

## AN APPLICATION TO MARKOV CHAIN

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**Abstract** - The Objective of this paper we study the Markov chain is a Stochastic Process that Satisfies the Markov property. It is an Mathematical model which is depend on the probabilities and is very useful in so many science field. In this paper we would given brief explanation about Markov chain.

**Key Words:** Transition matrix, Regular matrix.

### 1. INTRODUCTION

Suppose there is a physical or mathematical system that has 'n' possible states and at any one time, the system is in one and only one of its 'n' states. As well, assume that at a given observation period.  $K^{th}$  period, the probability of the system being in a particular state depends only on its status at the  $K-1^{st}$  period such system is called markov chain or markov rocess. For a markov chain with 'n' states, the state vector is a column vector whose  $i^{th}$  component represents the probability that the system is in the  $i^{th}$  stat at that time. Note that the sum of the entries of a state vector is 1. For example, vectors  $X_0$  and  $X_1$  are state vectors. If  $P_{ij}$  is the probability of transition from one state j to state i, then the matrix  $T = [P_{ij}]$  is called the transition matrix of the markov chain.

#### 1. Transition matrix

A transition matrix  $T$  is called regular if, for some integer  $r$ , all entries of  $T^r$  are strictly positive.

For example, the matrix

$$A = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 1 \\ 0 & 0.5 & 0 \end{bmatrix}$$

Is regular since  $A^4 = \begin{bmatrix} 0.4375 & 0.3125 & 0.5 \\ 0.3125 & 0.625 & 0.125 \\ 0.25 & 0.625 & 0.375 \end{bmatrix}$

A markov chain process is called regular if its transition matrix is regular.

Suppose  $A$  is a square matrix  $A$ ,

We say that the number is an eigen value of  $A$  satisfying  $AX = \lambda X$ . In this case, we say that  $X$  is an eigen vector of  $A$  corresponding to the eigen value:

### 2. APPLICATION:

If you have lived in Vijayawada for a while you must have realized that the weather is a main concern of the population. An unofficial study of the weather in the city in early spring yields the following observations.

- It is almost impossible to have two nice day in a row.
- If we have a nice day, we just as likely to have snow or rain the next day.
- If we have snow or rain, then we have on even chance to have the same the next day.
- If there is a change from snow or rain, only half of the time is this a change to a nice day

1. Write the transition matrix to model this system.
2. If it is nice today, what is the probability of being nice after one week ?
3. Find the long time behaviour of the weather.

**SOLUTION:**

Since the weather tomorrow depends on today, this is a markov chain process.

The transposition matrix of this system is

$$T = \begin{matrix} N \\ R \\ S \end{matrix} \begin{bmatrix} 0 & 0.25 & 0.25 \\ 0.5 & 0.5 & 0.25 \\ 0.5 & 0.25 & 0.5 \end{bmatrix}$$

We indicate  $N$  is Nice

$R$  is Rain

$S$  is Snow.

If it is nice today, then the initial state vector is

$$X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} N \\ R \\ S \end{matrix}$$

After seven days, the state vector

$$X_7 = T^7 X_0$$

$$= \begin{bmatrix} 0.199951 & 0.2000122 & 0.2000122 \\ 0.4000244 & 0.400244 & 0.3999633 \\ 0.4000244 & 0.3999633 & 0.4000244 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1999511 \\ 0.4000244 \\ 0.4000244 \end{bmatrix}$$

So, there is about 20% chance of being nice in one week. We are dealing with a regular markov chain since the transition matrix is regular, so we are sure that the steady-state vector exists.

To find it we solve the homogeneous system  $(T - I)X = 0$ .

Consider the following coefficient matrix.

$$\begin{bmatrix} -1 & 0.25 & 0.25 \\ 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \end{bmatrix}$$

Reducing to reduce echelon form

$$\begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

The general solution of this system is

$$\begin{bmatrix} 0.5t \\ t \\ t \end{bmatrix}, t \in \mathbb{R}$$

The sum of its component is 1.

$$0.5t + t + t = 1, \text{ given } t = 0.4.$$

Thus, the steady-state vector is

$$\rho = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$$

### 3. CONCLUSION

In the long term, there is 20% chance of getting a nice day 40% chance of having a rain day and 40% chance of having a snowy day.

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