

# Six Sigma Modified Quick Switching Variables Sampling System Indexed by Six Sigma AQL and Six Sigma AOQL: Sample Size Tightened

Dr. D. Senthilkumar<sup>1</sup>, Dr. B. Esha Raffie<sup>2</sup>

<sup>1</sup>Head & Associate Professor, Department of Statistic, PSG College of Arts & Science, Coimbatore – 641014.

<sup>2</sup> Assistant Professor, Department of Statistic, PSG College of Arts & Science, Coimbatore – 641014.

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**Abstract:-** In this article gives sample size tightened in Modified Quick Switching Variables Sampling System indexed by Six Sigma AQL and Six Sigma AOQL. The resulting system is referred to as a “Six Sigma Modified Quick Switching Variables Sampling System” (SSQSVSS- $r(n_{T\sigma}, n_{N\sigma}; k_{\sigma})$ ,  $r=2$  and  $3$ ). These procedures verified with practical applications and also constructed tables for easy selection of plans given indexed by six sigma quality levels.

**Keywords:** Modified Quick Switching Variables Sampling System, Operating Characteristic Curve, Six Sigma AQL and Six Sigma AOQL.

## Introduction

The construction procedure for the SSQSVSS ( $n_{T\sigma}, n_{N\sigma}; k_{\sigma}$ ),  $r=2$  and  $3$ , indexed by SSAQL and SSAOQL is based on Govindaraju (1990) procedures and tables, developing for the selection of single sampling plan for variables indexed by AQL and AOQL. Soundararajan (1981) has developed procedures and tables for the selection of single sampling plans for attributes for given AQL and AOQL. Govindaraju (1990) has developed procedures and tables for the selection of single sampling plans for variables indexed by AQL and AOQL. Later Soundarajan and Palanivel (2000) have developed procedures and tables for the selection of quick switching single sampling variables systems indexed by AQL and AOQL. Based on above article Senthilkumar and Esha Raffie (2017) have constructed SSQSVSS ( $n_{\sigma}; k_{T\sigma}, k_{N\sigma}$ ) indexed by Six Sigma AQL and Six Sigma AOQL. Senthilkumar and Esha Raffie (2016) have constructed six sigma modified quick switching variables sampling system [SSMQSVSS- $r(n_{T\sigma}, n_{N\sigma}; k_{\sigma})$ ,  $r=2$  and  $3$ ] indexed by six sigma quality levels of SSAQL and SSLQL. This concept can be extended to variables quality characteristics of the study, the resulting plan would be designated as SSQSVSS- $r$  and would be applied under the following conditions:

- The production is steady, so that results on current and preceding lots are broadly indicative of a continuous process.
- Lots are submitted substantially in the order of production.
- Inspection is by variables, with the quality being defined as the fraction of non-conforming.
- The sample units are selected from a large lot and production is continuous.
- The production process depends on automation and human involvement in the process is negligible.
- The industry may adopt system method with decision makers have an experience in adopting the six sigma quality initiatives.

## Basic Assumptions

- The quality characteristic is represented by a random variable  $X$  measurable on a continuous scale.
- Distribution of  $X$  is normal with mean and standard deviation.
- An upper limit  $U$ , has been specified and a product is qualified as defective when  $X > U$ . [when the lower limit  $L$  is Specified, the product is a defective one if  $X < L$ ].
- The Purpose of inspection is to control the fraction defective,  $p$  in the lot inspected.

When the conditions listed above are satisfied the fraction defective in a lot will be defined by  $p = 1 - F(v) = F(-v)$  with  $v = (U - \mu) / \sigma$  and

$$F(y) = \int_{-\infty}^y \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad (1)$$

where  $z \sim N(0, 1)$ . Here the decision criterion for the  $\sigma$ - method variables plan is to accept the lot if  $\bar{X} + k_{\sigma} \leq U$ , where  $U$  is the upper specification limit or if  $\bar{X} + k_{\sigma} \geq L$ , where  $L$  is the lower specification limit.

## SSQSVSS- $r((n_{\sigma}; k_{T\sigma}, k_{N\sigma})$ , where $r=2$ and $3$ ) with known $\sigma$ for given SSAQL and SSLQL

The Six Sigma Modified Quick Switching Variables Sampling System with known  $\sigma$  variables plan as the reference plan has following Operating Procedure

### Operating Procedure

Step 1: Draw a sample of size  $n_\sigma$  from the lot through normal inspection, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean  $\bar{X}$ .

Step 2: If  $\bar{X} + k_{N\sigma} \sigma \leq U$  or  $\bar{X} + k_{N\sigma} \sigma \geq L$  accept the lot and repeat Step 1 otherwise, go to Step 3.

Step 3: Under tightened inspection, draw a sample of size  $n_\sigma$  from the next lot inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean  $\bar{X}$ .

Step 4: If  $\bar{X} + k_{T\sigma} \sigma \leq U$  or  $\bar{X} + k_{T\sigma} \sigma \geq L$  accept the lot. When  $r$  consecutive lots are accepted, switch to Step 1, otherwise repeat Step 3.

where  $k_{N\sigma}$  and  $k_{T\sigma}$  are the acceptance criterion of the variable sampling plan under normal and tightened inspection respectively. Tightened inspection may be achieved by reducing  $k_N$  but leaving  $n_\sigma$  fixed. This moves the OC curve to the left, thus reducing the consumer's risk but increasing the producer's risk. Under  $\sigma$ -method  $\bar{X}$  and  $\sigma$  are the average quality characteristic and standard deviation respectively.

### Variable Sampling Plan and SSAOQL procedures

The fraction defective of SSQSVSS- $r(n_{T\sigma}, n_{N\sigma}; k_\sigma)$ ,  $r=2$  and  $3$  in a given lot is  $p = F(-v)$  with  $v = (U - \mu) / \sigma$  and its probability of acceptance has been in (2) and (3)

with 
$$w_N = (v - k_\sigma) \sqrt{n_{N\sigma}}$$

and 
$$w_T = (v - k_\sigma) \sqrt{n_{T\sigma}}$$

If the quality of the accepted lot is  $p$  and all defective units found in the rejected lots are replaced by non-defective units in a rectifying inspection plan, the Six Sigma average outgoing quality (SSAOQ) can be approximated as

$$SSAOQ = pP_a(p) \tag{2}$$

If  $p_m$  is the proportion nonconforming items at which SSAOQ is maximum, one has

$$SSAOQL = p_m P_a(p_m) \tag{3}$$

If SSAQL ( $p_1$ ) is prescribed, then the corresponding value of  $v_{SSAQL}$  or  $v_1$  will be fixed and if  $P_a(p)$  is fixed at 99.99966%, that is  $(1 - \alpha)$ . Where,  $\alpha = 0.0000034 \times 10^{-6}$ . Hence we have  $P_a(p_1) = (1 - \alpha)$  So that for given values of  $n_\sigma$ ,  $w_N$ ,  $w_T$  and SSAQL,  $k_{N\sigma}$ ,  $k_{T\sigma}$  are determined.

### Selection of known $\sigma$ SSQSVSS- $r(n_{T\sigma}, n_{N\sigma}; k_\sigma)$ , $r=2$ and $3$ , for given SSAQL and SSAOQL

Table 1 is used for selection of  $\sigma$ - method SSQSVSS-2( $n_{T\sigma}, n_{N\sigma}; k_\sigma$ ). For example, if the SSAQL is fixed at 0.00001 and the SSAOQL is fixed at 0.00003,  $m=2$ . Table 1 yields  $n_{N\sigma} = 159$  and  $k_\sigma = 3.664$ , which is associated with 3.5 sigma level of SSQSVSS-2( $n_{T\sigma}, n_{N\sigma}; k_\sigma$ ). The sample size  $n_{T\sigma} = m n_{N\sigma} = (2) (159) = 319$ . Thus, for the given requirement, the SSQSVSS-3( $n_{T\sigma}, n_{N\sigma}; k_\sigma$ ) is specified by the parameters  $n_{T\sigma} = 319$ ,  $n_{N\sigma} = 159$ , and  $k_\sigma = 3.664$  which is associated with 4.5 sigma level.

Table 2 is used for the selection of  $\sigma$ - method SSQSVSS-3( $n_{T\sigma}, n_{N\sigma}; k_\sigma$ ). For example, if the SSAQL is fixed at 0.000003 and the SSAOQL is fixed at 0.00001,  $m=2$ . Table 2 yields  $n_{N\sigma} = 256$  and  $k_\sigma = 4.089$ . The sample size  $n_{T\sigma} = m n_{N\sigma} = (2) (256) = 513$ . Thus, for the given requirement, the SSQSVSS-3( $n_{T\sigma}, n_{N\sigma}; k_\sigma$ ) is specified by the parameters  $n_{T\sigma} = 513$ ,  $n_{N\sigma} = 256$ , and  $k_\sigma = 4.089$  which is associated with 3.6 sigma level.

The user of Table 1 and Table 10 should understand the limitations of plans indexed by SSAOQL. Sampling with rectifying of rejected lots on the one hand reduces the average percentage of nonconforming items in the lots, but on the other hand introduces non-homogeneity in the series of lots finally accepted. That is, any particular lot will have a quality of  $p\%$  or  $0\%$  nonconforming depending on whether the lot is accepted or rectified. Thus the assumption underlying the SSAOQL principle is that the homogeneity in the qualities of individual lots is unimportant and only the average quality matters. For plans listed in Table 1 and Table 2, if the individual lot quality happens to be the product quality  $p_m$  at which SSAOQL occurs, then the associated probability of acceptance will be poor. Table 1 gives  $P_a(p_m)$  values of plans given in Table 1. For example, for SSAQL is 0.00001 and SSAOQL is 0.00005, Table 1 gives  $P_a(p_m) = 0.64$ . Then  $p_m = SSAOQL / P_a(p_m)$

= 0.00008 and Table 4 gives  $P_a(p_m)$  values of plans given in Table 2. For example, for SSAQL of  $p_1=0.00001$  and SSAOQL = 0.00004, Table 4 gives  $P_a(p_m) = 0.6$  Then  $p_m = \text{SSAOQL} / P_a(p_m) = 0.00006$ .

In order to avoid such inconvenience, the producer should maintain the process quality more or less at the SSAQL. The high rate of rejection of lots at  $p = p_m$  will also indirectly put pressure on the producer to improve the submitted quality.

**Selection of unknown  $\sigma$  SSQSVSS-r( $n_{Ts}, n_{Ns}; k_s$ ),  $r = 2$  and  $3$ , for given SSAQL and SSAOQL**

Table 1 also gives such matched S-method plan. For example, for given SSAQL is 0.000005 and SSAOQL is 0.00001,  $m=2$ , one obtains the parameters of the S-method plan from Table 1 to be  $n_{Ns} = 2809$  and  $k_s = 3.870$ , which is associated with 4.5 sigma level of SSQSVSS-2 ( $n_{Ts}, n_{Ns}; k_s$ ). The sample size  $n_{Ts} = m n_{Ns} = (2) (2809) = 5618$ . Thus, for the given requirement, the SSQSVSS-2( $n_{Ts}, n_{Ns}; k_s$ ) is specified by the parameters  $n_{Ts} = 5618$ ,  $n_{Ns} = 2809$ , and  $k_s = 3.870$  which is associated with 4.5 sigma level.

Table 2 also gives such matched S-method plan. For example, for given SSAQL is 0.000004 and SSAOQL is 0.00001,  $m=2$ , one obtains the parameters of the S-method plan from Table 2 to be  $n_{Ns} = 2600$  and  $k_s = 3.977$ , which is associated with 4.5 sigma level of SSQSVSS-3 ( $n_{Ts}, n_{Ns}; k_s$ ). The sample size  $n_{Ts} = m n_{Ns} = (2) (2600) = 5199$ . Thus, for the given requirement, the SSQSVSS-3( $n_{Ts}, n_{Ns}; k_s$ ) is specified by the parameters  $n_{Ts} = 5199$ ,  $n_{Ns} = 2600$ , and  $k_s = 3.977$  which is associated with 4.5 sigma level.

**Construction of Table 1 and Table 2**

For constructing Table 1 and 2, a trial value of  $p_m$  is assumed and the probability of acceptance at  $p_m$  is found using (2) as

$$P_a(p_m) = \text{SSAOQL} / p_m$$

The auxiliary variables  $v_m, w_{Nm}$  and  $w_{Tm}$  corresponding to the values of  $p_m$  and  $P_a(p_m)$  respectively, are found using (1), (2), and (3). For given values of  $p_1$ , determine the values of  $v_1, w_N$  and  $w_T$  using the approximation (Abramwitz and Stegun (1972)) for the ordinate of the cumulative normal distribution. With the values of  $v_m, w_{Nm}$  and  $w_{Tm}$ , the following equation is used for calculating  $n_{\sigma}$ .

SSQSVSS-2, formula of  $n_{\sigma}$  is

$$\sqrt{n_{N\sigma}} = (-\text{AOQL}) / (p_m^2 ((1 - P_N)(1 - P_N + 2P_T)) \sqrt{(\exp(v_m^2 - w_T^2)) + P_T^2 \sqrt{(\exp(v_m^2 - w_N^2))}} / (P_T^2 + (1 - P_N)(1 + P_T))^2) \tag{4}$$

and SSQSVSS-3, formula of  $n_{\sigma}$  is

$$\sqrt{n_{N\sigma}} = (-\text{AOQL}) / (p_m^2 ((X \sqrt{(\exp(v_m^2 - w_T^2))} + P_T^2 \sqrt{(\exp(v_m^2 - w_N^2))}) / Y^2) \tag{5}$$

where

$$X = (3P_T^2 + 2P_T - 2P_N - 3P_N P_T^2 + 2P_N^2 P_T - 4P_N P_T + 1)$$

$$Y = P_T^3 + (1 - P_N)(P_T^2 + P_T + 1)$$

with  $P_N = \Phi(w_N) = \text{pr}[(U - \bar{x}) / \sigma > k_{N\sigma}]$

and  $P_T = \Phi(w_T) = \text{pr}[(U - \bar{x}) / \sigma > k_{T\sigma}]$

Equation (4) and (5) are the formulae for finding the sample size of a known  $\sigma$  SSQSVSS-r( $n_{T\sigma}, n_{N\sigma}; k_{\sigma}$ ),  $r = 2$  and  $3$  system. For two points given on the OC curve it is then checked to see whether the assumed value of  $p_m$  corresponds to the proportion non-conforming at which the SSAOQL occurs or not. That is, it is checked to see whether or not the trial value of  $p_m$  satisfies the following conditions.

For SSQSVSS-2 condition is

$$\text{AOQL} - p_m^2 ((1 - P_N)(1 - P_N + 2P_T)) \sqrt{(n_{T\sigma} \exp(v_m^2 - w_T^2))} -$$

$$P_T^2 (\sqrt{(n_{N\sigma} \exp(v_m^2 - w_N^2))}) / (P_T^2 + (1 - P_N)(1 + P_T))^2 = 0 \tag{6}$$

and for SSQSVSS-3 condition is

$$AOQL / [p_m^2 ((X \sqrt{(n_{T\sigma} \exp(v_m^2 - w_T^2))}) + P_T^2 \sqrt{n_{N\sigma} (\exp(v_m^2 - w_N^2))}) / Y^2] = 0 \tag{7}$$

where

$$X = (3P_T^2 + 2P_T - 2P_N - 3P_N P_T^2 + 2P_N^2 P_T - 4P_N P_T + 1)$$

$$Y = P_T^3 + (1 - P_N)(P_T^2 + P_T + 1)$$

Equation (6) and (7) are obtained from the following relation

$$\frac{d(SSAOQ)}{dp} = P_a(p) + p \frac{dP_a(p)}{dp} = 0 \tag{8}$$

in which, for SSQSVSS-2

$$\frac{dP_a(P)}{dp} = ((1 - P_N)(1 - P_N + 2P_T) \sqrt{(n_{T\sigma} \exp(v_m^2 - w_T^2))} - P_T^2 (\sqrt{(n_{N\sigma} \exp(v_m^2 - w_N^2))}) / (P_T^2 + (1 - P_N)(1 + P_T))^2) \tag{9}$$

and for SSQSVSS-3

$$\frac{dP_a(P)}{dp} = [(X \sqrt{(n_{T\sigma} \exp(v_m^2 - w_T^2))}) + P_T^2 \sqrt{n_{N\sigma} (\exp(v_m^2 - w_N^2))}) / Y^2] \tag{10}$$

If assumed value of  $p_m$  does not satisfy (6) and (7), then another trial value of  $p_m$  is obtained from (6) and (7) by numerical methods. The methods of successive substitution is often found to give good results in equation (6) and (7) is rewritten for this purpose as

for SSQSVSS-2

$$p_m = AOQL / (p_m ((1 - P_N)(1 - P_N + 2P_T) \sqrt{(n_{T\sigma} \exp(v_m^2 - w_T^2))} + P_T^2 (\sqrt{(n_{N\sigma} \exp(v_m^2 - w_N^2))}) / (P_T^2 + (1 - P_N)(1 + P_T))^2)) \tag{11}$$

and for SSQSVSS-3

$$p_m = AOQL / [p_m (X \sqrt{(n_{T\sigma} \exp(v_m^2 - w_T^2))}) + P_T^2 \sqrt{n_{N\sigma} (\exp(v_m^2 - w_N^2))}) / Y^2] \tag{12}$$

After determining the next trial value of  $p_m$ , again the values of  $v_m$ ,  $w_{Nm}$ ,  $w_{Tm}$  and  $n_\sigma$  are found and the conditions (6) and (7) are rechecked.

For obtaining the values of  $v_1$ ,  $w_N$  and  $w_T$ , the approximation for the ordinate of the cumulative normal distribution available in Abramowitz and Stegun (1972) was used.

The S-method plans matching the  $\sigma$ -method plans were obtained using computer search routine through C++ programme. For selected combinations of SSAQL and SSAOQL, Table 1 and 2 was constructed following the above iterative procedure.

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**Table 1: SSQSVSS-2 with known and unknown  $\sigma$  indexed by SSAQL and SSAOQL**  
**( $n_{T\sigma} = m n_{N\sigma}$ , when  $m=2$ )**

| SSAQL    | SSAOQL   | $n_{T\sigma}$ | $n_{N\sigma}$ | $k_{\sigma}$ | $\sigma$ - level | $n_{TS}$ | $n_{NS}$ | $k_S$ | $\sigma$ - level |
|----------|----------|---------------|---------------|--------------|------------------|----------|----------|-------|------------------|
| 0.000001 | 0.000002 | 2724          | 1362          | 4.486        | 4.2              | 30134    | 15067    | 4.486 | 4.9              |
|          | 0.000003 | 1808          | 904           | 4.443        | 4.1              | 19654    | 9827     | 4.443 | 4.8              |
|          | 0.000004 | 1544          | 772           | 4.413        | 4.0              | 16579    | 8290     | 4.413 | 4.8              |
|          | 0.000005 | 1316          | 658           | 4.384        | 4.0              | 13963    | 6981     | 4.384 | 4.7              |
|          | 0.000006 | 1102          | 551           | 4.363        | 3.9              | 11591    | 5796     | 4.363 | 4.7              |
|          | 0.000007 | 890           | 445           | 4.347        | 3.8              | 9299     | 4650     | 4.347 | 4.6              |
|          | 0.000008 | 662           | 331           | 4.332        | 3.7              | 6874     | 3437     | 4.332 | 4.5              |
|          | 0.000009 | 454           | 227           | 4.318        | 3.6              | 4687     | 2343     | 4.319 | 4.4              |
|          | 0.00001  | 308           | 154           | 4.306        | 3.4              | 3164     | 1582     | 4.307 | 4.3              |
|          | 0.00002  | 220           | 110           | 4.224        | 3.3              | 2183     | 1091     | 4.225 | 4.2              |
|          | 0.00003  | 174           | 87            | 4.175        | 3.2              | 1691     | 845      | 4.176 | 4.1              |
| 0.000002 | 0.000003 | 2000          | 1000          | 4.331        | 4.1              | 20756    | 10378    | 4.331 | 4.8              |
|          | 0.000004 | 1709          | 855           | 4.301        | 4.1              | 17521    | 8760     | 4.301 | 4.8              |
|          | 0.000005 | 1456          | 728           | 4.272        | 4.0              | 14747    | 7373     | 4.272 | 4.7              |
|          | 0.000006 | 1212          | 606           | 4.251        | 4.0              | 12166    | 6083     | 4.251 | 4.7              |
|          | 0.000007 | 979           | 490           | 4.235        | 3.9              | 9759     | 4879     | 4.235 | 4.6              |
|          | 0.000008 | 950           | 475           | 4.220        | 3.9              | 9413     | 4707     | 4.220 | 4.6              |
|          | 0.000009 | 728           | 364           | 4.206        | 3.8              | 7170     | 3585     | 4.206 | 4.5              |
|          | 0.00004  | 203           | 102           | 3.951        | 3.3              | 1790     | 895      | 3.952 | 4.1              |
|          | 0.00005  | 203           | 102           | 3.914        | 3.3              | 1760     | 880      | 3.915 | 4.1              |
| 0.000005 | 0.000006 | 1613          | 807           | 3.936        | 4.1              | 14112    | 7056     | 3.936 | 4.8              |
|          | 0.000007 | 1303          | 652           | 3.915        | 4.0              | 11290    | 5645     | 3.915 | 4.7              |
|          | 0.000008 | 1265          | 632           | 3.899        | 4.0              | 10881    | 5440     | 3.899 | 4.7              |

|         |          |     |     |       |     |      |      |       |     |
|---------|----------|-----|-----|-------|-----|------|------|-------|-----|
|         | 0.000009 | 969 | 485 | 3.884 | 3.9 | 8280 | 4140 | 3.884 | 4.6 |
|         | 0.00001  | 662 | 331 | 3.870 | 3.8 | 5618 | 2809 | 3.870 | 4.5 |
| 0.00001 | 0.00002  | 354 | 177 | 3.746 | 3.5 | 2840 | 1420 | 3.747 | 4.3 |
|         | 0.00003  | 319 | 159 | 3.664 | 3.5 | 2459 | 1230 | 3.665 | 4.2 |
|         | 0.00004  | 271 | 135 | 3.615 | 3.4 | 2039 | 1019 | 3.616 | 4.2 |
|         | 0.00005  | 271 | 135 | 3.578 | 3.4 | 2003 | 1001 | 3.579 | 4.2 |
|         | 0.00006  | 229 | 114 | 3.560 | 3.4 | 1678 | 839  | 3.561 | 4.1 |
|         | 0.00007  | 187 | 93  | 3.538 | 3.3 | 1356 | 678  | 3.539 | 4.1 |
|         | 0.00008  | 164 | 82  | 3.518 | 3.2 | 1181 | 590  | 3.520 | 4.0 |
|         | 0.00009  | 119 | 60  | 3.484 | 3.1 | 843  | 421  | 3.486 | 3.9 |
| 0.00005 | 0.00006  | 252 | 126 | 3.372 | 3.4 | 1682 | 841  | 3.373 | 4.2 |
|         | 0.00007  | 206 | 103 | 3.350 | 3.3 | 1359 | 679  | 3.351 | 4.1 |
|         | 0.00008  | 181 | 90  | 3.330 | 3.3 | 1183 | 591  | 3.332 | 4.0 |
|         | 0.00009  | 131 | 66  | 3.296 | 3.1 | 843  | 422  | 3.298 | 3.9 |
|         | 0.0001   | 114 | 57  | 3.154 | 3.1 | 681  | 341  | 3.156 | 3.9 |
|         | 0.0002   | 84  | 42  | 3.002 | 3.0 | 463  | 231  | 3.005 | 3.7 |
|         | 0.0003   | 66  | 33  | 2.875 | 2.9 | 339  | 169  | 2.879 | 3.6 |

**Table 2: SSQSVSS-3 with known and unknown  $\sigma$  indexed by SSAQL and SSAOQL**  
 $(n_{T\sigma} = m n_{N\sigma}, \text{when } m=2)$

| SSAQL    | SSAOQL   | $n_{T\sigma}$ | $n_{N\sigma}$ | $k_{\sigma}$ | $\sigma$ - level | $n_{TS}$ | $n_{NS}$ | $k_S$ | $\sigma$ - level |
|----------|----------|---------------|---------------|--------------|------------------|----------|----------|-------|------------------|
| 0.000001 | 0.000002 | 2694          | 1347          | 4.481        | 4.2              | 29742    | 14871    | 4.481 | 4.9              |
|          | 0.000003 | 1784          | 892           | 4.438        | 4.1              | 19353    | 9677     | 4.438 | 4.8              |
|          | 0.000004 | 1510          | 755           | 4.408        | 4.0              | 16181    | 8090     | 4.408 | 4.8              |
|          | 0.000005 | 1282          | 641           | 4.379        | 4.0              | 13574    | 6787     | 4.379 | 4.7              |
|          | 0.000006 | 876           | 438           | 4.358        | 3.8              | 9195     | 4597     | 4.358 | 4.6              |
|          | 0.000007 | 872           | 436           | 4.342        | 3.8              | 9092     | 4546     | 4.342 | 4.6              |
|          | 0.000008 | 652           | 326           | 4.327        | 3.7              | 6756     | 3378     | 4.327 | 4.5              |
|          | 0.000009 | 424           | 212           | 4.313        | 3.5              | 4368     | 2184     | 4.314 | 4.4              |
|          | 0.00001  | 284           | 142           | 4.301        | 3.4              | 2911     | 1455     | 4.302 | 4.3              |
|          | 0.00002  | 186           | 93            | 4.219        | 3.2              | 1841     | 921      | 4.220 | 4.1              |
|          | 0.00003  | 140           | 70            | 4.170        | 3.1              | 1357     | 679      | 4.172 | 4.0              |
| 0.000004 | 0.000005 | 1732          | 866           | 4.072        | 4.1              | 16094    | 8047     | 4.072 | 4.8              |
|          | 0.000006 | 1443          | 721           | 4.043        | 4.0              | 13235    | 6617     | 4.043 | 4.7              |
|          | 0.000007 | 1151          | 575           | 4.022        | 4.0              | 10457    | 5229     | 4.022 | 4.7              |
|          | 0.000008 | 1116          | 558           | 4.006        | 3.9              | 10071    | 5036     | 4.006 | 4.7              |
|          | 0.000009 | 588           | 294           | 3.991        | 3.7              | 5268     | 2634     | 3.991 | 4.5              |
|          | 0.00001  | 584           | 292           | 3.977        | 3.7              | 5199     | 2600     | 3.977 | 4.5              |
|          | 0.00002  | 283           | 141           | 3.965        | 3.4              | 2506     | 1253     | 3.966 | 4.2              |
|          | 0.00003  | 234           | 117           | 3.883        | 3.3              | 1994     | 997      | 3.884 | 4.2              |

|          |          |      |     |       |     |       |      |       |     |
|----------|----------|------|-----|-------|-----|-------|------|-------|-----|
|          | 0.00004  | 200  | 100 | 3.834 | 3.3 | 1667  | 833  | 3.835 | 4.1 |
|          | 0.00005  | 190  | 95  | 3.797 | 3.3 | 1556  | 778  | 3.798 | 4.1 |
|          | 0.00006  | 155  | 78  | 3.779 | 3.2 | 1262  | 631  | 3.781 | 4.0 |
|          | 0.00007  | 122  | 61  | 3.757 | 3.0 | 981   | 491  | 3.759 | 3.9 |
|          | 0.00008  | 118  | 59  | 3.737 | 3.0 | 940   | 470  | 3.739 | 3.9 |
| 0.000005 | 0.000006 | 1579 | 790 | 3.931 | 4.1 | 13783 | 6892 | 3.931 | 4.8 |
|          | 0.000007 | 1251 | 625 | 3.910 | 4.0 | 10814 | 5407 | 3.910 | 4.7 |
|          | 0.000008 | 1247 | 623 | 3.894 | 4.0 | 10702 | 5351 | 3.894 | 4.7 |
|          | 0.000009 | 959  | 480 | 3.879 | 3.9 | 8176  | 4088 | 3.879 | 4.6 |
| 0.00001  | 0.00002  | 305  | 152 | 3.741 | 3.5 | 2438  | 1219 | 3.742 | 4.2 |
|          | 0.00003  | 301  | 150 | 3.659 | 3.5 | 2315  | 1158 | 3.660 | 4.2 |
|          | 0.00004  | 261  | 130 | 3.610 | 3.4 | 1959  | 979  | 3.611 | 4.2 |
|          | 0.00005  | 241  | 120 | 3.573 | 3.4 | 1776  | 888  | 3.574 | 4.2 |
|          | 0.00006  | 205  | 102 | 3.555 | 3.3 | 1498  | 749  | 3.556 | 4.1 |
|          | 0.00007  | 153  | 76  | 3.533 | 3.2 | 1107  | 553  | 3.535 | 4.0 |
|          | 0.00008  | 130  | 65  | 3.513 | 3.1 | 934   | 467  | 3.515 | 3.9 |
|          | 0.00009  | 85   | 43  | 3.479 | 2.9 | 601   | 300  | 3.482 | 3.8 |
| 0.00005  | 0.00006  | 222  | 111 | 3.367 | 3.4 | 1478  | 739  | 3.368 | 4.1 |
|          | 0.00007  | 182  | 91  | 3.345 | 3.3 | 1197  | 599  | 3.347 | 4.0 |
|          | 0.00008  | 147  | 73  | 3.325 | 3.2 | 958   | 479  | 3.327 | 4.0 |
|          | 0.00009  | 97   | 49  | 3.291 | 3.0 | 623   | 311  | 3.294 | 3.8 |

Table 3: SSQSVSS-2 known  $\sigma$  plans of  $P_a(p_m)$  Values

| SSAOQL   | SSAQL    |          |          |          |          |         |         |
|----------|----------|----------|----------|----------|----------|---------|---------|
|          | 0.000001 | 0.000002 | 0.000003 | 0.000004 | 0.000005 | 0.00001 | 0.00005 |
| 0.000002 | 0.91     |          |          |          |          |         |         |
| 0.000003 | 0.89     | 0.91     |          |          |          |         |         |
| 0.000004 | 0.84     | 0.86     | 0.89     |          |          |         |         |
| 0.000005 | 0.81     | 0.83     | 0.86     | 0.88     |          |         |         |
| 0.000006 | 0.78     | 0.80     | 0.83     | 0.85     | 0.88     |         |         |
| 0.000007 | 0.76     | 0.78     | 0.81     | 0.83     | 0.86     |         |         |
| 0.000008 | 0.72     | 0.74     | 0.77     | 0.79     | 0.82     |         |         |
| 0.000009 | 0.71     | 0.73     | 0.76     | 0.78     | 0.81     |         |         |
| 0.00001  | 0.63     | 0.65     | 0.68     | 0.70     | 0.73     |         |         |
| 0.00002  | 0.61     | 0.63     | 0.66     | 0.68     | 0.71     | 0.74    |         |
| 0.00003  | 0.58     | 0.60     | 0.63     | 0.65     | 0.68     | 0.71    |         |
| 0.00004  | 0.54     | 0.56     | 0.59     | 0.61     | 0.64     | 0.67    |         |
| 0.00005  | 0.51     | 0.53     | 0.56     | 0.58     | 0.61     | 0.64    |         |
| 0.00006  |          | 0.41     | 0.44     | 0.46     | 0.49     | 0.52    | 0.57    |

|         |  |  |      |      |      |      |      |
|---------|--|--|------|------|------|------|------|
| 0.00007 |  |  | 0.38 | 0.40 | 0.43 | 0.46 | 0.51 |
| 0.00008 |  |  |      | 0.35 | 0.38 | 0.41 | 0.46 |
| 0.00009 |  |  |      |      | 0.32 | 0.35 | 0.40 |

**Table 4: SSQSVSS-3 known  $\sigma$  plans of  $P_a(p_m)$  Values**

| SSAOQL   | SSAQL ( $p_i$ ) |          |          |          |          |         |         |
|----------|-----------------|----------|----------|----------|----------|---------|---------|
|          | 0.000001        | 0.000002 | 0.000003 | 0.000004 | 0.000005 | 0.00001 | 0.00005 |
| 0.000002 | 0.90            |          |          |          |          |         |         |
| 0.000003 | 0.88            | 0.90     |          |          |          |         |         |
| 0.000004 | 0.83            | 0.85     | 0.88     |          |          |         |         |
| 0.000005 | 0.80            | 0.82     | 0.85     | 0.88     |          |         |         |
| 0.000006 | 0.77            | 0.79     | 0.82     | 0.85     | 0.88     |         |         |
| 0.000007 | 0.75            | 0.77     | 0.80     | 0.83     | 0.86     |         |         |
| 0.000008 | 0.71            | 0.73     | 0.76     | 0.79     | 0.82     |         |         |
| 0.000009 | 0.70            | 0.72     | 0.75     | 0.78     | 0.81     |         |         |
| 0.00001  | 0.62            | 0.64     | 0.67     | 0.70     | 0.73     |         |         |
| 0.00002  | 0.60            | 0.62     | 0.65     | 0.68     | 0.71     | 0.74    |         |
| 0.00003  | 0.57            | 0.59     | 0.62     | 0.65     | 0.68     | 0.71    |         |
| 0.00004  | 0.53            | 0.55     | 0.58     | 0.61     | 0.64     | 0.67    |         |
| 0.00005  | 0.50            | 0.52     | 0.55     | 0.58     | 0.61     | 0.64    |         |
| 0.00006  |                 | 0.40     | 0.43     | 0.46     | 0.49     | 0.52    | 0.57    |
| 0.00007  |                 |          | 0.37     | 0.40     | 0.43     | 0.46    | 0.51    |
| 0.00008  |                 |          |          | 0.34     | 0.37     | 0.40    | 0.45    |
| 0.00009  |                 |          |          |          | 0.31     | 0.34    | 0.39    |