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Electro-Magnetic Wave Velocity as observed by two different Inertial Frames in Source Free and Material Free Region

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Abstract - *This article tries to provide the mathematical derivations behind a phenomenon that has been proven experimentally numerous times – The constancy of Speed of Light in vacuum. Here we use mathematics to calculate the velocity of the same Electromagnetic wave as observed by two different Inertial Frames residing in the same Source Free and Material Free region of Space.*

***Key Words*: Light, Speed, Constant, Two Differential Inertial Frames, Source-Free and Material-Free Region**

1. INTRODUCTION

Let us state two axioms of Physics which are accepted as Universal Truth:

1. The Equations of Maxwell are co-variant in nature. The word co-variant means while the physical entities present in the Equation may take different form in different Inertial Frames of Reference, the form of the Equation remains the same in all Inertial Frames of Reference.

2. The Universal Laws retain the same form in all Inertial Frames of Reference. [1]

Two Inertial Frames of Reference in a Source Free Region and Material Free Region. The diagram can be defined as follows:

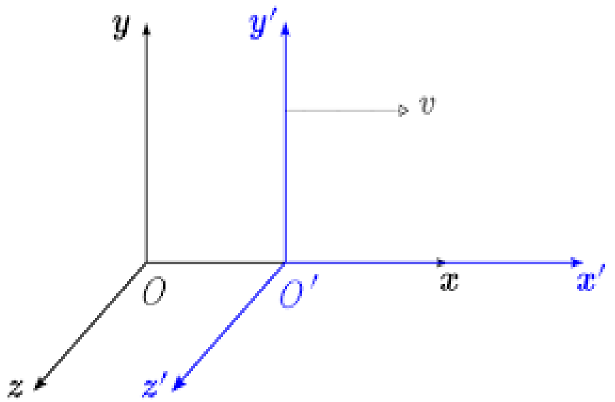

Fig -1: Two Inertial Frames of Reference moving with a velocity **v** with respect to each other.

The Frame with co-ordinates (x', y' z') – will be referred as Frame 2 and is moving with a constant velocity **v** from the frame with co-ordinates (x, y, z) which will be referred to as Frame 1

1.1 The Laws of Maxwell

Maxwell's Equations can be stated as follows:

$$\nabla . \boldsymbol{D} = \rho_V \tag{1}$$

In the above equation **D** is the Electric Flux Density and ρ_V is the Electric Charge Density. In a Source Free Region, the Electric Charge Density is 0. Thus, Equation (1) can be written as follows:

$$\nabla . \boldsymbol{D} = 0 \tag{2}$$

The relation between **D** and Electric Field, **E** can be written as: **D** = ε**E,** where ε is called permittivity and is constant for a medium. Therefore, Equation (2) can be written as follows:

$$\nabla . (\varepsilon \boldsymbol{E}) = 0$$

i.e, $\varepsilon\{\nabla . (\boldsymbol{E})\} = 0$

i.e, $\nabla . \boldsymbol{E} = 0$ (3)

$$\nabla . \boldsymbol{B} = 0 \tag{4}$$

B is the Magnetic Flux Density related to Magnetic Field **H** as follows: **B** = μ**H** where μ is permeability and is constant for any particular medium. Therefore, Equation (4) can be written as follows:

$$\nabla . \boldsymbol{H} = 0 \tag{5}$$

$$\nabla \text{ X E} = -\frac{\partial B}{\partial t} \tag{6}$$

$$\nabla \text{ X H} = \frac{\partial D}{\partial t} + J \tag{7}$$

where, **J** is the Current Density. In Source Free Region there is no source to generate Electric Current hence the Current Density (**J**) must be 0. That is **J** = 0. Thus Equation (7) becomes:

$$\nabla \text{ X H} = \frac{\partial D}{\partial t} \tag{8}$$

The Equations (1), (4), (6) and (7) are referred to as Maxwell's Laws.

2. The Kinematical Observations from Frame 1

Let the Electric Field observed by this frame be **E** and let the Magnetic Field observed by this frame be **H.** For a Vector Field **A**, the following Equation can be written:

$$\nabla \text{ X} \nabla \text{ X } \mathbf{A} = \nabla . (\nabla . \mathbf{A}) - \nabla^2 \mathbf{A} \tag{9}$$

The Equation (9) is a Universal Law true for all vector fields.

If we apply Equation (9) to the Magnetic Field **H** the following is obtained:

$\nabla \times \nabla \times \mathbf{H} = -\nabla^2\mathbf{H}$ (This is because in Equation 5, we obtained $\nabla . \boldsymbol{H} = 0$) (10)

The Equation (9) as applied to Electric Field **E** would give the following:

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2\mathbf{E} \quad (11)$$

(This follows from Equation 3).

In Frame 1, Maxwell-Faraday law is as follows:

$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t} \quad (12)$$

i.e, $\nabla \times \mathbf{E} = -\frac{\partial(\mu \mathrm{H})}{\partial t} = -\mu\frac{\partial H}{\partial t}$ (13)

i.e, $\nabla \times \nabla \times \mathbf{E} = \nabla \times (-\mu\frac{\partial H}{\partial t})$ (14)

i.e, i.e., $\nabla \times \nabla \times \mathbf{E} = -\mu \{\nabla \times (\frac{\partial H}{\partial t})\}$

i.e., $\nabla \times \nabla \times \mathbf{E} = -\mu\frac{\partial}{\partial t}(\nabla \times \mathbf{H})$ (15)

Following Equation (11), Equation (15) can be written as follows:

$$-\nabla^2\mathbf{E} = -\mu\frac{\partial}{\partial t}(\nabla \times \mathbf{H}) \quad (16)$$

Applying Equation (8) to Equation (16), the following is obtained:

$-\nabla^2\mathbf{E} = -\mu\frac{\partial}{\partial t}\{\frac{\partial(\varepsilon \mathrm{E})}{\partial t}\}$

i.e, $-\nabla^2\mathbf{E} = -\mu\varepsilon\frac{\partial}{\partial t}\{\frac{\partial(\mathrm{E})}{\partial t}\}$

i.e, $\nabla^2\mathbf{E} = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$ (17)

The Equation obtained in (17) is representing a Wave Equation which has 3 components, along direction x, along direction y and along direction z. To make calculation simple, we will convert the three-dimensional Wave Equation into one-dimension. Let it be assumed, that, the Field is polarized along x-direction and travelling along z-direction. Thus Equation (17) can be written as:

$$\frac{\partial^2 E(\text{polarized along the } x-\text{direction})}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E(\text{polarized along the } x-\text{direction})}{\partial t^2}$$

.....(18).

Equation (18) can be solved by the following:

E = f (z + ct), or E = f (z -ct).

Let us take E = f (z + ct) where c is the speed of the Wave. The left-hand side of Equation (18) can be evaluated as follows:

$$\frac{\partial^2 E}{\partial z^2} = \frac{\partial}{\partial z}(\frac{\partial E}{\partial z})$$

$$\frac{\partial E}{\partial z} = \frac{\partial\{f\ (z+ct)\}}{\partial z}$$

Let, f [z + ct] = f (k), where k = z + ct.

$$\frac{\partial E}{\partial z} = \frac{\partial\{f\ (k)\}}{\partial z} = \frac{\partial\{f\ (k)\}}{\partial k}\ X\ \frac{\partial k}{\partial z}$$

$$= f'(k)\ X\ \left\{\frac{\partial(z+ct)}{\partial z}\right\} = f'(k)\ X\ \left\{\frac{\partial(z)}{\partial z} + \frac{\partial(ct)}{\partial z}\right\}$$

$$= f'(k)\ X\ \{1+0\} = f'(k)$$

.....(19).

$$\frac{\partial^2 E}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial E}{\partial z}\right) = \frac{\partial}{\partial z}\{f'(k)\}$$

$$= \frac{\partial\{f'(k)\}}{\partial k}\ X\ \frac{\partial k}{\partial z} = f''(k)\ X\ \left\{\frac{\partial(z+ct)}{\partial z}\right\}$$

$$= f''(k)\ X\ \left\{\frac{\partial(z)}{\partial z} + \frac{\partial(ct)}{\partial z}\right\} = f''(k)\ X\ \{1+0\}$$

$$= f''(k)$$

.....(20). Thus, the left-hand side of Equation (18) represented by Equation (20) becomes equal to $f''(k) = f''(z+ct)$

Evaluating, the right-hand side of Equation (18) we get the following:

$$\mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \mu\varepsilon\ X\ \frac{\partial}{\partial t}(\frac{\partial E}{\partial t}) \quad (21)$$

Evaluating the term within the bracket, i.e., $\frac{\partial E}{\partial t}$

$$\frac{\partial E}{\partial t} = \frac{\partial\{f\ (z+ct)\}}{\partial t} = \frac{\partial\{f\ (k)\}}{\partial k}\ X\ \frac{\partial k}{\partial t}$$

$$= f'(k)\ X\ \frac{\partial(z+ct)}{\partial t} = f'(k)\ X\ \left\{\frac{\partial(z)}{\partial t} + \frac{\partial(ct)}{\partial t}\right\}$$

$$= f'(k)\ X\ \left\{0 + c\frac{\partial(t)}{\partial t}\right\} = cf'(k)$$

Evaluating the term $\frac{\partial}{\partial t}(\frac{\partial E}{\partial t})$:

$$\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) = \frac{\partial}{\partial t}\{cf'(k)\} = c\frac{\partial}{\partial t}\{f'(k)\}$$

$$= c\frac{\partial\{f'(k)\}}{\partial k}\ X\ \frac{\partial k}{\partial t} = cf''(k)\ X\ \frac{\partial(z+ct)}{\partial t}$$

$$= cf''(k)\ X\ \left\{\frac{\partial(z)}{\partial t} + \frac{\partial(ct)}{\partial t}\right\} = cf''(k)\ X\ \left\{0 + c\frac{\partial(t)}{\partial t}\right\}$$

$$= cf''(k)\ X\ \{0+c\} = c^2 f''(k) = c^2 f''(z+ct)$$

Therefore, Equation (21) can be written as:

$$\mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \mu\varepsilon c^2 f''(z+ct)$$

.....(22)

Equating Equation (20) and Equation (22), the following is obtained:

$f''(z+ct) = \mu\varepsilon c^2 f''(z+ct)$

i.e., $\mu\varepsilon c^2 = 1$

i.e., $c^2 = \frac{1}{\mu\varepsilon}$

i.e., $c = \frac{1}{\sqrt{\mu\varepsilon}}$ (23)

In material free space/ vacuum the following is obtained:

$\mu = \mu_0$ ($\mu_0 = 4\pi \times 10^{-7}$) H/m and,

$\varepsilon = \varepsilon_0$ ($\varepsilon_0 = 8.85 \times 10^{-12}$) F/m. If we put the values in Equation (23) we get the velocity of Electromagnetic wave as measured from Frame 1 as approximately 3 X 10^8 m/s and the formula for the velocity calculation as follows:

$$c = \frac{1}{\sqrt{\mu_0\ \varepsilon_0}} \quad (24)$$

3. The Kinematical Observations from Frame 2

Now, let us try to calculate the velocity of the same Electromagnetic wave from Frame 2. The Electric Field **E**, Electric Flux Density **D**, Magnetic Flux Density **B** and the Magnetic Field **H** as observed in Frame 1 will be observed as **E'**, **D'**, **B'** and **H'** in Frame 2. Let us recall the first Universal Truth we stated at the beginning of the paper:
The Equations of Maxwell are co-variant in nature. The word co-variant means while the physical entities present in the Equation may take different form in different Inertial Frames of Reference, the form of the Equation remains the same in all Inertial Frames of Reference. Therefore, Maxwell-Faraday Law in Frame 2 would be:

$$\nabla' \ X \ \boldsymbol{E}' = -\frac{\partial \boldsymbol{B}'}{\partial t'} \quad (25)$$

In a similar way, Maxwell-Ampere Law in Frame 2 would be:

$$\nabla' \ X \ \boldsymbol{H}' = \frac{\partial \boldsymbol{D}'}{\partial t'} + \boldsymbol{J}' \quad (26)$$

Let us recall the second Universal Truth we stated at the beginning of the paper: The Universal Laws retain the same form in all Inertial Frames of Reference. [2] This means, that while physical entities may appear differently for two different Inertial Frames of reference the Mathematical Laws and their formulations should remain the same in two Inertial Frames of Reference which are moving at a relative velocity of **v** with respect to each other. Therefore, Equation (9), which is a Universal Mathematical Law that can be applied to all vector Fields, can be written as:

$$\nabla' \ X \ \nabla' \ X \mathbf{A}' = \nabla' . (\nabla' . \mathbf{A}') - \nabla'^2 \mathbf{A}' \quad (27)$$

Since Frame 2 is also in the same Source Free and Material Free Region of Space as Frame 1, it can be concluded that:

$$\nabla' . \boldsymbol{H}' = 0 \quad (28)$$

$$\nabla' . \boldsymbol{E}' = 0 \quad (29)$$

Applying Equation (27) to the Magnetic Field **H'** and substituting Equation (28) the following is obtained:

$$\nabla' \ X \ \nabla' \ X \ \boldsymbol{H}' = -\nabla'^2 \boldsymbol{H}' \quad (30)$$

Similarly, the Equation for Electric Field as calculated from Frame 2 can be written as follows:

$$\nabla' \ X \ \nabla' \ X \ \boldsymbol{E}' = -\nabla'^2 \boldsymbol{E}' \quad (31)$$

In Frame (2) the relation between Magnetic Flux Density and Magnetic Field would be as follows:

$\boldsymbol{B}' = \mu \boldsymbol{H}'$, the value of μ does not change from Frame (1) to Frame (2) since μ is a constant for a medium of propagation and Frame (1) and Frame (2) are in the same medium.

$$\nabla' \ X \ \boldsymbol{E}' = -\frac{\partial(\mu \boldsymbol{H}')}{\partial t'} = -\mu \frac{\partial \boldsymbol{H}'}{\partial t'}$$

Taking curl of both sides this Equation will give us the following:

$$\nabla' \ X \ \nabla' \ X \ \boldsymbol{E}' = \nabla' \ X \ (-\mu \frac{\partial \boldsymbol{H}'}{\partial t'})$$

i.e., $-\nabla'^2 \boldsymbol{E}' = -\mu \{\nabla' \ X \ (\frac{\partial \boldsymbol{H}'}{\partial t'})\}$

$$-\nabla'^2 \boldsymbol{E}' = -\mu \frac{\partial}{\partial t'} (\nabla' \ X \ \boldsymbol{H}') \quad (32)$$

Substituting Maxwell-Ampere Law i.e. Equation (26) in Equation (32) and since we are in a Source Free Region, putting $\boldsymbol{J}' = 0$ and forming the relation between **D'** and **E'** as follows **D' = ε E'**, (the value of ε does not change from Frame (1) to Frame (2) since ε is a constant for a medium of propagation and Frame (1) and Frame (2) are in the same medium) we get the following:

$$\nabla'^2 \boldsymbol{E}' = \mu\varepsilon \frac{\partial^2 \boldsymbol{E}'}{\partial t'^2} \quad (33)$$

As we had assumed, in Frame 1 that the Field is polarized along the x-direction and moving along the z- direction, from Frame 2 we observe the Field polarized along the x' direction and travelling along the z' direction.

Thus, Equation (33) is reduced to the following format:

$$\frac{\partial^2 E'(\text{polarized along the } x' - \text{direction})}{\partial z'^2} = \mu\varepsilon \frac{\partial^2 E'(\text{polarized along the } x' - \text{direction})}{\partial t'^2}$$

.....(34)

Any equation of the form of Equation (34) can be solved with a function of the format F (z' + c't') where c' is the speed of the wave as measured from Frame 2.
Let us take, E' = F (z' + c't').
This can also be written as follows:
E' = F(k'), where,

k' = z' + c't'

Therefore,

$$\frac{\partial E'}{\partial z'} = \frac{\partial F}{\partial z'} = \frac{\partial F}{\partial k'} \ X \ \frac{\partial k'}{\partial z'}$$

$$\frac{\partial F}{\partial k'} = F'(k')$$

$$\frac{\partial k'}{\partial z'} = \frac{\partial (z' + c't')}{\partial z'} = \frac{\partial z'}{\partial z'} + \frac{\partial (c't')}{\partial z'} = 1 + 0 = 1$$

$$\frac{\partial E'}{\partial z'} = \frac{\partial F}{\partial z'} = F'(k') \ X \ 1 = F'(k') = F'(z' + c't')$$

.....(35)

$$\frac{\partial^2 E'}{\partial z'^2} = \frac{\partial}{\partial z'} \left(\frac{\partial E'}{\partial z'}\right) = \frac{\partial}{\partial z'} \{F'(k')\} = \frac{\partial F'(k')}{\partial k'} \ X \ \frac{\partial k'}{\partial z'}$$

$$\frac{\partial F'(k')}{\partial k'} = F''(k')$$

$$\frac{\partial k'}{\partial z'} = \frac{\partial (z' + c't')}{\partial z'} = \frac{\partial z'}{\partial z'} + \frac{\partial (c't')}{\partial z'} = 1 + 0 = 1$$

$$\frac{\partial^2 E'}{\partial z'^2} = F''(k') \ X \ 1 = F''(k')$$

.....(36)

Therefore, left-side of Equation (34) is F'' (z' + c't'). Using Chain Rule, we can evaluate the right-side of Equation (34). The value obtained would be

$c'^2 \ X \ F''(z' + c't') \ X \ \mu\varepsilon$

Equating the two sides of Equation (34) we get: $F''(z' + c't') = c'^2 \ X \ F''(z' + c't') \ X \ \mu\varepsilon$

i.e., $c'^2 \ X \ \mu\varepsilon = 1$

i.e., $c' = \frac{1}{\sqrt{\mu\varepsilon}}$

This c' is the speed of the same Electromagnetic wave as measured from Frame 2. As, Frame 2 is situated in the same material-free region/ vacuum as Frame 1 we get the following:

$\mu = \mu_0$ ($\mu_0 = 4\pi$ X 10^{-7}) H/m and,

$\varepsilon = \varepsilon_0$ ($\varepsilon_0 = 8.85$ X 10^{-12}) F/m.

Therefore c' becomes as follows:

$$c' = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad (37)$$

Comparing Equation (24) and Equation (37) we can see that: c (Velocity of Electromagnetic wave as measured from Frame 1) = c' (Velocity of Electromagnetic wave as measured from Frame 2).

Thus, mathematically we saw, that two different Inertial Frames moving at a velocity of v with respect to each other measure the same velocity of the Electromagnetic Wave.

4. CONCLUSION

The speed of the Electromagnetic Wave remains the same for different Inertial Frames. In a Material-Free Space the velocity of the Electromagnetic wave is approximately 3 X 10^8 m/s. This result has been confirmed by various Experiments over a century. Here, we simply try to develop a mathematical model from both Frames of References and conclude that despite Electric Field, Magnetic Flux Density and Magnetic Field appear different for the two inertial Frames of Refence, the final velocity of light should appear the same for both the Observers.

REFERENCE

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[2] A. Einstein, "DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?" Sept 27, 1905