Singular Identification of a Constrained Rigid Robot

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Abstract - *In this paper, a singular system identification procedure based on Strong equivalency is proposed for a constrained robot singular model. In fact, since constrained robot model is considered as a singular system, it requires to be transformed to an equivalent form before applying identification on it. This equivalent model plays an important role in the identification algorithm, since an inappropriate equivalency causes large identification error and sometimes identification divergence. The Strong equivalency applied in this paper keeps the important dynamics of the robot model while reducing the number of singular system's initial conditions. The Recursive Least Square identification algorithm is also applied in the next step on the Strong equivalent system. Indeed, Recursive Least Square identification procedure based on strong equivalent model is implemented for the first time for this constrained robot singular model, and can be generalized to all constrained robot models of this kind. Performance of this method on the robot is also validated by illustrating the simulations results of the technique on a robot singular model. According to the results, the model identification error convergence is improved significantly and also the output tracking is more satisfactory compared to the previous identification techniques for the constrained robot model.*

Key Words: Singular system identification, constrained robot, Strong equivalency, Recursive Least Square identification.

1. INTRODUCTION

Robot manipulators are widely being used in various industrial engineering areas and therefore, robot systems studies are under a significant attention by the scholars. Robot arm movement can be considered constrained or unconstrained. Unconstrained case happens when the robot arm moves in a free space without interacting with the environment. Constrained motion of robot case happens when the robot end-effectors interact mechanically with the environment. However, in practical tasks, the mechanical interactions of the robot arm with the environment or with the object being manipulated need to be considered. Therefore, in manufacturing environment, the constrained robot model is of greater importance than the unconstrained model [1].

Indeed, constrained robot model identification and estimation is regarded as an important issue in robotics since the robot parameters are not easily available (they are not usually provided by the robot manufacturers) and they are not directly measurable in practice due to the structural complexity and payload of robot manipulators [2], [3]. A large number of theoretical and experimental algorithms are done to deal with the identification of robot model parameters in this area and they resulted with different levels of accuracy. Although, in most of these algorithms, some dynamics of the robot are neglected in the process which is the main drawback of these approaches. So, considering that the constrained robot model is a singular model, the identification approach is regarded as a singular identification method.

Singular systems identification is of immerse interest among researchers, since these systems describe the dynamics of extensive number of systems such as power systems, electrical networks, chemical processes, economical systems, robotic systems, mathematics, neural networks, etc [4-8]. As a matter of fact, singular systems (also known as generalized state space systems) present a larger class of systems than the normal state space systems, and therefore singular theoretical properties had been studied as an active research topic by many researchers in the last decades, in [9-11]. Indeed, investigations on singular systems are being performed in two main areas; differential-algebraic equations theory of singular systems, and singular systems control theory. As far as the main focus in this paper is the singular equivalency and identification of robots, the researches in the singular systems theory are emphasized.

Actually, singular equivalency is considered as a crucial issue in singular systems studies, because the first step in all techniques dealing with this kind of systems is to find and equivalent regular state space model for them. Singular equivalency has been introduced as reduction methods for several years. These reduction methods have been widely studied in theory of singular matrices in [12]. Later, Polak introduced an algorithm to reduce a differential system to a linear time independent state form [13]. Moreover, Fettweis, Desoer and Dervisoglu explained a method for reducing the state equation of algebraic - differential systems in circuit theory [14], [15]. In parallel, Luenberger proposed an algorithm for the state equation reduction of singular discrete systems. Besides, [16-22] have made great progress in the field of controllability, observability and stability of singular systems [23]. All the studies mentioned used the regular theory to attain a result. The drawback of the regular theory is that it keeps the original system's dynamics and ignore the infinite impulse modes of the system. Neglecting the infinite modes is an important shortcoming for the constrained robot singular model, whereas it owns important infinite modes that by neglecting them, the system would be identified with low accuracy. Based on Strong equivalency model developed in [24], the singular identification through Least Square or any other on-line algorithms is improved significantly. This is also proved in another study on an electrical singular system in [23].

In this regard, the parameters identification of the constrained robot singular model is performed in this paper using Recursive Least Square algorithm as the identification method and Strong equivalent model as the equivalency approach. The objective is to show that the minimum estimation error and the best tracking of the parameters are obtained. The results of applying the proposed approach on the constrained robot model are found satisfactory in comparison with the previous robot identification methods. Moreover, by using the proposed approach for this model, the difficulties in singular identification are handled to some degrees.

The rest of the paper is organized as follows: in section 2 the constrained robot model and its specifications are presented. Section 3 describes the Strong equivalency approach for singular systems. The Recursive Least Square estimation algorithm is briefly explained in section 4, and section 5 contains the proposed method simulations and numerical results on the constrained robot model. Eventually, the work is finished by the conclusion in section 6.

2.CONSTRAINED ROBOT MODEL AND ITS SPECIFICATIONS

Without considering friction and other disturbances, the dynamics of an n-link constrained rigid robot manipulator can be described as follows:

$$M(q)\ddot{q} + C(q,\dot{q}) + g(q) = f + u \tag{1}$$

In the above equation, $q \epsilon R^n$ denotes the vector of generalized displacements in robot coordinates, $f \in R^n$ denotes the vector of generalized constraint forces in robot coordinates, and $u \in R^n$ denotes the vector of control inputs in robot coordinates. $M(q) \in R^{n\times n}$ is the symmetric positive definite manipulator inertia matrix, $C(q,\dot{q}) \in R^n$ is the vector of centripetal torques and $g(q) \in R^n$ is the vector of gravitational torques. It is assumed that the robot is non-redundant and equipped of joint position and velocity sensors and a force sensor at its end-effector [25-27].

From a proper definition of matrix $C(q,\dot{q})$, $H(q)$, and $C(q,\dot{q})$ in equation (1), the following equation is attained.

$$x^T(\dot{H} - 2C)x = 0, \qquad \forall x \in R^n \tag{2}$$

So, $\dot{H} - 2C$ is a skew-symmetric matrix.

The dynamic equation (1) can be expressed as linear in terms of a suitable selected set of robot and load parameters.

$$M(q)\ddot{q} + C(q,\dot{q}) + g(q) = \psi(q,\dot{q},\ddot{q})\theta \tag{3}$$

where $\psi(q,\dot{q},\ddot{q}) \epsilon R^{n\times P}$ and $\theta \in R^P$ are the vectors including the unknown manipulator and load parameters.

Also, a positive constant α exists in the following equation.

$$\alpha I_n \le M(q) \ , \quad \forall q \in R^n \tag{4}$$

such that I_n is the $n \times n$ identity matrix. Matrix $M^{-1}(q)$ exists and is positive definite and bounded.

Because of the existence of the impulsive modes in the model, the robot system is considered as a singular system. One robot arm model can be linearized around the operating point, to get the following linear singular state space general formulation as its model representation.

$$E(\theta)\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + w(t) \ , \ t \ge 0 \tag{5}$$

$$y(t) = C(\theta)x(t) + v(t)$$

In the above equation, $y(t)$ is the output vector of length k. $x(t)$ is a vector of system state variables with dimension n, and $u(t)$ is an m dimensional vector of input. E , A, B, C are real matrices, considering that E is the singular matrix. θ is also the regression parameter vector of unknown parameters of the system matrices. $w(t)$ and $v(t)$ are process and output noises that are not considered in this work.

The impulsive modes cause dependent state space equations. Dependent states generate a big trouble in identification process because this will result in initial conditions in the algebraic differential equations and it is required to estimate the initial conditions throughout the identification procedure. Thus, estimating the initial conditions randomly may result in high identification error or even divergence.

3.STRONG EQUIVALENCY

To overcome the problem of initial conditions mentioned in the previous section, the number of initial conditions can be reduced; in other words, the number of dependent state equations can be reduced. For this aim, the original model needs to be transformed to an equivalent reduced model in the first step before applying identification algorithm. So, this section is mainly focused on describing the Strong equivalency approach on a singular system.

Most of the singular equivalency methods reduce the model to an equivalent model without considering the infinite dynamics of the model. Since most of these approaches are developed based on the regular theory, they are not the most efficient equivalencies to be chosen for the singular model [12-22]. Based on these approaches, the singular system is transformed to the following equation [23].

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}\bar{x}(t) \quad (6)$$

$$y(t) = \bar{C}\bar{x}(t) + D(p)u(t)$$

with $p = d/dt$ and $D(s)$ as the polynomial part of G(s).

Therefore, the reduction method deletes some information of the system which generates huge errors in the original system identification process.

By the new Strong equivalency which is based on the Canonical Kronecker form of the $(sE - A)$, this problem is settled. The new equivalency is based on the Restricted System Equivalency, introduced by Rosenbrock [20]. The Canonical Krocker form of $(sE - A)$ can be described based on the non-singular matrices M and N as follows.

$$M(sE - A)N = \begin{bmatrix} sI_r - \bar{A} & 0 \\ 0 & I_{n-r} - s\tilde{E} \end{bmatrix} ,$$

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} , \quad N = [N_1 \quad N_2] \quad (7)$$

n is the model regular degree of freedom and $\tilde{E}$ is the nilpotent matrix with the singular index equal to $k = n - r$. Therefore, the state vector is divided into two parts; regular and singular subsystem state vectors respectively, as follows.

$$x(t) = N\begin{bmatrix} \bar{x}(t) \\ \tilde{x}(t) \end{bmatrix}$$
(8)

The singular system definition (5) is then converted to its Laplace form as follows.

$$\begin{bmatrix} sE - A & -B \\ C & 0 \end{bmatrix}\begin{bmatrix} X(s) \\ U(s) \end{bmatrix} = \begin{bmatrix} Ex(0^-) \\ Y(s) \end{bmatrix}$$
(9)

From the Restricted System equivalency of [19], equation (9) can be converted to equation (10) as the equivalent model.

$$\begin{bmatrix} sI_n - \bar{A} & 0 & -\bar{B} \\ 0 & I_{r-n} - s\tilde{E} & -\tilde{B} \\ \bar{C} & \tilde{C} & 0 \end{bmatrix}\begin{bmatrix} \bar{X}(s) \\ \tilde{X}(s) \\ U(s) \end{bmatrix} = \begin{bmatrix} \bar{x}(0^-) \\ -\tilde{E}\tilde{x}(0^-) \\ Y(s) \end{bmatrix}$$
(10)

In the above equation, $\tilde{B}$ and $\bar{B}$ are the sub blocks of MB, and $\tilde{C}$ and $\bar{C}$ are the sub blocks of CN.

Thus, in this way the original system is divided into two sub systems and all the properties of the original system are kept in the equivalent system.

Later, this approach is modified by the Strong equivalency, since there is a need to estimate the two sub systems parameters separately and it is inconvenient.

Based on the Strong equivalency, the Laplace description of the model (9) can be converted to the following model.

$$\begin{bmatrix} M & 0 \\ Q & I \end{bmatrix}\begin{bmatrix} sE - A & -B \\ C & D \end{bmatrix}\begin{bmatrix} N & R \\ 0 & I \end{bmatrix} = \begin{bmatrix} sE_1 - A_1 & -B_1 \\ C_1 & D_1 \end{bmatrix} \quad (11)$$

In the above equation, $QE = ER = 0$.

However, this transformation imposes extra constraints (Q and R matrices) over the equivalency method. This strong equivalency is more convenient because this method provides a more integrated model rather than two separate sub models as in equation (10), and it is a significant superiority of this equivalency in identification applications [29].

In this paper, the Strong equivalency described in this section is used in the first step before applying Recursive Least Square identification algorithm on the robot system.

4.IDENTIFICATION

The well-known Recursive Least Square (RLS) is chosen as the identification algorithm in this work. This algorithm is an adaptive filter that recursively updates to find the coefficients that minimizes the weighted least squares cost function. This algorithm is proved to converge fast and this is the main reason for adopting it in this research.

Here, the main stages associated with RLS for identification of the strong equivalent model of equation (11) are described:

First: Updating the unknown vector of parameters;

$$\hat{\theta}(t) = \theta(t-1) + K(t)(y(t) - \varphi^T(t)\theta(t-1))$$
(12)

Second: Reconstructing the Gain matrix $K(t)$;

$$K(t) = P(t)\varphi(t) = P(t-1)\varphi(t)(I + \varphi^T(t)\hat{\theta}(t-1))$$
(13)

Third: Updating the Covariance matrix;

$$P(t) = P(t-1) - P(t-1)\varphi(t)\big(I + \varphi^T(t)P(t-1)\varphi(t)\big)^{-1}\varphi^T(t)P(t-1)) = (I - K(t)\varphi^T(t))P(t-1))$$
(14)

The parameters summary used in the above algorithm are as follows.

$\hat{\theta}(t) = estimated\ parameter\ vector\ values\ at\ time\ t$

$\varphi(t) = regression\ vector$

$P = covariance\ matrice$

$K = gain\ matrice$

In this algorithm, the initial conditions for covariance matrix and estimated parameters vector are defined in the first stage. Then, P, K, and $\hat{\theta}(t)$ are updated in each iteration through the above formulations until the algorithm converges. Initial conditions of the states are also involved in this algorithm, because the error of estimation is required to be calculated in each stage considering the output values. In the next section, it is shown that the results of RLS on the Strong equivalent model are satisfactory.

5.SIMULATIONS AND RESULTS

The original formulation (1) of the robot manipulator is linearized around its operating points through MATLAB code and the following model is attained in the form of a singular system. Also, consider that the model is attained for one arm of the robot [28].

$$\begin{bmatrix} M(q) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} u - g(q) \\ f \end{bmatrix} \quad (15)$$

So, with the aid of Strong equivalency, model of equation (15) can be converted to its equivalent form by the following process.

By choosing q, $\dot{q}$, $\ddot{q}$ as the singular system states, the singular system model matrices A, B, C, E are stated as equation (16).

$$A(t) = \begin{bmatrix} 0 & 0 & e^{-t} \\ -1 & e^{2t} & 1 \\ -2t & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 1 \quad 0], \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (16)$$

$$x(t) = \begin{bmatrix} q \\ \dot{q} \\ \ddot{q} \end{bmatrix}$$

With the aim of attaining the equivalent model of the system, the following calculations for analyzing the modes of the system are done.

$$rank\ [\lambda E - A \quad B] = rank \begin{bmatrix} \lambda & 0 & -e^{-t} & 0 \\ 1 & \lambda - e^{2t} & -1 & 0 \\ 2t & 1 & -1 & 1 \end{bmatrix}$$

$$rank \begin{bmatrix} \lambda E - A \\ c \end{bmatrix} = rank \begin{bmatrix} \lambda & 0 & -e^{-t} \\ 1 & \lambda & -1 \\ 2t & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

So from the above ranks, it is resulted that the impulsive modes of the system which arise because of the dependent states are neither controllable nor observable. These cause problem during the identification because of the initial conditions arising.

From equation (11), the following equivalent model is resulted for this system (the suffix SE in the equation (17) is short for Strong equivalency).

$$A_{SE}(t) = \begin{bmatrix} 0 & 0 & 0 \\ -1 & e^{2t} & 1 \\ -2t & -e^{-t} & 1 \end{bmatrix}, \quad B_{SE} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

$$C_{SE} = [0.5 \quad 1 \quad 0], \quad E_{SE} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Also, note that in the above equation the state variables change; the new state variables are $q - \dot{q}$, $q + \ddot{q}$, and $\dot{q} + \ddot{q}$. Therefore, the initial conditions of the state become zero in this case, and there is no need to estimate the initial conditions value while applying the identification algorithm.

Parameters of the Strong equivalent model are then identified through the 3-steo identification algorithm of section 4. The number of inputs and outputs used for the identification process is 600, with P matrix equal to *1000I*. The initial states values are chosen as zero regarding the Strong equivalency property which is independent of initial values.

The following results are attained for eight model parameters estimation. Also, the estimation error results are shown in the continuation.

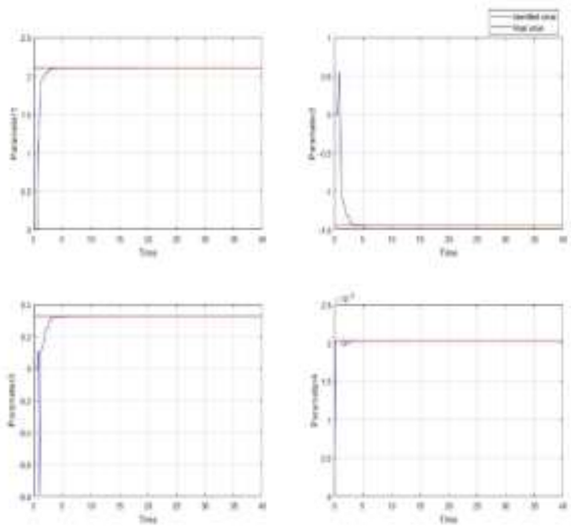

Fig -1: First four model parameters; identified values and real values using the proposed method

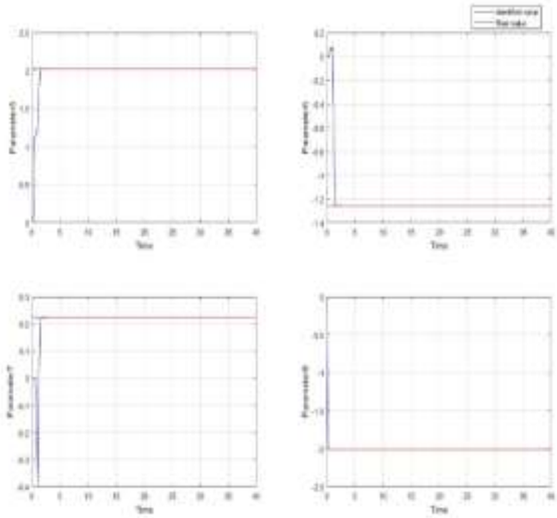

Fig -2: Second four model parameters; identified values and real values using the proposed method

From the results of Fig -1, it can be realized that the first four identified parameters of the system converged soon after less than 3 seconds to the real values. However, by zooming the figure it is obvious that the estimated values fluctuate around the real value with very small fluctuations and eventually converges exactly to the real point after less than 30 seconds. The same thing happened also for the second four model parameters in Fig -2. This is considered as a significant improvement in robot model identification compared to the previous works. Moreover, using Strong equivalency in identification process through the proposed method, the estimation convergence is guaranteed, which is not done in previous studies. Also, computation time of the simulations is significantly less than identification process using previous singular equivalencies.

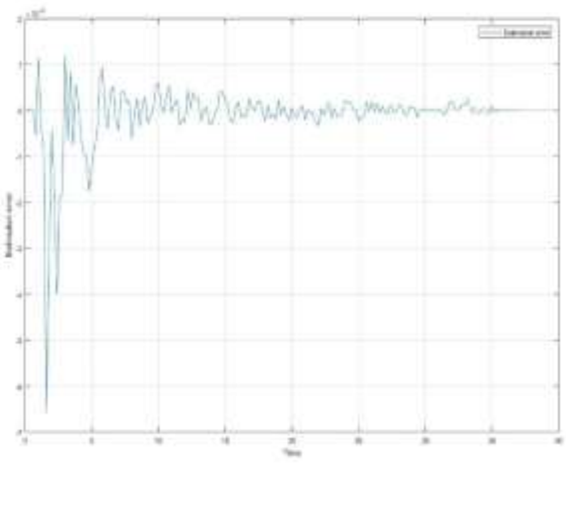

Fig -3: Estimation error using the proposed method

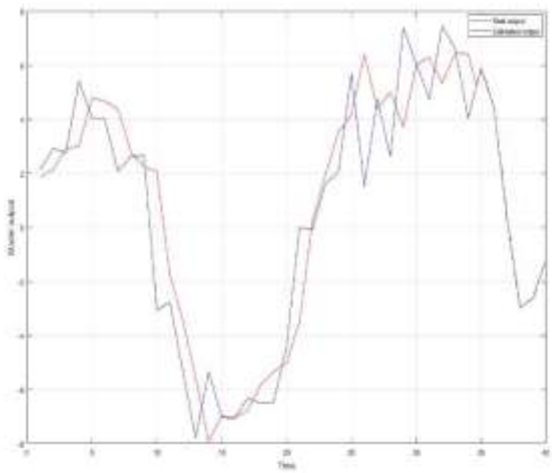

Fig -4: Model real and identified outputs using the proposed method

According to Fig -3, the estimation error of the system output is not even large at the beginning of the identification. This estimation error converges to exactly zero after around 37 seconds. Also, in Fig -4 the estimated system output tracks the real system output with high accuracy level. The estimated output even merges with the real output after some seconds, at time 35.

Overall, from the results, it can be realized that the estimation error for the robot model is significantly small and the identification results are satisfactory. In fact, the estimated parameters of the system are attained with a high accuracy close to the original parameters very fast.

6.CONCLUSION

In this paper, the problem of identification of a constrained rigid robot arm is studied. The constrained robot is a singular system and therefore the singular equivalency method is applied in the first stage of the identification algorithm. The proposed method in this work is the Strong equivalency combined with the Recursive Least Square algorithm. The simulations on the robot model are performed in MATLAB and the resulted figures proved the efficiency of the proposed approach. The benefit of this approach compared to the previous identifications on this robot model is that the Strong equivalency in this work keeps all the system dynamics without any loss of information in the system. Meanwhile, the number of non-zero initial conditions is reduced in the equivalent model, which is the main objective of all the equivalency approaches. Therefore, the estimation accuracy is high with low estimation error. Besides, the identified parameters of the model and its estimated output converged exactly to real values after short computation time.

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