Energy Harvested and Achievable Rate of Massive MIMO under Channel Reciprocity Error

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Abstract - *This paper investigates the harvested energy and downlink achievable rates of massive multiple-input-multiple-output (MIMO) enabled simultaneous wireless information and power transfer (SWIPT) systems. Since the transmission range of massive MIMO SWIPT systems is quite short, the transmission channels are generally of line-of-sight. Therefore, by assuming Rician fading channels, this paper theoretically derives the approximate expressions of harvested energy and achievable rate. Our main objective is to design the most energy efficient and information transfer communication system between base station and users under the effect of channel reciprocity error. In this paper, we model and analyze the impact of RF mismatches on the performance of linear precoding in a TDD multi-user massive MIMO system, by taking the channel estimation error into considerations. We use the truncated Gaussian distribution to model the RF mismatch.*

Key Words: **Massive MIMO, Energy Efficiency, linear precoding, channel reciprocity error, RF mis match, imperfect channel estimation, Achievable rate.**

1.INTRODUCTION

MASSIVE (or large scale) MIMO (multiple-input-multiple-output) systems have been identified as enabling technologies for the 5th Generation (5G) of wireless systems]. Such systems propose the use of a large number of antennas at the base station (BS) side. A notable advantage of this approach is that it allows the use of simple processing at both uplink (UL) and downlink (DL) directions. For example, for the DL transmission, two commonly known linear precoding schemes, i.e., maximum ratio transmission (MRT) and zero-forcing (ZF), have been extensively investigated in the context of massive MIMO systems. Most prior studies assume perfect channel reciprocity by constraining that the time delay from the UL channel estimation to the DL transmission is less than the coherence time of the channel. Such an assumption ignores two key facts: 1) UL and DL radio-frequency (RF) chains are separate circuits with random impacts on the transmitted and received signals 2) the interference profile at the BS and UT sides may be significantly different. The former phenomenon is known as RF mismatch, which is the main focus of this paper. RF mismatches can cause random deviations of the estimated values of the UL channel from the actual values of the DL channel within the coherent time of the channel. Such deviations are known as reciprocity errors that invalidate the assumption of perfect reciprocity. The existing works on studying reciprocity errors can be divided into two categories. In the first category, reciprocity errors are considered as an additive random uncertainty to the channel coefficients. However, it is shown that additive modelling of the reciprocity errors is inadequate in capturing the full impact of RF mismatches. Therefore, the recent works consider multiplicative reciprocity errors where the channel coefficients are multiplied by random complex numbers representing the reciprocity errors.

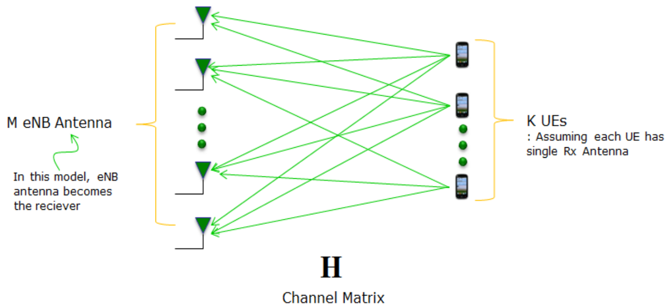

Fig -1: A Massive MIMO system

2. SYSTEM MODEL

A single cell multi-user MIMO system is considered, where M is the number of antennas at the base station and K is the number of single antennas at the user's end. All the antennas work in same time-frequency band. We have assumed that, (i.e. M >> K). The system is modeled in such a way that transmission in one coherent time interval T, ƮT is divided for Training phase, and the remaining time (1 -Ʈ) T is used for transmitting information and Power simultaneously, where,Ʈ ϵ (0,1). We have considered the channel vector between the kth user and the base station as Rician fading model of Rician factor K_k and large scale fading βk, is represented as

$$h_k = \sqrt{\beta_k * \frac{k_k}{k_k + 1}}\, a_k + \sqrt{\frac{\beta_k}{k_k + 1}}\, z_k \tag{1}$$

Where,

$$z_k \sim N(0, I)$$

denotes random component of the k_{th} user. a_k is a deterministic vector. Mathematically, it is given by:

$$a_k = e^{\left\{-j(m-1)\left(\frac{2\pi d}{\lambda}\right)\text{Sin}(\theta_k)\right\}} \quad (2)$$

where, d is antenna spacing, θ_k is the arrival angles of M different signals to the k_{th} user antenna. λ is the wavelength and d= $\lambda/2$, for simplicity, $d_k = \sqrt{\frac{\beta_k}{k_k+1}}$

2.1 CHANNEL RECIPROCITY ERROR MODELLING

Due to the fact that the imperfection of the channel reciprocity at the single-antenna UT side has a trivial impact

on the system performance, we focus on the reciprocity errors at the BS side. The channel reciprocity matrix, H_{bt} represents effective response at the BS. It is represented as follows:

$$H_{bt} = \text{diag}\left[h_{\{bt,1\}}, h_{\{bt,2\}}, h_{\{bt,3\}}, \dots h_{\{bt,m\}}, \dots h_{\{bt,M\}}\right] \quad (3)$$

Where, $h_{\{bt,m\}}$ represents the response matrix of the m_{th}

antenna at the BS to the single antenna user. (The subscript b is for base station, t for the transmitter end of base station while the second subscript m is for assigning the antenna.) Mathematically,

$$h_{\{bt,m\}} = A_{bt,m} * e^{\{j\phi_{bt,m}\}} \quad (4)$$

$A_{bt,m}$ represents the amplitude response and $\phi_{bt,m}$ represents the phase response of the k_{th} antenna at the BS. We have taken $A_{bt,m}$ and $\phi_{bt,m}$ as truncated Gaussian since it is more realistic in comparison to the uniformly distributed error model.

$$A_{bt,m} \sim NT(\mu, 0, \sigma^2), A_{bt,m} \in [a, b] \quad (5)$$

$$\phi_{bt,m} \sim NT(\mu, 0, \sigma^2), \phi_{bt,m} \in [\phi_1, \phi_2] \quad (6)$$

Therefore, the channel matrix for the k_{th} user under consideration of channel reciprocity is given by:

$$H = \text{diag}(h_k) * H_{bt} \quad (7)$$

Now, channel reciprocity has been taken into consideration for the channel matrix of the k_{th} user.

2.2 UPLINK CHANNEL ESTIMATION

A set of mutually orthogonal pilot sequence vector, each of length L, is assigned randomly to all the users for uplink pilot transmission (to BS). The pilot sequence matrix is of the size L*K. The estimated channel after minimum mean square error (MMSE) is given by:

$$\widehat{h_k} = \sqrt{d_k K_k} a_k + d_k \sqrt{\{L\, p_k^u\}}\, (d_k/d_k L\, p_k + \sigma^2) \times$$
$$\left(\sqrt{d_k L\, p_k^u}\, \{z_k\} + n_k \phi_k\right) \quad (8)$$

Where,

$$n_k \sim CN(\mu, \sigma^2) \text{ and } \phi_k \sim CN(\mu, \sigma^2)$$

n_k represents white Gaussian noise distribution and ϕ_k is the uplink power of the k_{th} user antenna.

$$\phi = [\phi_1 \phi_2 \phi_3 \dots \dots \dots \dots \phi_k \dots \dots \dots \phi_K] \quad (9)$$

L is the length of each pilot sequence. ϕ_k represents the assigned pilot sequence of the k_{th} user.

$\widehat{h_k}$ is a complex gaussian matrix, represented by $\widehat{h_k} \sim CN(\overline{h_k}, \gamma_k I_M)$, where,

$$\overline{h_k} = \sqrt{\{d_k k\_k\}} a_k,\ \gamma_k = d_k^2\, L\, p_k\, (d_k/d_k L\, p_k^u + \sigma^2)$$

2.2 DOWNLINK INFORMATION AND ENERGY TRANSMISSION

During the downlink, information symbols are sent from the base station to each user. Length of each symbol is L. The mean and variance of the symbols are 0 and 1 respectively. Let the symbol vector of k_{th} user be S_k.Then the received signal at the k_{th} user is given by the following equation:

$$y_k = \sqrt{\{p_k\}} h_k^H w_k s_k + \sum_{\{t \neq k\}}^{\{K\}} \sqrt{\{p_t\}}\, h_k^H w_t s_t + n_k \quad (10)$$

where p_k is the transmitted power to the k_{th} user, h_k^H is the conjugate Transpose of h_k (Hermitian of h_k Matrix) and w_k is the MRT precoding matrix of the k_{th} user. Mathematically, w_k is given by

$$w_k = \frac{\{\widehat{h_k}\}}{\sqrt{E\left|\left|\widehat{h_k}\right|\right|^2}} \quad (11)$$

n_k is an additive complex white Gaussian noise of

$$E|n_k| = 0 \text{ and } \sigma^2 = 1$$

The system adopts a power splitter at each user antenna such that it splits the power into two: (i) ρ_k part of the power is used for decoding the received signal from the BS and (ii) (1- ρ_k) part of the power is used for harvesting energy, $\rho_k \in [0,1]$. If we neglect the power of the noise and assume that the battery storage is large enough for accommodating all the harvested energy, then the expected harvested energy by the k_{th} user is given by

$$Q_k = \delta_k E[|\, h_k^H \sum_{\{t=1\}}^{\{K\}} \sqrt{p_t}\, w_t|^2] \quad (12)$$

Here,

$$\delta_k = \eta_k(1-\rho_k)(1-\tau) \quad (13)$$

And,

$$\eta_k \in [0,1]$$

2.4 SINR AND ACHIEVABLE RATE

As the signal arrives to the k_{th} user, ρ_k fraction of power is used for the decoding information and (1- ρ_k) is used for harvesting energy. The received signal y_k is expressed as:

$$y_k = \sqrt{\rho_k}\,(\sqrt{p_k} h_k^H w_t s_t + \sum_{t\neq k}^{K} \sqrt{p_k}\,(h_k^H w_t s_t + n_k) + n_{p,k} \quad (14)$$

Where, $n_{p,k}$ is the noise signal added to the received signal

$$n_{p,k} \sim CN(0, \sigma_p^2\}$$

SINR can be calculated by the ratio of the signal power and sum of the power of the interference signal and the noise signal:

$$SINR = \frac{E|\sqrt{\rho_k p_k} h_k^H w_k s_k|^2}{E|(\sum_{t\neq k}^{K} \sqrt{p_k}\, h_k^H w_t s_t + n_k) + n_{p,k}|^2} \quad (15)$$

Hence, Achievable Rate of the k_{th} user is given by:

$$R = (1-\tau) \times \log_2(1 + (SINR)) \quad (16)$$

3. RESULTS AND ANALYSIS

A single cell massive MIMO system with 5 users is simulated. During the simulation, the frame length is fixed as 300 symbols; T is normalized to be 1 second, and the pilot length L equals to K. For simplicity, the same power split coefficients and Rician factors are chosen for all users. The energy conversion efficiency is all 1. The accuracy of the proposed approximate and asymptotic expressions of the harvested energy are first investigated, and the results are shown. It is observed that for the harvested energy the numerical results match the simulated results greatly. The results also indicate that the amount of the harvested energy increases approximate linearly with the number of antennas. The harvested energy is simulated for different truncation limit in dB and for the different variance of noise or interference signal. Also, the achievable rate and harvested energy is plotted with reciprocity error and without reciprocity error, Because of non-reciprocal system the performance has decreased significantly. Results indicate that in all cases the approximate and asymptotic expressions of achievable rates are almost the same, and both of them are coherent with the simulated rates

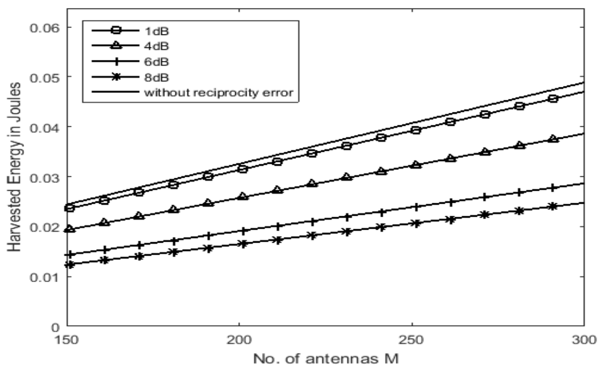

Fig -2: Average harvested energy V/S number of antennas for different values of truncation limit of channel

In addition, the achievable rate over Rician fading channels outperforms that over Rayleigh fading channels, and has a noticeable growth as M increase.

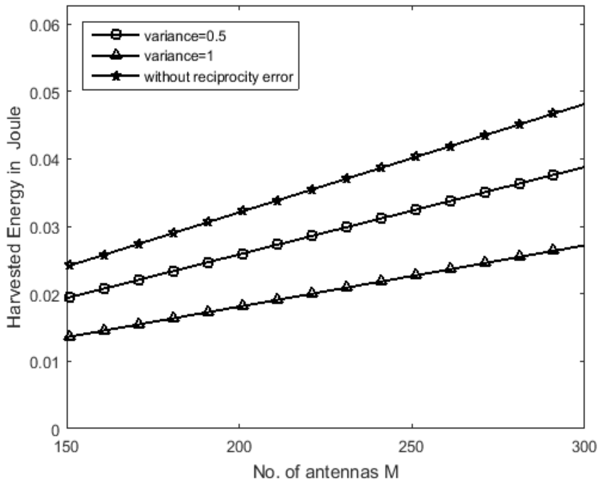

Fig -3: Average harvested energy V/S number of antennas for different variance.

Since the distance between users and BS is usually small in practical SWIPT systems, the power split coefficient can be properly designed to keep the accuracy of asymptotic achievable rate without impacting the system achievable rate.

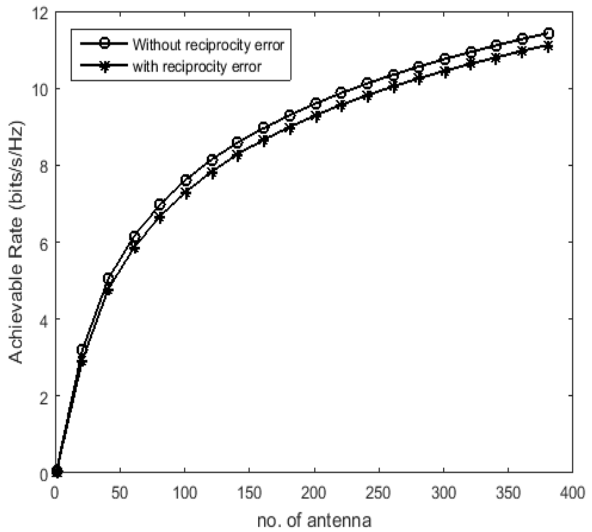

Fig -4: Average Achievable Rate V/S number of antennas

After the massive MIMO regime (a few hundreds of antennas) achievable rate is tending to be constant. Rician fading channel are more general and accurate for massive MIMO enabled SWIPT systems. The optimum number of base station antenna is around 300 to 600 antennae i.e., is massive MIMO regime. Hence, the massive MIMO is most energy efficient system.

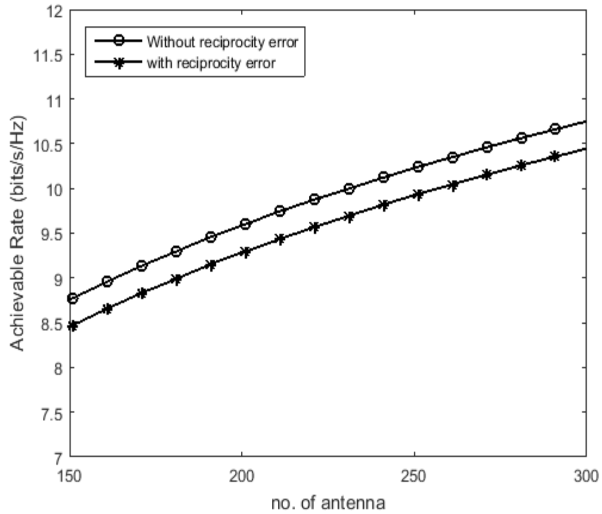

Fig -5: Average Achievable Rate V/S number of antennas

3. CONCLUSION

Harvested energy and the achievable rate have been derived for massive MIMO enabled SWIPT systems over Rician fading channels under the channel reciprocity error. In this paper, we have analyzed the impact of the channel reciprocity error caused by the RF mismatches, on the performance of linear precoding schemes such as MRT and ZF in TDD massive MU-MIMO systems with imperfect channel estimation. Considering the reciprocity errors as multiplicative uncertainties in the channel matrix with truncated Gaussian amplitude and phase errors, we have derived analytical expressions of the output SINR for MRT and ZF in the presence of the channel estimation error, and analyzed the asymptotic behavior of the system when the number of antennas at the BS is large. The perfect match has been found between the analytical and simulated results in the cases with the practical and asymptotically large values of the BS antennas, which verifies that our analytical results can be utilized to effectively evaluate the performance of the considered system. Computer simulations are done and results have validated the accuracy of the derived expressions. Especially, the concise asymptotic achievable rate can be applied to the design of SWIPT systems. Hence, the reciprocity error in channel has decreased the harvested energy and achievable rate in the massive MIMO system, in some case it may be coherent with the reciprocal system and it may be advantageous for the system.

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Abhishek Thakur is currently studying in IIIT Manipur in the department of Electronics and Communication Engineering. His research interests include Massive-MIMO, Wireless communications and signal processing.