Mathematical and Computational Modelling of Dispersion Characterization for Dielectric Tube Waveguide Filled with Plasma

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Abstract - *A Dielectric open waveguide structure supports both guided waves and leaky waves. In this paper, analytical theory of guided and leaky mode has been formulated for dielectric tube waveguide filled with plasma (DTWFP). This structure is analyzed for lower order symmetric transverse magnetic mode (TM). In this paper, the numerical and computational results for guided and leaky mode of dispersion characteristics has been compared. The guided mode is existed when the normalized phase constant value is greater than 1 and below that leaky mode is existed which is known as antenna mode. By filling plasma in the core region, it makes waveguide or antenna reconfigurable by changing plasma frequency.*

Key Words: Dielectric Tube, Transverse Mode, Dispersion, Plasma, Muller Method

1. INTRODUCTION

Above 30GHz dielectric waveguide shave attracted much more attention over metallic waveguide [1]. Most millimeter dielectric waveguide is open wave-guiding structure; power is leaked out when uniformity of this guided wave is perturbed [2]. Due to low loss wave-guiding structure leaky wave antennas are more popular in practical application. Many researchers have done analysis based on different structure of leaky wave antenna [3-6].

Plasma exhibits unique property which depends on plasma density; which in turns defines plasma permittivity, so applying plasma to a waveguide can change field distribution of the electromagnetic wave in the waveguide. At higher frequency it is best candidate to transfer EM energy with less attenuation and low losses. There are different types of structure investigated by many authors like high gain printed leaky antenna [3], plasma column [4], plasma toroidal column [5], dielectric waveguide filled with plasma [6]. The dispersion property of a waveguide is an integral part of the various waveguide. The fundamental characteristics of dielectric waveguide is dispersion relation which determine the field distribution, mode classification, wave velocity characteristics, such as the phase and group velocities. These characteristics are also important when designing waveguides.

In this paper, dispersion characterization of DTWFP structure is simulated to design electrically reconfigured plasma waveguide. The numerical results computed with MATLAB and computational results are compared with CST Microwave studio. The main aim to validate the numerical and computational results for dispersion cutoff frequency for fundamental TM_{01} mode.

2. CONSTRUCTION OF DIELECTRIC TUBE WAVEGUIDE FILLED WITH PLASMA (DTWFP)

In the existing model, DTWFP is considered for electromagnetic wave modal characteristics study. Here three regions of consideration which is shown in Fig.1. The core region is plasma medium, Second region is dielectric and Outer region ($r \rightarrow \infty$) is the free space region extending up to infinity. Here regions are assumed to be homogeneous and lossless.

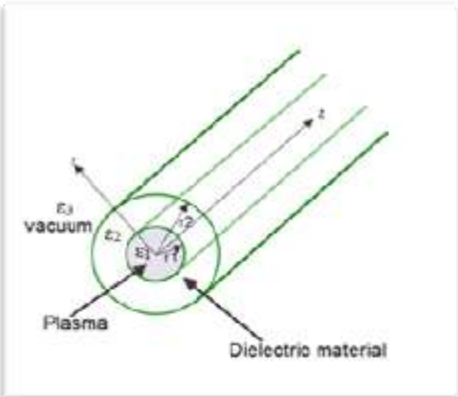

Fig. 1 A schematic diagram of dielectric tube filled with plasma

For simple modeling phenomenon, plasma exhibits dielectric constant between zero and unity or it can have a negative value. To propagate EM waves through plasma, it is essential to have real transverse propagation constant, when the operating frequency is greater than plasma frequency ω_p. For collision less plasma, permeability μ_P is equal to unity and permittivity εp as given in [7],

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$$

Where ω the wave angular frequency, ω_P is the angular plasma frequency and n, e and m are the density, charge and mass of electrons respectively.

Plasma can be generated by filling ionizing the gas like neon, argon in the tube structure. Energizing the plasma can be accomplished with microwaves, optical fibers, RF heating. Plasma density, shape of the tube and current distribution or EM fields propagate in structure helps to define radiation pattern. Turn off nature of plasma makes it electrically invisible to nearby antenna coupling and scattering effect can be reduced [7].

3. MATHEMATICAL MODELLING OF DTWLP

Here we consider the dielectric waveguide with circular cross section. For this analysis cylindrical coordinate system (r, θ, z) are required, propagation direction is taken as z. Axial electric and magnetic field components in each region according to [8, 9] can be expressed as

$$E_{z1} = A_1 J_m(K_1 r) Um$$

$$H_{z1} = B_1 J_m(K_1 r) Vm$$

$$E_{z2} = [A_2 H_m^{(1)}(K_2 r) + A_3 H_m^{(2)}(K_2 r)] Um$$

$$H_{z2} = [B_2 H_m^{(1)}(K_2 r) + B_3 H_m^{(2)}(K_2 r)] Vm$$

$$E_{z3} = A_4 H_m^{(2)}(K_3 r) Um$$

$$H_{z3} = B_4 H_m^{(1)}(K_3 r) Vm$$

Um and Vm is sine and cosine function. J_m is the Bessel function of first kind, $H_m^{(1)}$ and $H_m^{(2)}$ are the Hankel functions of first and second kind respectively [12]. Here, 'm' is the azimuthally Eigen value. A_1, B_1, A_2, B_2, A_3, B_3, A_4 and B_4 are the constants to be determined. Propagation vectors in transverse direction are given as [8, 9].

When m=0, characteristics equation splits into two equations which is circularly symmetrical TE and TM modes.

For TM mode:

$$\varepsilon_1 \eta_1 (\varepsilon_2 \Delta_2 - \varepsilon_3 \Delta_5) - \varepsilon_2^2 \Delta_3 - \varepsilon_2 \varepsilon_3 \Delta_1 \eta_6 = 0 \quad (9)$$

3. COMPUTATIONAL RESULTS AND SIMULATIONS

To obtain the normalized propagation constant (β/K_0) as a function of frequency. The dispersion relation equation is solved numerically by complex root search Muller method using MATLAB. The guided dispersion characteristics for lower order symmetric TM mode is plotted in Fig. 2. The cut off frequency for guided mode TM_{01} is 35GHz and before 15.5 GHz plasma wave is existed when wave is oscillated inside the waveguide. Leaky mode is existed between 1 to 15.5 GHz. The leaky mode with complex propagation constants are found below the guided mode region. For leaky mode region, $\bar{\beta} < \sqrt{\mu_3 \varepsilon_3}$ where $\bar{\beta} = \beta/K_0$ is normalized propagation constant in axial direction. Here plasma permittivity is taken in terms of normalized plasma wavenumber (Kpr1=2).

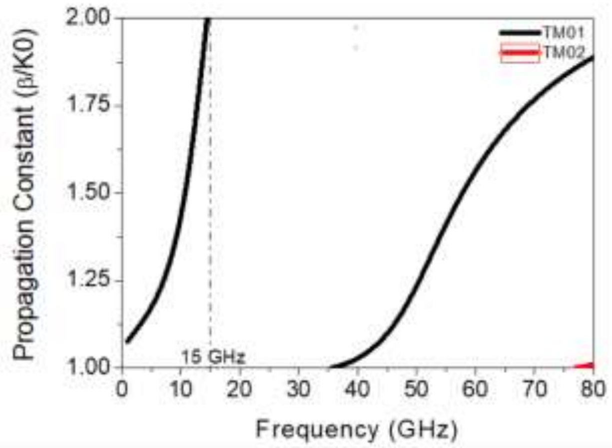

Fig. 2. Guided Dispersion diagram of DTWLP structure in MATLAB

In the following section, DTWFP is simulated in CST Microwave studio. Here there are two ports are presented. There are three solvers available in CST MW to analyze the electromagnetic structure. 1. Eigen Mode 2. Transient Mode. 3. Frequency Mode. Based on solver results, a Scattering parameter, E field and H field results are generated using transient solver. Dispersion diagram of frequency dependent plasma material is solved using frequency solver. Mode cutoff frequency is calculated by the Eigen mode solver. To model this structure, we have considered the waveguide core region is made of plasma medium cladding region is of dielectric (having permittivity 2.1).

DTWFP structure support, leaky mode propagation of a number of modes such as TM and HE modes. However, in the present paper our discussion is limited to the fundamental TM01 mode.

Now, we are comparing the analytically solved guided mode dispersion equation in MATLAB and computational result solved in CST. The guided mode wave propagation characteristics of DTWLP structure is shown in Fig. 2.

Now, the same design parameter is taken into consideration for DTWLP structure. It was simulated in Eigen mode solver of CST microwave studio to figure out the phase velocity for a different frequency. The phase velocity equation is $v_p = \omega/k$. Based on the value of phase velocity, we can define the normalized phase constant value which is shown in Table 1.

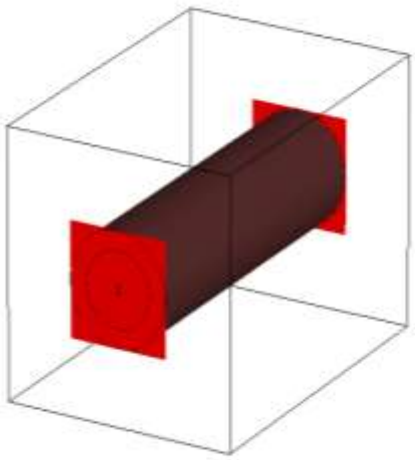

Fig. 3. Schematic diagram of DTWFP in CST simulation

Table -1: Data for Leaky Mode Extracted from CST Simulation

Frequency (GHz)	Phase Velocity	Normalized Phase Constant
6.41	8.73	0.07
8.54	69.81	0.39
9.53	130.90	0.66
10.92	191.99	0.84
12.55	253.07	0.96
13.37	314.16	0.98
16.30	475.26	1.12

For the guided mode the normalized phase constant value should be greater than 1. Therefore, from the Table-1 it is concluded that below 16GHz leaky mode propagation is presented which has the same value as per the MATLAB simulation results

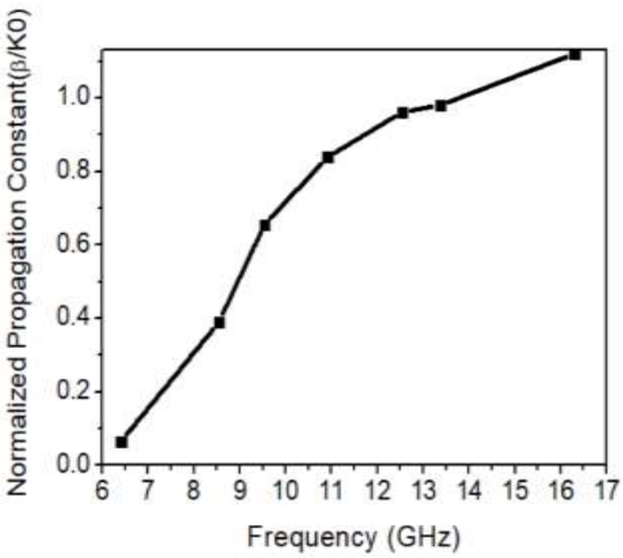

Fig. 4. Leaky Dispersion diagram of DTWLP structure in CST

For TM01 mode Electric field confinement in the axial direction is calculated at a different frequency and plotted in Fig. 5. As already discussed below, 10 GHz guides

Support leaky mode propagation due to the plasma frequency lesser field is confined in cladding and core region and most of the power leaks out in free-space due to the high value of attenuation coefficient. Opposite to that an increase in frequency, the field is more confined in the cladding region as compared to free space due to the reduced value of attenuation coefficient. It can be seen that at 20 GHz in Fig. 6. most of the field confined in cladding region no any power leak out.

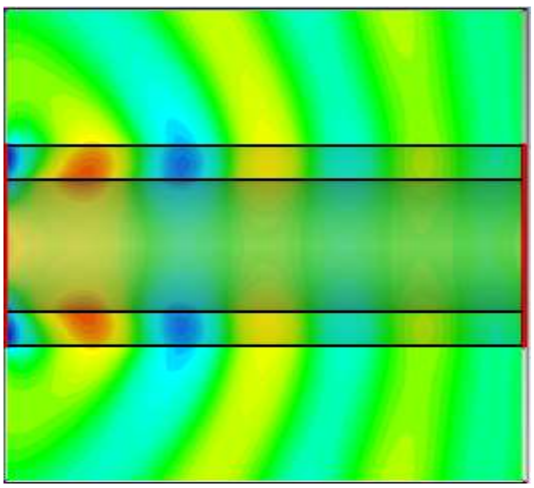

Fig. 5 Electric Field distribution at 10GHz (Leaky Mode)

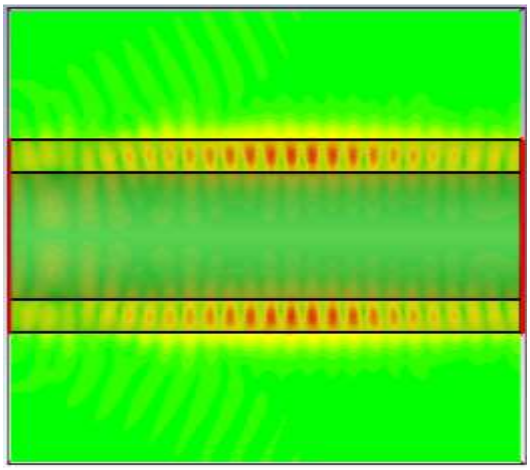

Fig. 6 Electric Field distribution at 20GHz (Guided Mode)

3. CONCLUSIONS

From the numerical and computational investigation of Guided and leaky mode field theory, the cutoff frequency can be distinguished with the dispersion equation. Leaky mode exists below guided mode cutoff value and in these region attenuation constant increases as frequency decreases. By filling plasma in waveguide, it can be electrically reconfigurable antenna via changing plasma density. Here normalized plasma wavenumber ($K_p r_1$=2) then the cutoff frequency of fundamental TM_{01} mode is 15.5 GHz in MATLAB and 16GHz in CST Simulation results.

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