

GLOBAL ACCURATE DOMINATION IN JUMP GRAPH

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ABSTRACT: A dominating set D of a jump graph is an accurate dominating set. If $V-D$ has no dominating set of cardinality $|D|$. An accurate dominating set D of a graph G is also an accurate dominating set of \bar{G} . The global accurate dominating number $\sqrt{ga}(J(G))$ are obtained and exact values of $\sqrt{ga}(J(G))$ for some standard graphs are found. Also a Nordhaus-Gaddum type results established.

Key words: accurate dominating set, global accurate dominating set, global accurate domination number.

Mathematics subject-Classification:05C

I Introduction

All graphs considered here are finite, undirected without loops and multiple edges. Any undefined terms in this paper may found in kulli [1]

A set D of vertices in a jump graph is a dominating set of $J(G)$, if every vertex not in D is adjacent to a vertex in D . The domination number of a jump graph is denoted by $\sqrt{j}(G)$ is the minimum cardinality of a dominating set in $J(G)$.

A dominating set D of a jump graph $J(G)$ is accurate dominating set. If $V(J(G))-D$ has no domination set of cardinality $|D|$. The accurate domination number $\sqrt{a}(J(G))$ of $J(G)$ is the minimum cardinality of an accurate dominating set. This concept was introduced by kulli and kattimani in [2]

A dominating set D of a jump graph $J(G)$ is a global dominating set. If D is also a dominating set of $J(\bar{G})$. The global domination number $\sqrt{g}(J(G))$ of $J(G)$ is the minimum cardinality of a global dominating set [5].

In [4] kulli and kattimani introduced the concept of global accurate domination as follows.

An accurate dominating set D of a graph G is a global accurate dominating set, if D is also an accurate dominating set of \bar{G} . The global accurate domination number $\sqrt{ga}(G)$ of G is the minimum cardinality of a global accurate dominating set. Analogously, a set d of a jump graph $J(G)$ is a global accurate dominating set if D is also an accurate dominating set of $J(\bar{G})$. The global accurate domination number $\sqrt{ga}(J(G))$ of $J(G)$ is minimum cardinality of a global accurate dominating set.

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x . A \sqrt{a} -set is minimum accurate dominating set.

2. Results

We characterize accurate dominating set which are global accurate dominating sets.

Theorem 1: An accurate dominating set D of a jump graph $J(G)$ is a global accurate dominating set if and only if the following condition holds.

For each vertex $v \in V(J(G))-D$, there exists vertex $u \in D$ such that u is not adjacent to v . There exists a vertex $w \in D$ such that w is adjacent to all vertices in $V(J(G))-D$.

Theorem2. Let $J(G)$ be a jump graph such that neither $J(G)$ nor $J(\bar{G})$ has an isolated vertex, Then

$$\sqrt{ga}(J(G)) = \sqrt{ga}(J(\bar{G}))$$

$$\sqrt{a}(J(G)) + \sqrt{a}(J(\bar{G})) \leq \sqrt{a}(J(G)) \leq$$

$$2 \sqrt{a}(J(G)) + \sqrt{a}(J(\bar{G}))$$

Theorem 3. Let $J(G)$ be a jump graph such that neither $J(G)$ nor $J(\bar{G})$ have an isolated vertex then

$$\sqrt{a}(J(G)) \leq \sqrt{ga}(J(G))$$

Proof; Every global accurate dominating set is an accurate dominating set then above inequality holds.

Theorem 4: Let $j(G)$ be a jump graph such that neither $j(G)$ nor $J(\bar{G})$ have an isolated vertex then

$$\sqrt{g}(J(G)) \leq \sqrt{ga}(J(G))$$

Proof; Every global accurate dominating set is an accurate dominating set then above inequality holds.

Exact values of $\sqrt{ga}(J(G))$ for some standard graphs are given in Theorem 5.

Theorem 5;

$$\begin{aligned} \sqrt{ga}(J(K_p)) &= p \\ \sqrt{ga}(J(C_p)) &= \lfloor \frac{p}{2} \rfloor + 1 \quad \text{if } p \geq 3 \end{aligned}$$

$$\sqrt{ga}(J(P_p)) = \lfloor \frac{p}{2} \rfloor + 1 \quad \text{if } p \geq 2$$

$$\sqrt{ga}(J(K_{m,n})) = m+1 \quad \text{if } m \leq n$$

$$\sqrt{ga}(J(W_p)) = \lfloor \frac{p}{2} \rfloor + 1 \quad \text{if } p \geq 5$$

For any regular jump graph $J(G) = \lfloor \frac{p}{2} \rfloor + 1 \quad \text{if } p \geq 2$

Now we obtain an upper bound for $\sqrt{ga}(J(G))$

Theorem 6.; Let $J(G)$ has two non adjacent vertices u and v such that u is adjacent to some vertex in $V(J(G)) - u$ this implies that $V(J(G)) - \{u\}$ is a global accurate dominating set of G Thus

$$\sqrt{ga}(J(G)) \leq |V(J(G)) - \{u\}| \text{ or}$$

Proof. Suppose result holds. Assume that $J(G) \neq K_p, \bar{K}_p$. Then $J(G)$ has at least three vertices u, v , and w such that u and v are adjacent and w is not u . Then this implies that $V(J(G)) - \{u\}$ is a global accurate dominating set of $J(G)$. This proves necessity.

Converse is obvious.

Theorem8. Let D be an accurate dominating set of $J(G)$ if there exists two vertices $u \in V(J(G)) - D$ and $v \in D$ such that u is adjacent only to the vertices of D and v is adjacent to the vertices of $V(J(G)) - D$. Then

$$\sqrt{ga}(J(G)) \leq \sqrt{a}(J(G)) +$$

Proof: Let D be a \sqrt{a} -set of $J(G)$ if there exists a vertex $u \in V(J(G)) - D$. such that u is adjacent only to the vertices of D then $D \cup \{u\}$ is a global accurate dominating set of g , thus

$$\sqrt{ga}(J(G)) \leq |D \cup \{u\}|$$

$$\leq |D| + 1$$

Or
$$\sqrt{ga}(J(G)) \leq \sqrt{a}(J(G)) + 1$$

In jump graph $J(G)$, a vertex and an edge incident with it are said to cover each other. A set of vertices that cover all the edges of $J(G)$ is a vertex cover of $J(G)$. The vertex covering number $\alpha_0(J(G))$ of jump graph $J(G)$ is the minimum number of vertices in a vertex cover. A set S of vertices in $J(G)$ is independent if no two vertices in S are adjacent. The independence number $\alpha_0(J(G))$ of $J(G)$ is the maximum cardinality of an independent set of vertices. The Clique number $\beta_0(J(G))$ of $J(G)$ is the maximum order among the complete sub graph of $J(G)$.

Theorem 9: Let $J(G)$ be a jump graph without isolated vertices then

$$\sqrt{ga}(J(G)) \leq \alpha_0(J(G)) + 1$$

Proof: Let s be a maximum independent set of vertices in $J(G)$. Then for any vertex $v \in S$, $\{V(J(G)) - S\} \cup \{v\}$ is a global accurate dominating set of $J(G)$ thus

$$\sqrt{ga}(J(G)) \leq |\{V(J(G)) - S\} \cup \{v\}|$$

$$\leq |V-S| + 1$$

$$\leq p - \beta_0(J(G)) + 1$$

Or
$$\sqrt{ga}(J(G)) \leq \alpha_0(J(G)) + 1$$

We obtain a Nordhus - gaddum type result

Theorem 10: Let $J(G)$ be a jump graph such that neither $j(G)$ nor $J(\bar{G})$ have an isolated vertex Then,

$$\sqrt{ga}(J(G)) + \sqrt{ga}(J(\bar{G})) \leq p + \sqrt{o}(J(G)) - w(J(G)) + 2$$

Proof: By theorem 9
$$\sqrt{ga}(J(G)) \leq \alpha_0(J(G)) + 1$$

Therefore
$$\sqrt{ga}(J(\bar{G})) \leq \alpha_0(J(\bar{G})) + 1$$

$$\leq p - \beta_0(J(\bar{G})) + 1$$

$$\leq p - w(J(\bar{G})) + 1$$

Hence
$$\sqrt{ga}(J(G)) + \sqrt{ga}(J(\bar{G})) \leq p + \sqrt{o}(J(G)) - w(J(G)) + 2$$

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