Core Power Receding Horizon Control (RHC) Approach for Pressurized Water Reactor (PWR) Nuclear Power Plants

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Abstract - *To track load changes, a well-performed core power control is essential in pressurized water reactors (PWR) and nuclear power stations. Keeping the core power stable at the required values within the acceptable range of error bands to ensure the safety demands of the PWR is very difficult due to the nuclear reactors sensitivity. Since nuclear reactors are extremely sensitive and time-varying, creating/developing and designing a well-performed core power control system is challenging. PID control methods have an abundance of benefits and advantages, yet it is problematic for PID control methods to satisfy the requirements and needs for controlling core power in a fleet and precise way [1]. Several methods have been applied by researchers to aid in the control of the core power in pressurized water reactors (PWR) power stations, some of which may include constant axial offset strategy [2], fuzzy logic methods [3], neural network methods [4], robust optimal control systems [5], and finally intelligent controls systems [6]. Moreover, the researchers may have succeeded with some of these methods, yet there are some difficulties in controlling the core power due to the nuclear reactor's sensitivity. Hence, there are various opportunities for additional methods to be applied to control the core power in PWR with both accuracy and speed. In this paper, the Receding Horizon Control (RHC) is examined in detail and the results are compared against the classical Proportional Integral Derivative (PID) controller.*

Key Words: **Receding Horizon Control (RHC), PID Control, Pressurized Water Reactor (PWR), Quadratic Programming (QP), Nuclear Power Plant, Optimal Control**

1. INTRODUCTION

The need for efficient, clean, and a sustainable energy source is the ultimate quest of the 21st century, where nuclear power plants have been playing a major role in fulfilling this mission since the 20th century, with continuous focused research on methods for improving this technology to better serve humanity. According to the International Atomic Energy Agency (IAEA), there are 449 nuclear power reactors in operation with a total net installed capacity of 391,744 MWe and 56 under construction with a total net installed capacity of 57,192 MWe around the world [7].

1.1 Types of Nuclear Power Plants

Generally, there are three main purposes for nuclear reactors: civilian use, military use, and research use. Civilian reactors (the focus of this paper) are mainly used to generate electricity, where the reactor design relies heavily on the main purpose for nuclear reactor which in turn affects the type of fuel, coolant, and other design details [8].

There are currently four generations of nuclear power plants. The first generation (1950-1979), the experimental prototypes, include the UK's Magnox Reactors and the US's Dresden. The second generation (1970-1990), consisting of commercial power reactors much like the uranium enriched Light Water-cooled Reactors (LWRs), as well as the Boiling Water Reactor (BWR), and the Pressurized Water Reactor (PWR). The third generation (1990-2030), including the Advanced Light Water Reactors (ALWRs), containing of System 80+ and the Advanced Boiling Water Reactor (ABWR) were used in East Asia, and by 2010-2030, new designs like the European EPR and the Westinghouse Advanced Passive AP600 and AP1000 are expected to be installed, since they promote safety and contribute to finances [9]. The fourth generation (2030 and beyond), these reactors are expected to surpass all previous generations, meeting with upgraded reliability and safety demands, economic competitiveness, proliferation resistance, as well as the reduction of radioactive waste in the future when it is fully built [10].

1.2 Pressurized Water Nuclear Power Plants

In a Pressurized Water Reactor (PWR), heat is generated inside the core within the reactor vessel and then carried to the steam generator via the pressurized water in the primary coolant loop. Steam is then generated when water is vaporized from the primary coolant loop to a secondary loop within the steam generator, and this steam is guided to the main turbine, resulting in the spinning of the turbine generator, producing electricity. The liquid form of the water in the reactor vessel is enforced and maintained through high pressure, whereas steam that is used to spin the turbine is created in a separate steam generator [11].

The reactor core is located in the heart of the reactor, found in the reactor pressure vessel, which encompasses three vital mechanisms including the fuel in the reactor core (found in small fuel rods). The second is the surrounding water which plays the part of the moderator, heat-transfer agent, and a coolant. The third mechanism comprises the control rods that maintain the reactivity at a certain level and shut down the reactor in emergencies [12]. Primary water originating from the core is pumped at high pressures through heat exchanger pipes in the steam generator, where water is directed into the secondary side of the steam generator, which is heated into steam that is used to spin the turbine. They are then sent to a condenser where they are cooled in a second heat exchanger before returning once again to the steam generator. The cooler, cold side of the condenser heat exchanger characterizes the PWR tertiary loop, although in principle it does not need to be closed, as the cooling water from the condenser could circulate from and to a water source, such as an ocean or a river. However, you can typically find that the condenser output is sent to circulate via a cooling tower, where it can be cooled by evaporation. The water lost from becoming steam in the cooling tower can be replaced by a water source that is drawn from a river or an ocean [13].

1.3 Control Rods

Control rods are vital in every nuclear power plant holding two main objectives, firstly being safety in terms of controlling the divergence of the chain reaction from the fission process and second, controlling the amount of output power from the reactor. When operating a nuclear plant, a complicated task with a simple requirement is to be maintained, keeping the chain reaction running smoothly for months without interruption while sustaining the reactor from reaching explosion. This balance can be achieved if the reactor operator maintains the reactor criticality at the value of 1 (number of secondary neutrons triggering a new fission). The laws of physics would naturally apply in correcting small variations in power, but the reactor designers must decide on the equilibrium point that will result in natural regulation of criticality. Doppler effect plays a major role in the self-regulation mechanism, where increasing the temperature in the reactor core results in increasing the nuclei's thermal agitation [10]. The chain reaction slows down with the temperature dropping gradually as a result.

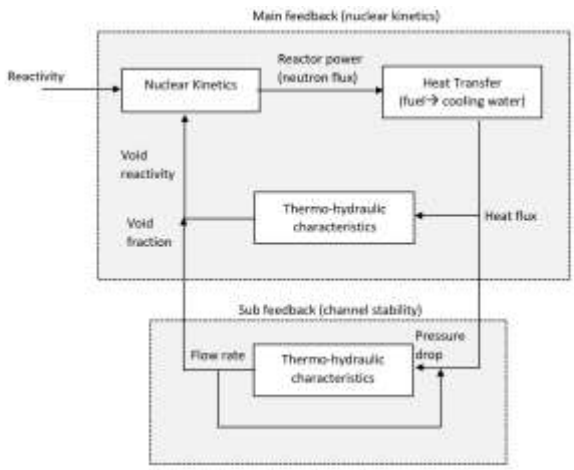
The rod control system (RCS) regulates the neutrons motion while absorbing the full-length rods. The system works by moving the rods, as a response to the demand signals generated from the reactor operator during the start-up phase, shutdown phase, during power operations, or even from the automatic rod control system (ARCS) in order to preserve the reactor's average temperature in the coolant system (T_{avg}). Furthermore, in response to the manual/automatic reactor trip signals, the control system releases the rods, allowing them to fall into the core to shut down the reactors. The rod control system deals with short-term reactivity changes due to fuel depletion and xenon transient changes, and to compensate for long term effects, the system adjusts the boron concentration levels within the reactor coolant. The automatic rod control system maintains the primary coolant average temperature between power levels of 15% and 100% at the pre-programed values through adjustments to the reactivity within core [14].

When the average temperature deviates from the program temperature by an amount greater than the preselected quantity, it results in the movement of the automatic rod in order to return the T_{avg} to the program temperature. The speed of the rod varies depending on the size of the temperature deviation, and the direction of rod movement is fully dependent on the average temperature of the reactor coolant in certain conditions where the program temperature is higher or lower than the initial program temperature [14]. The generated heat from the fission process boils the water to turn it into steam that drives the turbine-generator system to convert mechanical energy into electrical energy. The main components of a nuclear power plant are: fuel, control rods, moderator, coolant, turbine-generator system, and reactor vessel.

2. MODELING

The core power control model is based on a combination of several mathematical models that include: neutron dynamics models, thermal hydraulic models, and reactivity models [15].

Figure 1: Main Feedback (Nuclear Kinetics)



2.1 Neutron Dynamics Model

In order to describe the reactor kinetics, the number of neutrons and the number of delayed neutron precursors that change with time are considered. The point reactor kinetics model refers to grouping the energy of the neutrons and

ignoring the space dependence of variables. Hence, the reactor is not considered as a single point but the assumption is made that the space distribution of parameters does not change with time. The point reactor kinetics model can be used by weighting the reactivity feedback amount determined with the importance function when a slow disturbance is treated in spatial asymmetry. In general, the point reactor approximation can be used to approximate a slow change of the space distribution of parameters. It can be applied to many transient events that contain the disturbance to be handled by reactor control [15].
The reactivity is the degree of deviation from the critical state, and it can be defined as follows:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{1}$$

(ρ : reactivity, k_{eff}: effective neutron multiplication factor)

$$k_{eff} = \frac{Neutron\ production\ rate}{Neutron\ loss\ rate} \tag{2}$$

If the reactor is supercritical, $k_{eff} > 1$ and the value of ρ is positive. If the reactor is subcritical, $k_{eff} < 1$ and the value of ρ is negative. ρ takes a value within the range of $-1 < \rho < 1$. The reactivity is expressed as a numerical value or a percentage. The prompt neutron generation time can be defined by the following equation:

$$\Lambda = \frac{Total\ number\ of\ neutrons\ in\ reactor}{Neutron\ generation\ rate} \tag{3}$$

Point-reactor kinetic equations of multigroup delayed neutrons will cause a heavy calculation workload, so these equations can be simplified by multigroup delayed neutrons being equivalent to one single group of delayed neutrons. The simplified kinetic equations are as follows:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c \tag{4}$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c \tag{5}$$

(n : neutron density, t : time, ρ : total reactivity, β : total fraction of effective delayed neutrons, Λ : time of neutron generation, λ : decay constant of delayed neutron precursors, c : concentration of delayed neutron precursors)

Using normalization method to re-write equations (4) and (5) as follows:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \frac{\beta}{\Lambda} c \tag{6}$$

$$\frac{dc}{dt} = \lambda n - \lambda c \tag{7}$$

The core power is proportional to the neutron density which can be described as follows:

$$P_a(t) = n\, P_0 \tag{8}$$

($P_a(t)$: actual core power, P_0 : nominal core power, P_0 is constant which means n represents relative core power.)

2.2 Thermal Hydraulic Models

In relation to the law of conservation of energy, the equations are as follows:

$$P_c(t) = \Omega\,(T_f - T_c) \tag{9}$$

$$P_e(t) = M\,(T_l - T_e) \tag{10}$$

($P_c(t)$: heat quantity transferred from fuel to cooling water, $P_e(t)$: heat quantity transferred from cooling water to the secondary circuit, Ω : heat transfer coefficient between fuel and cooling water, M: heat capacity of mass flow rate of cooling water, T_f : average temperature of fuel, T_c : average temperature of cooling water, T_l : outlet temperature of cooling water, T_e : inlet temperature of cooling water)

The inlet temperature of cooling water is mainly constant and stable at 300˚C. The difference between the inlet and the outlet temperature of cooling water is roughly 30˚C. The physical parameters of cooling water were assumed to be constant during heat exchange between fuel and cooling water. Thus, the resulting equations are as follows:

$$T_c = \frac{1}{2}(T_l + T_e) \tag{11}$$

$$\delta T_e = 0 \tag{12}$$

(δ : deviation value relative to the balance point)

Therefore, the thermal hydraulic models in PWRs are as follows:

$$f_f P_a(t) = \mu_f \frac{dT_f}{dt} + P_c(t) \tag{13}$$

$$(1 - f_f)\, P_a(t) + P_c(t) = \mu_c \frac{dT_l}{dt} + P_e(t) \tag{14}$$

(f_f: fraction of reactor power deposited in fuel, μ_f: heat capacity of fuel, μ_c: heat capacity of cooling water)

2.3 Reactivity Models

The reactivity models are as follows:

$$\delta\rho = \delta\rho_r + \alpha_f(T_f - T_{f0}) + \frac{\alpha_c(T_l - T_{l0})}{2} + \frac{\alpha_c(T_e - T_{e0})}{2} \tag{15}$$

$$\frac{d\delta\rho_r}{dt} = G_r Z_r \tag{16}$$

($\delta\rho_r$: reactivity produced by the movement of control rod, α_f: reactivity coefficient of fuel, α_c: reactivity coefficient of cooling water, T_{f0}: initial steady-state fuel temperature, T_{l0}: initial outlet temperature of cooling water, T_{e0}: initial inlet temperature of cooling water, G_r : total reactivity worth of control rod, Z_r: velocity of the control rod)

This following equation was obtained from equations (11), (12) and (15):

$$\delta\rho = \delta\rho_r + \alpha_f(T_f - T_{f0}) + \frac{\alpha_c(T_l - T_{l0})}{2} \tag{17}$$

2.4 Modeling of Reactor Core Power – State Space Model

The state-space model for PWRs can be obtained as follows:

$$\begin{cases} \dot{x} = A_d x + B_d u \\ y = Cx + Du \end{cases} \tag{18}$$

($\dot{x}$: derivative of x, matrix x : variable of the state space, y : output quantity of the state space, A_d, B_d, C, and D : coefficient matrixes, u : controlled quantity of the state space)

The value of δn is much smaller than n_0, according to the linearized theory of slow perturbation around the balance point. Therefore, the neutron density can be described as follows [15]:

$$n = n_0 + \delta n \tag{19}$$

(n : neutron density, n_0 : balance value of neutron density, δn : deviation value of neutron density relative to the balance point)

By linearizing and simplifying equation (6):

$$\frac{d\delta n}{dt} = \frac{-\beta}{\Lambda}\delta n + \frac{\beta}{\Lambda}\delta c + \frac{\delta\rho}{\Lambda} n_0 \tag{20}$$

For the model, the variable, the output quantity, and the controlled quantity of the state space were selected as follows:

$$\begin{cases} x = [\delta n \quad \delta c \quad \delta T_f \quad \delta T_l \quad \delta\rho_r]^T \\ y = [\delta n] \\ u = [Z_r] \end{cases} \tag{21}$$

The state space model in equation (18) is solved using linear algebra and differential equations based on equations (7), (8), (9), (10), (13), (14), (16), (17), (20), and (21) to obtain the state space matrices A_d, B_d, C, and D as follows:

$$A_d = \begin{pmatrix} \frac{-\beta}{\Lambda} & \frac{\beta}{\Lambda} & \frac{\alpha_f}{\Lambda} n_0 & \frac{\alpha_c}{2\Lambda} n_0 & \frac{n_0}{\Lambda} \\ \lambda & -\lambda & 0 & 0 & 0 \\ \frac{f_f}{u_f} P_0 & 0 & \frac{-\Omega}{u_f} & \frac{\Omega}{2u_f} & 0 \\ \frac{1-f_f}{u_f} P_0 & 0 & \frac{\Omega}{u_c} & \frac{-2M+\Omega}{2u_c} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{22}$$

$$B_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ G_r \end{bmatrix} \tag{23}$$

$$C = [1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{24}$$

$$D = [0] \tag{25}$$

Table 1 below lists the parameters along with their values for typical PWRs. These values were substituted in the state space model matrices A_d, B_d, C, and D in the simulation of the reactor core power model in MATLAB.

Table 1: Typical Parameters' Values for PWRs

Parameter	Description	Value	Unit
β	Total fraction of effective delayed neutrons	0.006019	-
Λ	Neutron generation time	0.00002	s
α_f	Reactivity temperature coefficient of fuel	$(n_0-4.24)\times10^{-5}$	$°C^{-1}$
n_0	Initial equilibrium relative neutron density	1	m^{-3}
α_c	Reactivity temperature coefficient of coolant	$(-4n_0-17.3)\times10^{-5}$	$°C^{-1}$
λ	Decay constant of delayed neutron precursors	0.15	s^{-1}
f_f	Fraction of reactor power deposited in fuel	0.92	-
u_f	Thermal capacity of fuel	26.3	$MW \cdot s \cdot °C^{-1}$
P_0	Nominal core power	25000	MW
Ω	Heat transfer coefficient between fuel and coolant	$(\frac{5}{3}n_0 + 4.9333)$	$MW \cdot s \cdot °C^{-1}$
u_c	Heat capacity of coolant	$(\frac{160}{9}n_0 + 54.002)$	$MW \cdot s \cdot °C^{-1}$
M	Heat capacity of mass flow rate of coolant	$(28n_0 + 74)$	$MW \cdot s \cdot °C^{-1}$
G_r	Total reactivity worth of control rod	0.0145	-

2.5 Receding Horizon Control (RHC)

Receding Horizon Control (RHC), also commonly known as Moving Horizon Control (MHC) or Model Predictive Control (MPC) is an important advanced optimal control technique used for difficult multivariable control problems. The history of the RHC dates back to the early 1970s when Shell Oil engineers developed this technique and deployed it specifically to control large interactive Multi-Input-Multi-Output (MIMO) processes in the refinery distillation columns [13]. RHC makes use of explicit dynamic plant model to predict the effect of future reactions of the manipulated variables on the output and the control signal obtained by minimizing the cost function [13].

Seborg summarizes the basic concept of RHC as follows: suppose that we wish to control a multiple-input, multiple-output process while satisfying inequality constraints on the input and output variables [13]. If a reasonably accurate dynamic model of the process is available, model and current measurements can be used to predict future values of the outputs. Then the appropriate changes in the input variables can be calculated based on both predictions and measurements [13]. In essence, the changes in the individual input variables are coordinated after considering the input-output relationships represented by the process mode [13]. In RHC applications, the output variables are also referred to as controlled variables or CVs, while the input variables are also called manipulated variables or MVs. Measured disturbance variables are called DVs or feedforward variables [13].

RHC is a powerful tool due to the fact that the prediction capability allows solving optimal control problems online, where tracking error, namely the difference between the predicted output and the desired reference, is minimized over a future horizon possibly subject to constraints on the manipulated inputs, outputs, and states [13]. It is worthy to note that the success of RHC heavily relies on the correctness of the process model. There are many great advantages for using RHC, which include [13]:

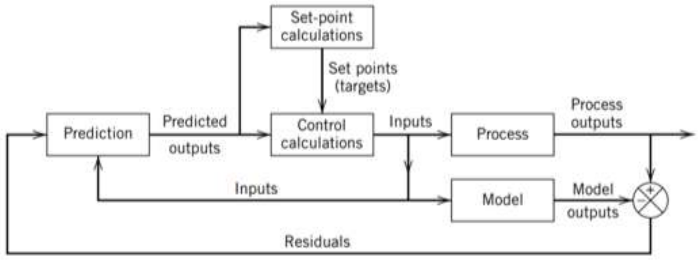
1. The dynamic and static interactions between input, output, and disturbance variables are captured by the process model
2. Both inputs and outputs constraints are considered systematically
3. The optimum set points calculations can be coordinated with the control calculations
4. The accurate model predictions can become a hazard sign for potential problems

The objectives of the RHC can be summarized as follows:

a) Prevent violations of input and output constraints
b) Drive some output variables to their optimal set points, while maintaining other outputs within specified ranges
c) Prevent excessive movement of the input variables
d) Control as many process variables as possible when a sensor or actuator is not available

The RHC block diagram is shown in Figure 2 [13]. A process model is used to predict the current values of the output variables. The residuals, the differences between the actual and predicted outputs, serve as the feedback signal to a Prediction block. The predictions are used in two types of RHC calculations that are performed at each sampling instant: set-point calculations and control calculations. Inequality constraints on the input and output variables, such as upper and lower limits, can be included in either type of calculation.

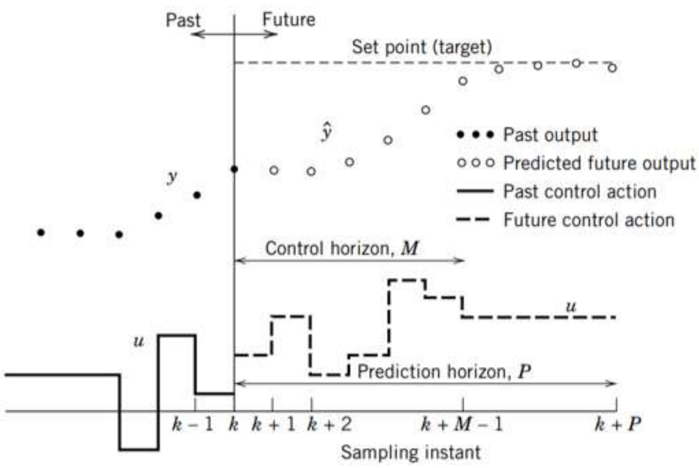
Figure 2: RHC Block Diagram



The objective of the RHC control calculations is to determine a sequence of control moves (that is, manipulated input changes) so that the predicted response moves to the set point in an optimal manner. The actual output y, predicted output $\hat{y}$, and manipulated input u for SISO control are shown in Figure 3 [13]. At the current sampling instant, denoted by k, the RHC strategy calculates a set of M values of the input {u (k +i-1), i=1, 2,..., M}. The set consists of the current input u(k) and M-1 future inputs. The input is held constant after the M control moves. The inputs are calculated so that a set of P predicted outputs $\hat{y}$ (k+i), i=1, 2,..., P} reaches the set point in an optimal manner. The control calculations are based on optimizing an objective function. The number of predictions P is referred to as the prediction horizon while the number of control moves M is called the control horizon [13].

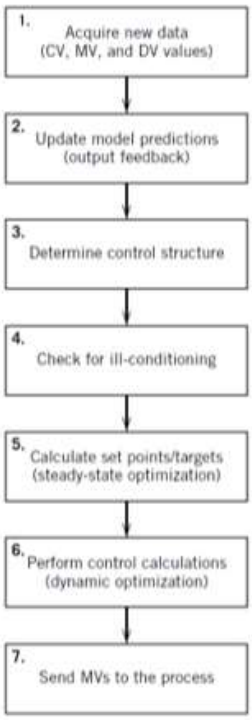
A distinguishing feature of RHC is its receding horizon approach. Although a sequence of M control moves is calculated at each sampling instant, only the first move is actually implemented. Then a new sequence is calculated at the next sampling instant, after new measurements become available; again only the first input move is implemented. This procedure is repeated at each sampling instant. The analysis for SISO systems can be generalized to MIMO systems by using the Principle of Superposition [13].

Figure 3: Basic Concept of RHC



The flowchart in Figure 4 provides an overview of the RHC calculations [13]. The seven steps are shown in the order they are performed at each control execution time. For simplicity, we assume that the control execution times coincide with the measurement sampling instants. In RHC applications, the calculated MV moves are usually implemented as set points for regulatory control loops at the Distributed Control System (DCS) level, such as flow control loops. If a DCS control loop has been disabled or placed in manual, the MV is no longer available for control, thus in this situation, the control degrees of freedom are reduced by one. Even though an MV is unavailable for control, it can serve as a disturbance variable if it is measured [13].

Figure 4: RHC Calculations Flowchart



2.6 Quadratic Programming (QP)

A quadratic program (QP) is an optimization problem with a quadratic objective function and linear constraints. The general quadratic program (QP) is as follows [16]:

$$\begin{aligned} \min \quad & q(x) = \frac{1}{2}x^T G x + x^T c \\ \text{subject to} \quad & a_i^T x = b_i, \quad i \in \varepsilon, \\ & a_i^T x \geq b_i, \quad i \in I \end{aligned} \tag{26}$$

(G : symmetric $n \times n$ matrix, ε and I : finite sets of indices; c, x, and $\{a_i\}$, $i \in \varepsilon \cup I$: vectors in $\boldsymbol{R}^n$)

Quadratic Programs can be solved in a finite amount of computation, but the effort required to find a solution depends on the characteristics of the objective function and the number of inequality constraints [16].

If the Hessian matrix G is positive semidefinite, then it is a convex QP, and in this case the problem is similar in difficulty to a linear program. Nonconvex QP, in which G is an indefinite matrix, is challenging because it can have several stationary points and local minima [16].

The gradient projection method allows the active set to change rapidly from iteration to iteration. It is most efficient when the constraints are simple in form when there are only bounds on the variables. Therefore, the problem becomes [16]

$$\begin{aligned} \min \quad & q(x) = \frac{1}{2}x^T G x + x^T c \\ \text{subject to} \quad & l \leq x \leq u \end{aligned} \tag{27}$$

(G : is symmetric, l and u : vectors of lower and upper bounds on the components of x)

Each iteration of the gradient projection algorithm consists of two stages. The first stage involves searching along the steepest descent direction from the current point x, that is, the direction $-g$, where $g = Gx + c$. Whenever a bound is encountered, the search direction is "bent" so that it stays feasible. Next is searching along the resulting piecewise-linear path and locate the first local minimizer of q, which is denoted by x^c and is referred to as the 'Cauchy point'. The working set is defined to be the set of bound constraints that are active at the Cauchy point, denoted by $A(x^c)$. In the second stage of each gradient projection iteration, it is required to explore the face of the feasible box on which the Cauchy point lies by solving a subproblem in which the active components x_i for $i \in A(x^c)$ are fixed at the values x_i^c [16].

After the Cauchy point x^c has been computed, the components of x^c that are at their lower or upper bounds define the active set [16]:

$$A(x^c) = \{i \mid x_i^c = l_i \text{ or } x_i^c = u_i\} \tag{28}$$

In the second stage of the gradient projection iteration, approximately solving the QP obtained by fixing the components x_i for $i \in A(x^c)$ at the values x_i^c. The remaining co]:

$$\begin{aligned} \min \quad & q(x) = \frac{1}{2}x^T G x + x^T c \\ \text{subject to} \quad & x_i = x_i^c, \quad i \in A(x^c), \\ & l_i \leq x_i \leq u_i, \quad i \notin A(x^c) \end{aligned} \tag{29}$$

The algorithm for gradient projection method is as follows [16]:

Compute a feasible starting point x^0;
for $k = 0, 1, 2, \ \ldots$
if x^k satisfies the KKT conditions
stop with solution $x^* = x^k$;
Set $x = x^k$ and find the Cauchy point x^c;
Find an approximate solution x^+ such that $q(x^+) \le q(x^c)$
and x^+ is feasible;
$x^{k+1} \leftarrow x^+$;
end (for)

2.7 RHC Model

By applying the RHC concept, the state space model in equation (18) is discretized as follows:

$$\begin{cases} x[k+1|k] = Ax[k] + B\Delta u[k] \\ y[k+1|k] = Cx[k+1|k] + Du[k] \end{cases} \quad (30)$$

Equation (30) can be further simplified as follows:

$$\begin{cases} x[k+1|k] = Ax[k] + B\Delta u[k] \\ y[k+1|k] = CAx[k] + CB\Delta u[k] \end{cases} \quad (31)$$

(A : discrete matrix form of A_d, B : discrete matrix form of B_d, $x[k]$: value of the variable of the state space at current sampling time k, $[k+1|k]$: predictive value of the next sampling time $k+1$, Δ : increment)

Following the RHC concept, $\Delta u[k] = 0$ which indicates that $u[k]$ is constant and this means that it is out of the control horizon.

The receding horizon strategy objective function is as follows:

$$J = (R_s - Y)^T(R_s - Y) + \Delta U^T R_w \Delta U \quad (32)$$

with,

$$Y = (y[k+1|k] \ \ y[k+2|k] \ \ y[k+3|k] \ldots y[k+Np|k])^T \quad (33)$$

$$R_s = [1\,1\,1 \ldots 1]^T r[k] \quad (34)$$

$$\Delta U = (\Delta u[k] \ \ \Delta u[k+1] \ \ \Delta u[k+2] \ldots \Delta u[k+Nc-1]])^T \quad (35)$$

$$R_w = R_1 \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \quad (36)$$

(R_s : column vector with Np elements, $r[k]$: reference trajectory, Np : prediction horizon, Nc : control horizon, R_w : diagonal matrix and the weight matrix with Nc x Nc dimensions)

Rewriting equation (33) as follows:

$$Y = Fx[k] + \varphi \Delta U \quad (37)$$

with,

$$\varphi = \begin{pmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ CA^{Np-1}B & CA^{Np-2}B & CA^{Np-3}B & \cdots & CA^{Np-Nc}B \end{pmatrix} \quad (38)$$

$$F = \begin{pmatrix} CA \\ CA^2 \\ \vdots \\ CA^{Np} \end{pmatrix} \quad (39)$$

In order to attain the optimum solution of the RHC system, the objective function in equation (32) should reaches to its minimum value. To achieve that, it is necessary to take the derivative of the objective function with respect to ΔU, that is $\frac{df}{d\Delta U}$.

To reach to the extreme value of the objective function, then $\frac{df}{d\Delta U} = 0$. Accordingly, equations (32) and (37) can be solved to obtain the only optimum solution of the RHC system:

$$\Delta U = (\varphi^T \varphi + R_w)^{-1} \varphi^T (R_s - Fx[k]) \quad (40)$$

Considering a realistic system for the core power of the PWR, there are some mechanical constraints that limits the optimal solution. Usually, the control rod's maximum speed is 72 steps/min with the position range being 0 to 1 and it takes 300 steps to move from 0 to 1. Therefore, the range for $u[k]$ is $-0.004 \le u[k] \le 0.004$.

With the introduction of this new linear constraint to the quadratic problem, the optimum solution problem is altered to a constraint optimum solution problem which would require Quadratic Programming (QP) to approach it.

By returning to the objective function in equations (23) and (37), the objective function can be re-written as follows:

$$J = (R_s - Fx[k])^T(R_s - Fx[k]) - \Delta U^T \varphi^T (R_s - Fx[k]) \\ -(\Delta U^T \varphi^T (R_s - Fx[k]))^T + \Delta U^T (\varphi^T \varphi + R_w) \Delta U \quad (41)$$

considering that:
$\Delta U^T \varphi^T (R_s - Fx[k]) = (\Delta U^T \varphi^T (R_s - Fx[k]))^T$, since $\Delta U^T \varphi^T (R_s - Fx[k])$ has one element only, the objective function can be written as follows:

$$J = (R_s - Fx[k])^T(R_s - Fx[k]) - 2(\varphi^T (R_s - Fx[k]))^T \Delta U \\ + \Delta U^T (\varphi^T \varphi + R_w) \Delta U \quad (42)$$

To proceed with the Quadratic Programming, it is essential to re-write the objective function in equation (42) in terms of a new objective function J_{QP} as follows:

$$J_{QP} = \frac{1}{2}\Delta U^T(\varphi^T\varphi + R_w)\Delta U + (\varphi^T(R_s - Fx[k]))^T\Delta U \quad (43)$$

By RHC concept and extreme value theory, when $\frac{df}{d\Delta U} = 0$, then $\frac{dJ_{QP}}{d\Delta U} = 0$. This indicates that J_{QP} is qualitatively analogous to J which in turn designate that when J_{QP} reaches its minimum value, then the optimum solution of the RHC is attained. Further to the RHC theory, the range of U is defined as:

$$-M \le U \le M \quad (44)$$

with both U and M being column vectors with Nc elements as follows:

$$U = (u[k] \quad u[k+1] \quad u[k+2] \quad \cdots \quad u[k+Nc-1])^T \quad (45)$$

$$M = (0.004 \quad 0.004 \quad 0.004 \quad \cdots \quad 0.004)^T \quad (46)$$

which can be further expanded as follows in accordance to RHC theory

$$\begin{cases} u[k] = u[k-1] + \Delta u[k] \\ u[k+1] = u[k-1] + \Delta u[k] + \Delta u[k+1] \\ \vdots \\ u[k+Nc-1] = u[k-1] + \Delta u[k] + \cdots + \Delta u[k+Nc-1] \end{cases} \quad (47)$$

equation (47) can be re-written in the following form

$$U = A_{QP}\Delta U + u[k-1]B_{QP} \quad (48)$$

with A_{QP} being a triangular matrix having Nc x Nc dimensions and B_{QP} being a column vector with Nc elements:

$$A_{QP} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix} \quad (49)$$

$$B_{QP} = (1 \quad 1 \quad 1 \quad \cdots \quad 1)^T \quad (50)$$

The following equation is obtained from equations (44), (45), (46), (48), (49), and (50):

$$\begin{cases} -A_{QP}\Delta U \le M + u[k-1]B_{QP} \\ A_{QP}\Delta U \le M - u[k-1]B_{QP} \end{cases} \quad (51)$$

From equations (43) and (51), the following equation is obtained:

$$J_{QP} = \frac{1}{2}\Delta U^T H \Delta U + W^T\Delta U \quad (52)$$

$$G_{QP}\Delta U \le T_{QP} \quad (53)$$

$$H = (\varphi^T\varphi + R_w) \quad (54)$$

$$W = \varphi^T(R_s - Fx[k]) \quad (55)$$

$$G_{QP} = \begin{pmatrix} -A_{QP} \\ A_{QP} \end{pmatrix} \quad (56)$$

$$T_{QP} = \begin{pmatrix} M + u[k-1]B_{QP} \\ M - u[k-1]B_{QP} \end{pmatrix} \quad (57)$$

To find the optimum solution of the control system problem, it was essential to engage the QP to solve the constraint optimum solution problem, which was demonstrated in equations (52), (53), (54), (55), (56), and (57). Accordingly, the QP optimum problem can be summarized as:

$$\begin{aligned} &Maximize \quad J_{QP} = \frac{1}{2}\Delta U^T H\Delta U + W^T\Delta U \\ &Subject\ to \quad G_{QP}\Delta U \le T_{QP} \end{aligned} \quad (58)$$

3. CONTROLLER DESIGN

3.1 PID Controller

The Proportional Integral Derivative (PID) controller is a control loop feedback mechanism. The PID algorithm consists of three basic coefficients: proportional, integral, and derivative, which are varied to get optimal response. The proportional corrects instances of error, the integral corrects accumulation of error, and the derivative corrects present error versus error the last time it was checked. The effect of the derivative is to counteract the overshoot caused by P and I. When the error is large, the P and the I will push the controller output; this controller response makes error change quickly, which in turn causes the derivative to more aggressively counteract the P and the I. Tuning a control loop is the adjustment of its control parameters (gain/proportional band, integral gain/reset, derivative gain/rate) to optimum values for a target response.

High K_p will lead to oscillation in values and will tend to generate an offset, hence, K_I will counteract the offset. Higher value of K_I implies that the setpoint will reach the process variable too fast. If this action is very fast, the process variable is prone to be unsteady, as such K_D keeps this under control. PID tuning is the process of finding the values of proportional, integral, and derivative gains of a PID controller to achieve desired performance and meet design requirements. MATLAB is used to automatically tune the PID controller gains and to achieve the optimal system design to meet the design requirements.

3.2 RHC Controller

The basic concept behind RHC is to start with a model of the open-loop process that describes the dynamical relationships between the system's variables that includes command inputs, internal states, and measured outputs. After that, the system variables' constraint specifications are included, such as input limitations and desired ranges [17].

The control problem setup is completed by adding the desired performance specifications and are typically articulated through different weights on actuator efforts and tracking errors as in the case of linear quadratic regulation, while the remaining design of the RHC is automatic. The first step is to construct and translate the optimal control problem based on the given model, constraints, and weights, into an equivalent optimization problem, that depends on the reference signals and the initial state [17]. Then for each sampling time the optimization problem is solved by taking the current measured state as the initial state of the optimal control problem.

That is why this approach is said to be predictive, as the optimal control problem is formulated over a time-interval that starts at the current time up to a certain time in the future. Hence, the optimal sequence of future control moves is the result of the optimization. It is worth to note that only the first sample of this sequence is actually applied to the process while the remaining moves are discarded. For the next time step, a new optimal control problem based on new measurements is solved over a shifted prediction horizon.

This is a way of transforming an open-loop design methodology i.e., optimal control into a feedback one, as at every time step the input applied to the process depends on the most recent measurements by such a receding-horizon mechanism. Given that the performance index and constraints express true performance objectives as well as if the model is accurate enough, the RHC provides near-optimal performance.

It is important to be aware of the existence of a trade-off between complexity of the optimization and the model accuracy; hence, the simpler the model and performance index/constraints are, the easier it is to solve the optimization. The typical standard way of computing the linear RHC control action, which is implemented in most commercial RHC packages, is to solve the QP problem on line at each time t. Linear RHC controllers can be therefore embedded in arbitrarily complex MATLAB programs, with maximum versatility.

4. MATLAB SIMULATION RESULTS

4.1 PID Controller Results

From MATLAB simulation results, for the desired core power level to change from 50% to 60% to 50% nominal core power and 100% to 60% to 100% nominal core power, the PID controller load tracking performance is shown in Figures 5 and 6 respectively. The PID can successfully track the load changes although the overall performance is lacking in terms of providing the optimum solution. The overshoot is too large, the settling time is also large and the general performance is not up to the expectations. It takes much longer for the PID controller to stabilize at the desired core power after the load changes. This performance is expected from the PID as it was explained earlier that there are drawbacks and limitations for the PID controllers, however in terms of finding optimum solution, it is evident that the advancement in research led to various methodologies that could optimize the problem, one of which is the RHC controller.

Figure 5: The relative core power output of the PID controlled system at 100% → 60% → 100% of the desired core power level changes.

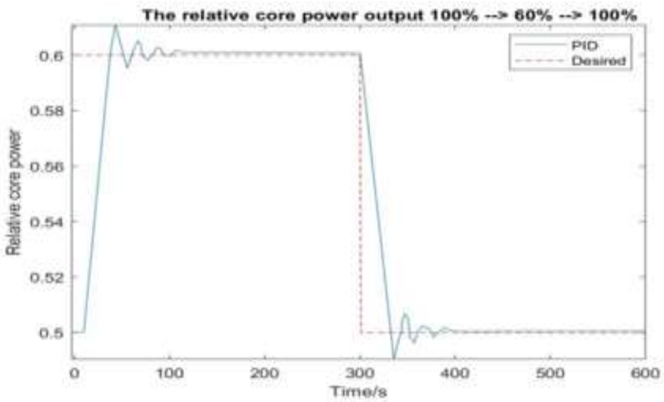
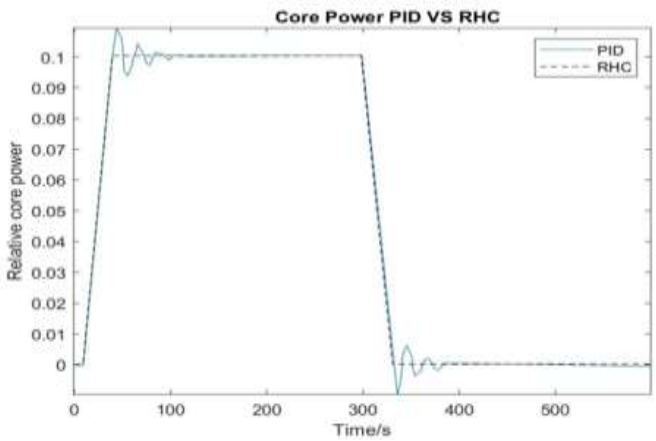


Figure 6: The relative core power output of the RHC Vs. PID controlled system at 50% → 60% → 50% of the desired core power level changes



4.2 RHC Controller Results

From MATLAB simulation results, for the desired core power level to change from 50% to 60% to 50% nominal core power and 100% to 60% to 100% nominal core power, the RHC controller load tracking performance is shown in Figures 6 and 7 respectively. The RHC can successfully track the load changes but the overall performance is evidently better than PID in terms of providing the optimum solution. The overshoot is minimized drastically, the settling time is greatly improved and the general performance is closer to the desired. It takes less time for the RHC controller to stabilize at the desired core power after the load changes. This performance is better than the PID as it was explained earlier that there are drawbacks and limitations for the PID controllers in comparison to the optimal RHC controller.

Figure 7: The relative core power output of the RHC controlled system at 100% → 60% → 100% of the desired core power level changes

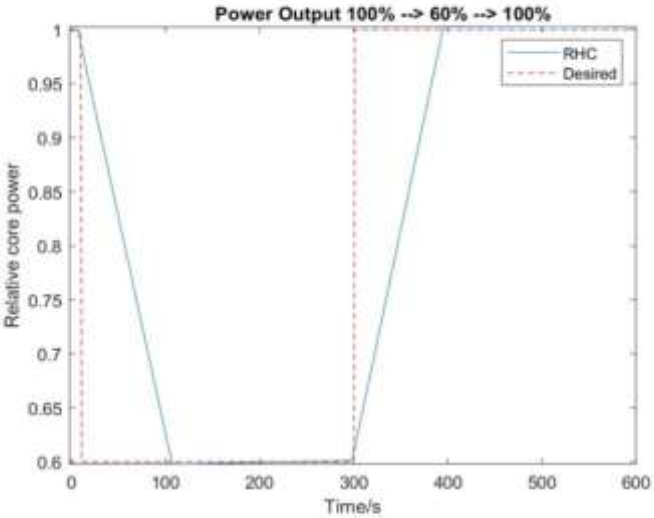
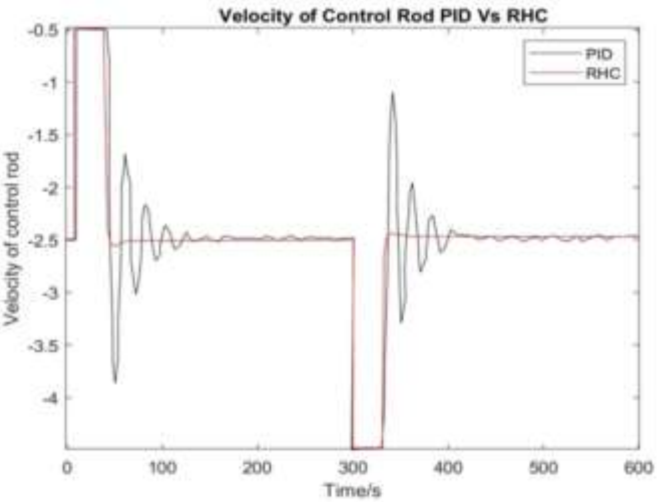
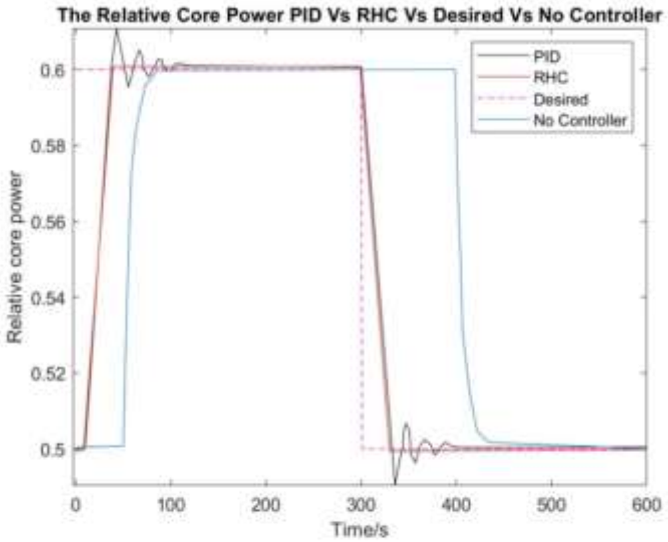


Figure 8: The velocity of the control rod of the RHC Vs. PID controlled system at 50% → 60% → 50% of the desired core power level changes



To have a clear comparison between PID and RHC controller's outputs, Figures 6 and 8 show both PID and RHC performances (control rod velocity and core power output) on one plot. It is quite clear that there are more oscillations for the PID controller and the settling time is large as the system takes more time to stabilize. Figure 9 on the other hand displays the plots of the core power output for the desired, no controller, PID and RHC cases. Clearly, RHC is the closest to the desired output and it has less overshoot and less settling time with a smooth output plot.

Figure 9: The relative core power output of the Desired, No Controller, RHC, and PID controlled system at 50% → 60% → 50% of the desired core power level changes



5. CONCLUSIONS

In this paper, the state space RHC methodology was applied to control the core power in a PWR nuclear power plant and was compared against the classical PID control methodology. RHC is one of the many methodologies used for core power control in PWRs, as it is challenging for the classical control methods such as PID control methods to control core power while maintaining prompt response to load changes that guarantees immediate stability. The proposed RHC control method was mainly constructed on the mathematical models of the reactor core, the state-space MPC model, and Quadratic Programming. The MATLAB simulation results indicate the effectiveness and the high performance of the proposed state space RHC method for load tracking of the core power of PWR nuclear power plant. After the load changes, the RHC control system reflected swiftly and stabilized at the desired core power rapidly and efficiently. The advantages of the state space RHC methodology are verified by the comparison between the state space RHC method and the PID control methodology. In addition, the RHC control system also possesses strong robustness. RHC is a powerful tool due to the fact that the prediction capability allows solving optimal control problems online, where tracking error, namely the difference between the predicted output and the desired reference, is minimized over a future horizon possibly subject to constraints on the manipulated inputs, outputs, and states.

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