamma_{sns}(J(G)) \leq p - w(J(G)) + 1$

This proves the result.

Corollary 5.1: let J(G) be a graph with w(J(G)) ≥ □(J(G)) then $\gamma_{sns}(J(G) \leq p - \gamma(J(G))$ where $\gamma(J(G))$ is the minimum degree of J(G). Further, the bound is attained if and only if one of the following is satisfied.

i) $W(J(G) = \gamma(J(G))$

ii) $W(J(G)) = \gamma(J(G)) + 1$and every S contains a vertex not adjacent to an vertex of V-S.

Proof : suppose w(J(G))n≥ γ (J(G)) + 1 then by above theorems

γ_{sns}(J(G)) ≤ p - γ (J(G)) suppose w(J(G)) = γ (J(G)) as let S be a w-set of J(G). Then V-S is a strong non split dominating set of J(G) and hence γ_{sns}(J(G)) ≤ p - γ (J(G)).

Now we prove the second part.

Suppose one of the given conditions is satisfied then from the above results it is easy to see that γ_{sns}(J(G)) ≤ p - γ (J(G)).

Suppose the bound is attained. Then again from the above w(J(G)) = γ(J(G)) or w(J(G) = ?(J(G)) + 1. Suppose there exists a w-set S with |S|= γ(J(G)) + 1 such that every vertex in S is adjacent to some vertex in V-S. Then V-S is a strong non split dominating set of J(G), and hence γ_{sns}(J(G)) ≤ p - γ(J(G))-1 which a contradiction. Hence one of the given conditions is satisfied.

In the next result we list the exact values of γ_{sns}(J(G)) for some standard graphs.

Proposition 6 :

i) For any complete jump graph J(K_p) with p ≥ 2 vertices γ_{sns}J(K_p)=1
ii) For any complete bipartite jump graph $K_{m,n}$ with 2 ≤ m ≤ n
γ_{sns}(J($K_{m,n}$)) =m + n-2
iii) For any cycle J(C_p)) with p ≥3 vertices γ_{sns}(J(C_p)) = p – 2
iv) For any path J(P_p) with p ≥ 4 vertices γ_{sns}(J(P_p)) =p -2
v) For any wheel J(w_p) with p ≥ 4 vertices γ_{sns}(J(w_p)) = p – 3.

A set D of vertices in a jump graph J(G)=(V,E) is a vertex set dominating set if for any set S ⊆ V-D, there exists a vertex v∈ D such that the induced sub graph

<S∪{v} > is connected. The vertex set domination number γ_{vs} (J(G)) of J(G) is the minimum cardinality of a vertex set dominating set[8]

Theorem 7 : If a graph J(G) has independent strong non split dominating set then diam (J(G)) ≤ 3 where diam (J(G)) is the diameter of J(G).

Proof: Let D be an independent strong non split dominating set of J(G)

We consider the following s Cases,

Case(i): u, v ∈ V-D then d(u,v)=1

Case (ii) let u ∈ D and v ∈ V-D. Since is independent, there exist a vertex w∈V-D such that u is adjacent to w , Thus d(u,v) ≤ d(u,w) + d(w,v) ≤ 2

Case(iii) Let u, v ∈ D As above there exists two vertices w_1, w_2 ∈ V –D such that u is adjacent w_1 and v is adjacent to w_2 Thus, d(u,v) ≤ d(u,w_1) + d(w_1, w_2) + d(w_2, v) ≤ 3

Thus for all vertices u,v ∈V, d(u,v) ≤ 3

Hence diam (J(G)) ≤ 3

Corllory 7.1: if γ (J(G)) = γ_{sns}(J(G)) then diam (J(G)) ≤ 3

Proof : Let D be a γ_{sns}-set of J(G).Since D is also a γ-set every vertex v∈D is adjacent to at least one vertex u ∈ V-D. s in the proof of theorem 7 one can show that d(u,v) ≤ 3 for all vertices u,v ∈ V.

Thus diam (J(G)) ≤ 3.

Theorem 8: Let D be an independent set of vertices in G if |D| < 1 - Δ(J(G)) then V-D is a strong non split dominating set of J($\bar{G}$) where J($\bar{G}$) is the complement of J(G).

Proof : Since each vertex v∈ D is not adjacent to at least one vertex in V-D,it implies that V-D is a dominating set og J($\bar{G}$) and further it is a strong non split dominating set as <D> is complete in J($\bar{G}$).

A dominating set D of a connected graph J(G) is a split dominating set, if the induced sub graph <V-D> is connected[4]. In [5] Kulli and Janakiram extended the concept of split domination to strong split domination as follows.

A dominating set D of a connected graph J(G) is a strong split dominating set if induced sub graph <V-D> is totally disconnected with at least two vertices. The strong split number $\gamma_{sns(}$J($\bar{G}$)) of J($\bar{G}$) is minimum cardinality of a strong split dominating set.

Theorem 9 : Let d be a γ_{sns}-set of J(G) by (1) and (5) V-D has at least two vertices. Also by (ii) every vertex in V-d is not adjacent to at least one vertex in D. This implies that D is a dominating set of J($\bar{G}$) and further it is a strong split dominating set as <V-D> is totally disconnected with at least two vertices in J($\bar{G}$).

Thus γ_{ss}(J($\bar{G}$)) ≤ γ_{sns}(J(G))

Kulli and Janakiram [6] introduced the concept,

A dominating set D of a graph J(G)=(V,E) is a regular set dominating set if for any set I⊆ V-D, there exists a se S⊆ D such that the induced sub graph < I ∪D> is regular. The regular set domination number ⍰$_{rs}$(J(G) is the minimum cardinality of a regular set dominating set.

Theorem 10 : For any graph J(G)

γ_{ns}(J(G)) ≤ γ_{sns}(J(G)) + 1.

Proof : Let D be a γ_{sns}-set of J(G).Since <V-D> is complete for any vertex u∈ V-D, D ∪ {u} is a regular set dominating set of J(G).This proves the result.

Theorem 11 : If diam(J(G)) ≤ 3 then

γ_{sns}(J(G)) ≤ p −m where m is the number of cut vertices of J(G)

Proof : If J(G) has no cut vertices, then the result is trivial

Let S be the set of all cut vertices with |S|=m, let u,v∈ S suppose u and v are not adjacent. Since there exists two vertices u_1 and v_1 sch that u_1 is adjacent to u and v_1 is adjacent to v, it implies that d(u_1, v_1) ≥ 4 a contradiction. Hence every two vertices in S are adjacent and every vertex in S is adjacent to at least one vertex in V-S. This proves that V-S is a strong non split dominating set of J(G).

Hence the result.

Theorem 12 : Let J(G) be a graph such that every vertex of J(G) is either a cut vertex or an end vertex of w(J(G)) then

γ_{ns}(J(G)) = γ_{sns}(J(G)) = p − m where m is the number of cut vertices of J(G)

Proof : Let S be the set of all cut vertices with |S|=m since w(J(G))=m it implies that every two vertices in S are adjacent and hence every vertex in S is adjacent to an end vertex. This proves that V-S is a γ_{ns}-set of J(G) and further it is a

γ_{sns}(J(G))-set as <S> is complete

Hence the result.

The following definition is used to prove one next result.

A dominating set D of a graph J(G) is an efficient dominating set if every vertex in V-D is adjacent to exactly one vertex in D. This concept was introduced by Cockayne et.al.,

Theorem 13 : Let J(G) be a n-regular graph with 2n vertices. If D is an efficient dominating set of J(G) with n-vertices then both D and V-D are strong non split dominating sets of J(G)

Proof: Since every vertex in V-D is adjacent to exactly one vertex in D. It implies that every two vertices in V-D and adjacent. As J(G) is n-regular every vertex in D is adjacent to some vertex in V-D. Suppose that there exists a vertex u∈D such that u is adjacent to two or more vertices in V-D. The there exists a vertex v∈ D such that deg(v) ≤ n-1 a contradiction. Hence every vertex in D is adjacent to exactly one vertex in V-D. Thus as above every two vertices in D are adjacent. Hence D and V-D are strong non split dominating set of J(G).

Theorem 14: Let J(G) be a graph with Δ(J(G)) ≤ p – 2 when D be a strong non split dominating set of J(G) such that <D> is complete and |D|≤ γ (J(G)) then

i) D is minimal
ii) V-D is also a minimal strong non split dominating set of J(G)

Proof: Since <D> is complete, it implies that for end vertex v∈ D there exists a vertex u ∈ V-D such that v is not adjacent to u. Thus by theorem 2 D is minimal.

|D| ≤ γ (J(G)), it implies that every vertex in D is adjacent to some vertex in V-D. Then V-D is strong non split dominating set of J(G) and further as above it is minimal.

Theorem15 : if Δ(J(G) < α_0(J(G)) then γ_{sns}(J(($\bar{G}$))= p – w(J($\bar{G}$)) where α_0(J(G)) is the vertex covering number of J(G).

Proof : Let S be a vertex cover of J(G) with |S| = α_0(J(G)). Since Δ(J(G) < α_0(J(G)), J(G) ≠ J(K_p) and V-S is an independent set with at least two vertices such that every vertex in V-S is not adjacent to at least one vertex in S. This proves that S is a strong non split dominating set of J(($\bar{G}$)

Thus γ_{sns}(J(($\bar{G}$)) ≤ |S|

≤ α_0(J(G))

≤ p – β_0(J(G))

≤ p – w(J(($\bar{G}$))

Result follows from theorem 5.

Next we obtain Nordhus Gaddum type results[7]

Theorem 16 : Let J(G) be a graph such that both J(G) and J(($\bar{G}$)) are connected. W(J(G) ≥ γ (J(G)) and w(J(($\bar{G}$))) ≥ γ(J(($\bar{G}$))) then

γ_{sns}(J(G)) + γ_{sns}(J(($\bar{G}$))) ≤ p + 1 + Δ(J(G) - γ (J(G)).

Proof: b By corollary 5.1 γ_{sns}(J(G)) ≤ p - γ(J(G)), and

γ_{sns}(J(($\bar{G}$)) ≤ p - γ(J(($\bar{G}$)) ≤ 1 + Δ(J(G))

Hence the result

ACKNOWLEDGEMENT;

The author thankful to the referees for their valuable comments and suggestions.

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