Analytical modeling for vibration analysis of partially cracked Cylindrical Shell

Prakash Tiwari[1], P.V.Joshi[2]

[1]Pursuing M. Tech (Machine Design) from SSGI, Bhilai, Chhattisgarh, India
[2] Senior Associate professor , Department of Mechanical Engg. SSGI, Bhilai, C.G., India

---***---

Abstract:

In this thesis we determined the effect of crack in homogeneous isotropic partially cracked cylindrical shell on natural frequency. Crack is taken along the axis of the shell. Relation between the tensile stress and bending stress at far side and at the crack location is applied by using line spring method. With taking the crack into account, we derived the governing equation of partially cracked cylindrical shell for axisymmetric vibration. The governing equation is solved by analytical methods for simply supported cylindrical shell and by varying various parameters first mode natural frequencies are calculated for simply supported boundary condition. We used line spring model to calculate crack compliance coefficient. Effects of the shell properties such as length, radius as well as the effects of the crack characteristics such as its length on the natural frequencies of the cracked shell are analyzed in this study. Also we used method of multiple scales to solve duffing equation and modal amplitude response is plotted with detuning parameter.

1. Introduction :

Thin shell is now extensively used in nuclear power plant, large length roofs , water tanks and in many engineering application. In mechanical engineering, shell forms are used in piping systems, cylindrical walls, silos, turbine disks, and pressure vessels technology, Aircrafts, missiles, rockets, ships, and submarines. Another application of shell engineering is in the field of biomechanics: shells are found in various biological forms, such as the eye and the skull, and plant and animal shapes.

Rice and levy[1] developed line-spring model Which is more accurate method than FEM . Line spring model can be applied to plate and shell.

Delale and Erdogan[2] used line-spring model developed by rice and levy to obtain approximate solution for a cylindrical shell containing Semi elliptical part through internal and external surface crack. They used reissner type theory to formulate shell problem. Reissner type theory was used to account for transverse shear deformation. R.B.King[3] Developed simplified line spring model by changing the crack front with constant depth and assuming ligament spring as “elastic perfectly plastic” and treating crack of constant depth. Raju and newman [4] founded stress intensity factors for semi elliptical cracks in pipes and rod. It is assumed that the configuration is subjected to either bending load or remote tension. D.M.Parks[5] extended line spring model developed by rice and levy to estimate J-integral and crack tip opening displacement for plates and shell with crack. Asif Irasar[6] developed analytical model for the effect of crack in structural plate and panels . He generated reduced order analytical model for the behavior of cracked plate subjected to a force which cause it to vibrate
K. Moazzez, H., Saeidi Googarchin, S.M.H. Sharifi[7] Developed a new analytical method for determination of a long cylindrical shell with semi elliptical crack. Equation used are based on DMV theory.

2. Analytical Modelling

2.1 Problem Identification:

Cylindrical shells are now extensively used in engineering application. Generally if the shell is cracked then its natural frequency is not same as the intact plate. Natural frequency of cracked shell is generally less than the intact shell. It can result the failure much before the expected time.
In the thesis we will establish relation for natural frequency of cracked shell.

2.2 Problem Formulation

2.2.1 Assumption

Material is homogeneous and isotropic, crack brought the change only in the moment perpendicular to the crack. Except moment crack will not affect other parameters like force. Solution is determined for the simply supported shell only. Only axisymmetric vibration is considered.

2.2.2 Derivation

From[15] the general equation for intact cylindrical shell for axisymmetric vibration is

$$\boxed{\frac{d^2w}{dx^4}+4\beta^4 w=\frac{1}{D}\left(p_3+\frac{\nu N_1}{R}\right)}$$

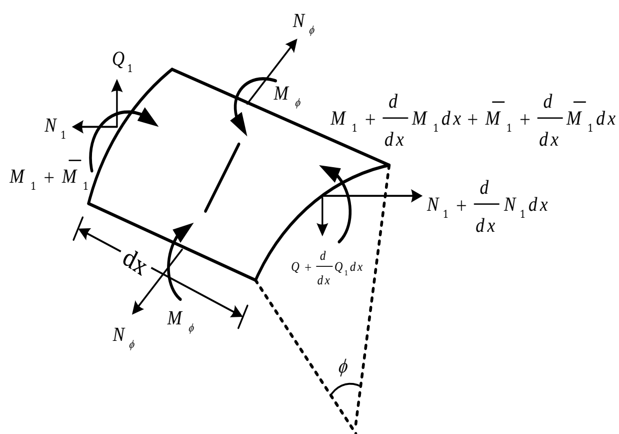

Fig. 1 Force and moment in partially cracked cylindrical shell

Equation of static equilibrium:

$$\frac{dN_1}{dx}+p_1=0 \qquad (2.1)$$

$$\frac{dQ_1}{dx}+N_2\frac{1}{R}+p_3=0 \qquad (2.2)$$

$$\frac{dM_1}{dx}+\frac{d\bar{M}_1}{dx}-Q_1=0 \qquad (2.3)$$

Eliminating Q from equation (2.2) and (2.3)

$$\frac{d^2M_1}{dx^2}+\frac{d^2\bar{M}_1}{dx^2}+\frac{N_2}{R}+p_3=0 \qquad (2.4)$$

Now the stress resultant and stress couple relation takes the form

$$M_1=-D\frac{d^2w}{dx^2} \quad M_1=-D\nu\frac{d^2w}{dx^2}=\nu M_1$$

(2.5)

After simplication we have

$$\boxed{\frac{d^2w}{dx^4}+\frac{1}{D}\frac{d^2\bar{M}_1}{dx^2}+4\beta^4 w=\frac{1}{D}\left(p_3+\frac{\nu N_1}{R}\right)}$$

(2.6)

Where

$$\beta^4=\frac{Eh}{4R^2D}=\frac{3(1-\nu^2)}{R^2h^2}$$

β is a geometric parameter of [length]$^{-1}$

Equations (2.6) represent the governing differential equation of asymmetrically loaded cylindrical shell.

Now by adding inertia forces to given external loads

$$\frac{1}{D}\frac{d^2\bar{M}_1}{dx^2}+\frac{\partial^4 w}{\partial x^4}+4\beta^4 w=\frac{-\rho h}{D}\frac{\partial^2 w}{\partial t^2} \qquad (2.7)$$

From equation (2.14) we have, values of $\bar{M}_1$ in terms of M_1 for different values of crack length. If

$$c=\frac{2a}{3\left(\left(\frac{\alpha_{bt}}{6}\right)+\alpha_{bb}\right)(3+\nu)(1-\nu)h+2a}$$

Let $\bar{M}_1=cM_1$ Putting the values in eq. (2.7) we have

$$\boxed{(1-c)\frac{d^4w}{dx^4}+4\beta^4 w=-\frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2}} \qquad (2.8)$$

Where deflection w is a function of x coordinate and time t $w=w(x,t)$

The solution of simply supported shell of length L assuming simple harmonic equation in time is sought in following forms

$$w(x,t)=\sum_{n=1}^{\infty}A_n\sin\left(\frac{n\pi x}{L}\right)\sin\omega t \qquad (2.9)$$

After putting the values and simplifying we get the following relation

$$\boxed{\omega^2 = \frac{D}{\rho h}\left[(1-c)\left(\frac{n\pi}{L}\right)^4 + 4\beta^4\right]} \tag{2.10}$$

Where $\beta^4 = \dfrac{Eh}{4R^2 D}$ and $D = \dfrac{Eh^3}{12(1-\nu^2)}$

Crack term formulation:

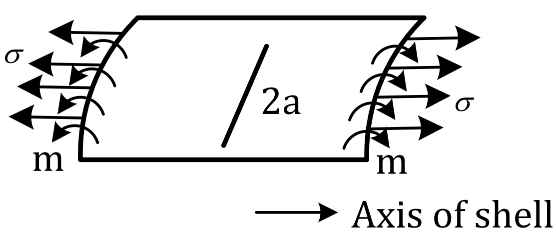

Fig 2 Tensile stress and bending Stress at far side of the shell

From [5] we have the following relation between tensile and bending stresses at crack at far side

$$\boxed{\bar{\sigma} = \frac{2a}{(6\alpha_{tb} + \alpha_{tt})(1-\nu^2)h + 2a}\sigma}$$

(2.11)

And

$$\boxed{\bar{m} = \frac{2a}{3\left(\left(\frac{\alpha_{bt}}{6}\right) + \alpha_{bb}\right)(3+\nu)(1-\nu)h + 2a}m}$$

(2.12)

Similarly same relation holds good for Force and moments at far side and at crack side

$$\bar{N} = \frac{2a}{(6\alpha_{tb} + \alpha_{tt})(1-\nu^2)h + 2a}N \tag{2.13}$$

$$\bar{M} = \frac{2a}{3\left(\left(\frac{\alpha_{bt}}{6}\right) + \alpha_{bb}\right)(3+\nu)(1-\nu)h + 2a}M \tag{2.14}$$

Value of $\alpha_{tt}, \alpha_{bb}, \alpha_{bt}, \alpha_{tb}$ for shell can be determined from the following equation taken from [2]

$$\alpha_{tt} = \xi^2 \sum_{n=0}^{12} C_{tt}^{(n)} \xi^{2n}$$

$$\alpha_{bb} = \xi^2 \sum_{n=0}^{12} C_{bb}^{(n)} \xi^{n}$$

$$\alpha_{bt} = \alpha_{tb} = \xi^2 \sum_{n=0}^{18} C_{bb}^{(n)} \xi^{n}$$

$$\boxed{\begin{aligned}\alpha_{tt} = \xi^2 (&1.9761\xi^0 + 11.4870\xi^2 + 7.7086\xi^4 + 15.0143\xi^6 + 280.1207\xi^8 - 1099.72\xi^{10} + 3418.9795\xi^{12} \\ &- 7686.9237\xi^{14} + 12794.1279\xi^{16} - 13185.0403\xi^{18} + 7868.2682\xi^{20} - 1740.2463\xi^{22} + 124.1360\xi^{24})\end{aligned}}$$

$$\boxed{\begin{aligned}\alpha_{bt} = \alpha_{tb} = \xi^2 (&1.9735\xi^0 - 2.2166\xi^1 + 21.6051\xi^2 - 69.3133\xi^3 + 196.3\xi^4 - 406.2608\xi^5 + 644.9350\xi^6 \\ &- 408.9569\xi^7 - 159.6927\xi^8 - 988.9879\xi^9 + 4266.5487\xi^{10} - 2997.1408\xi^{11} - 6050.7849\xi^{12} \\ &+ 8855.3615\xi^{13} + 3515.4345\xi^{14} - 11744.1116\xi^{15} + 4727.9784\xi^{16} - 1685.6087\xi^{17} - 845.8958\xi^{18})\end{aligned}}$$

$$\boxed{\begin{aligned}\alpha_{bb} = \xi^2 (&1.9710\xi^0 - 4.4277\xi^1 + 34.4952\xi^2 - 165.7321\xi^3 + 626.3926\xi^4 - 2144.4561\xi^5 + 7043.4169\xi^6 \\ &- 19003.2199\xi^7 + 37853.3028\xi^8 - 52595.4681\xi^9 + 48079.2948\xi^{10} - 25980.1559\xi^{11} + 6334.2425\xi^{12})\end{aligned}}$$

From equation (2.10) we have

$$(1-c)\frac{d^4 w}{dx^4} + 4\beta^4 w = -\frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2}$$

Now let

$$\frac{\partial^2 \omega}{\partial t^2} = A_n \sin\left\{\frac{n\pi x}{L}\right\}\ddot{\phi}(t)$$

After simplifying we get

$$\frac{\rho h}{D}\ddot{\phi} + \left\{(1-c)\left(\frac{n\pi}{L}\right)^4 + 4\beta^4\right\}\phi(t) = 0$$

Comparing above eq. with the governing eq. of free vibration we have Equivalent mass $M = \frac{\rho h}{D}$ and equivalent stiffness $K = \left\{(1-c)\left(\frac{n\pi}{L}\right)^4 + 4\beta^4\right\}$

By method of multiple scale we have

Frequency amplitude response equation

$$\boxed{\sigma_{pq} = \pm\sqrt{\frac{\lambda_{pq}^2 p^2}{4\omega_{pq}^2 D^2 d^2}}}$$

And peak amplitude

$$\boxed{d_p = \frac{\lambda_{pq}}{2\omega_{pq}\mu D} p}$$

3. Result and discussion:

We made the calculation for material constant $E = 6Gpa$, $\rho = 2660\ kg/m^3$, $\nu = 0.3$ $\mu = 0.08$. All the values of results are for simply supported beam.

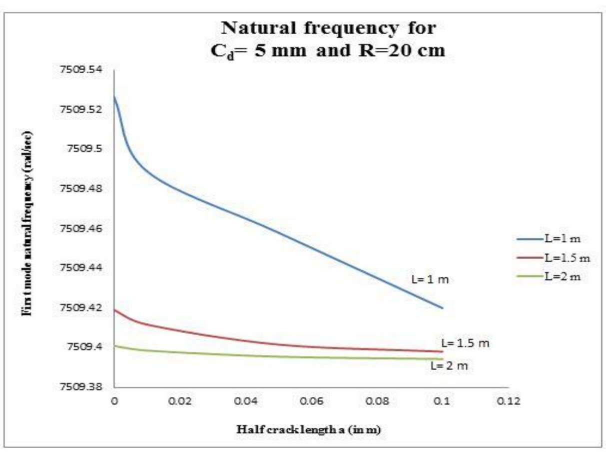

Chart -1: variation of (ω_n) with half crack length (a) for different values of L

The fig. shows natural frequency for length 1m , 1.5m , 2m . We can see that as we increase the length the frequency decreases. The result is conforming with equation (3.10) where the natural frequency of is inversely proportional to forth order of length of shell. Hence we can conclude that to increase natural frequency we should keep the length as minimum as possible.

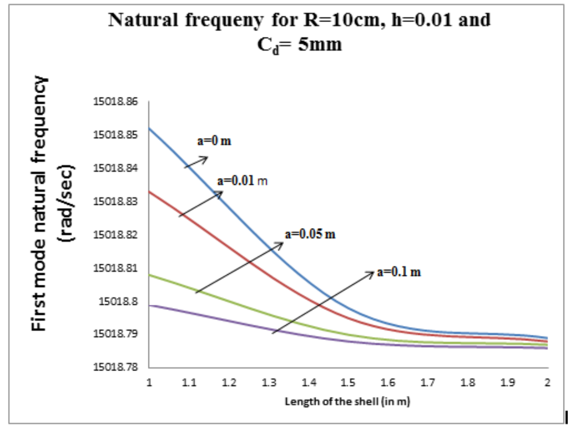

Chart 2 : variation of (ω_n) with Length of shell (L) for different values of half crack length (a)

Crack length $a = 0$ shows intact shell. From the result we can say that the highest value of natural frequency occurs for intact shell and as the crack length increases the natural frequency decreases. So we should avoid crack as far as possible to keep the value of natural frequency on higher side.

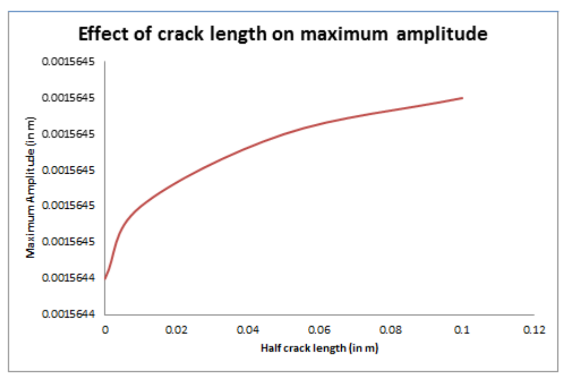

Chart 3: Effect of crack length on maximum amplitude

we can see that as maximum amplitude of vibration is directly proportional to the crack length. As we increase the crack length the value of amplitude of vibration increases which is not good for any machine. Hence to keep the value of amplitude of vibration low we should try to avoid cracking of shell. For small value of crack length amplitude increases rapidly but after a=0.005 m it increases steadily

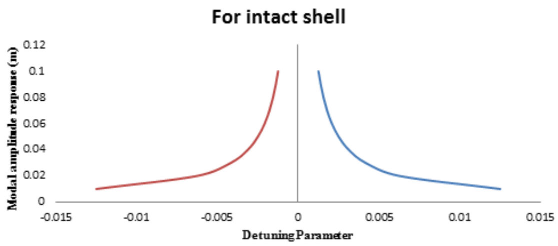

Chart 4: Graph between Detuning parameter and modal amplitude response for intact shell

For $E = 6Gpa$, $x_0 = 0.5m$, $L = 1m$, $R = 10cm$, $\rho = 2660kg / m^3$, $h = 0.01m$, $\mu = 0.08, \nu = 0.3$

Shows detuning parameter for different values of b and other fixed value mentioned above. We found the graph symmetric and conforming to [6]

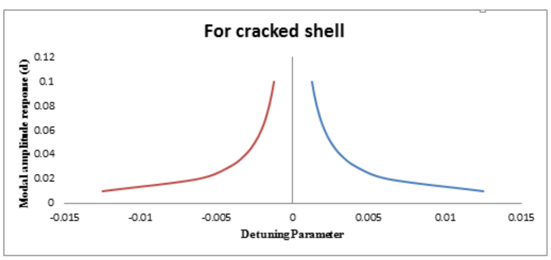

Chart 5: Graph between Detuning parameter and modal amplitude response for intact shell

We did the same work as intact shell and used the same data except value of half crack length i.e. taken 0.01m. For cracked shell we obtained value of Detuning parameter for different modal amplitude response.

Now for $C_d = 5mm$,$h = 0.05m, E = 6Gpa$, $x_0 = 0.5m$, $L = 1m$, $\rho = 2660kg / m^3$, $h = 0.01m$, $\mu = 0.08, \nu = 0.3$ we have the following table for values of natural frequency

Table 1 : Natural frequencies of isotropic shell

Natural frequency of isotropic shell			
S. No.	**Radius (cm)**	**First mode natural frequency (rad/sec)**	
		Intact shell	**Cracked shell (a=0.01 m)**
1.	10cm	15020.447	15019.978
2.	15cm	10015.026	10014.323
3.	20cm	7512.7347	7511.7969

From table 1 we can conclude that the values of natural frequency of intact shell for any radius are more than the corresponding values of cracked shell. Also as we increase the radius the value of natural frequency decreases.

4. Conclusion

After observing the results we arrived at the following conclusion :

1. Presence of crack affects the natural frequency.
2. First mode natural frequency (ω_n) of the homogeneous, isotropic shell decreases with increase in length.
3. Increase in crack length results a increase in natural frequency (ω_n).
4. The natural frequency (ω_n) increases with shell thickness.
5. With increase in radius of the shell the value of natural frequency decreases .
6. Peak amplitude of the shell increases with increase in crack length which is not desirable.
7. The curve between detuning parameter and modal amplitude response is linear and symmetric.

5. Acknowledgement:

We want to sincerely acknowledge our director Dr. P. B. Deshmukh and HOD Jeetendra Kumar Tiwari for inspiring us for successful progression of research work .

6. References :

[1] Rice J.R. and Levy N., "The part through surface crack in an Elastic plate", journal of applied mechanics, vol 39, pp. 185-194 , 1972

[2] F. Delale and F. Erdogan , "Application of linespring model to a cylindrical shell containing circumferential or axial part through crack", Journal of applied mechanics, vol. 49, pp. 97-102 ,1982

[3] R.B. King, "Elastic and plastic analysis of surface flaw using a simplified linespring model", Engineering fracture mechanics, vol. 18, pp. 217-231,1983

[4] S. raju and J.C newman "Stress intensity factors for circumferential pipes and rods under tension and bending stress", 1985

[5] D.M. Parks, "The Inelastic Line-Spring Estimates of Elastic-Plastic Fracture Mechanics Parameters for Surface-Cracked Plates and Shells", "Journal of pressure vessel technology", vol. 103, pp. 246-254, 1981

[6] Asif Irasar, " vibration analysis of cracked aluminium plate", University of Glasgow,2008

[7] K. Moazzez, H., Saeidi Googarchin, S.M.H. Sharifi, "Natural frequency analysis of a cylindrical shell containing a variably oriented surface crack utilizing Line-Spring model", Thin walled structure, vol. 125, pp. 63-75, 2018

[8] Parks, D.M. "Inelastic analysis of surface flaws using the linespring model", proceedings of the 5th international conference on fracture , France, 1981.

[9] Tanish Dey and L.S. Ramchandra, "Nonlinear vibration analysis of laminated composite circular cylindrical shell", composite structures, vol 163, pp. 89-100, 2016

[10] Nied , H.F., " A hollow cylinder with axisymmetric internal or surface crack under non axisymmetric Arbitrary loading", Ph.D. Dissertation , Lehigh University , 1981

[11] Jay C. Chen and Charles D. Bebcock, "Nonlinear vibration of cylindrical shell", California institute of technology, Calif, 1975

[12] Reissner, E. and wan, F.Y.M., " On the equation of linear shallow shell theory", Study in applied mathematics, vol. 48, pp. 132-145, 1969.

Nomenclature:

p_1, p_2, p_3 : Surface load in three mutual perpendicular axis

N_1 and N_2 :Internal forces in x and y direction at far side of the shell

Q_1 : Shear force

β : Geometric parameter

$\bar{\sigma}$ and $\bar{m}$: Tensile and bending stress at crack

σ and m : Tensile and bending stress at far side

$\alpha_{tt}, \alpha_{bb}, \alpha_{tb} = \alpha_{bt}$:Compliance coefficient

X_m : Modal Function

ψ_{pq} :Time dependent modal coordinate

d_p : Peak amplitude

σ_{pq} : Modal amplitude response

E : young's modulus of elasticity

L : Length of the shell

w : Displacement perpendicular to shell axis

h : Thickness of shell

R : Radius of shell