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Comparison Between Thin Plate And Thick Plate From Navier Solution Using Matlab Software

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Abstract - *The basic theory available for the analysis of the plate is thin plate theory but we do have some other theories like Ressiner's theory and Mindlin theory which are advances for the plate analysis. The basic Kirchhoff's theory independent of thickness term whereas the advance theory contains thickness term in the equation. So it is interesting to note the variation of stress resultants with the thickness. In the present work the expression for bending moment, twisting moment and shear forces are derived for both Kirchhoff's thin plate theory and Ressiner's thick plate theory using Naviour's approach. Thus the well-known Navier's solution for the analysis of the simply supported plates is used to compare the two theories. Detailed program of Ressiner's thick plate and Kirchhoff's thin plate theory are written in Matlab7. Convergence of both the series is studied by taking summation of number of terms and the converged results obtained are discussed. The variations of bending moments, twisting moments and shear forces across the plate are calculated and plotted to compare the two theories. The results obtained from the program are discussed.*

***Key Words*: Kirchhoff's thin plate theory, Ressiner's thick plate theory , Navier's solution, Bending Moment, Twisting Moment, Shear Force.**

1. INTRODUCTION

Plates are plane structural elements whose thickness is small compared to length and width. The plate edges can be simply supported, fixed, point or elastically restrained. Plate whose thickness is divided into uniform halves by a flat surface being equidistant to its faces is called middle surface of the plate. Due to the geometry of the mid plane, the shape of the plate is defined. Plates are usually flat structure, which develops flexible moments, twisting moments and shear forces. Loads acting on plate are perpendicular to the plate surface. These loads are carried by bending, torsional moments and by transverse shear force. Plates show wide applications in bridges, architectural structures, hydraulic structures etc. Plates are further classified as thick and thin plates based on thickness. The typical plate structure diagram is as shown below.

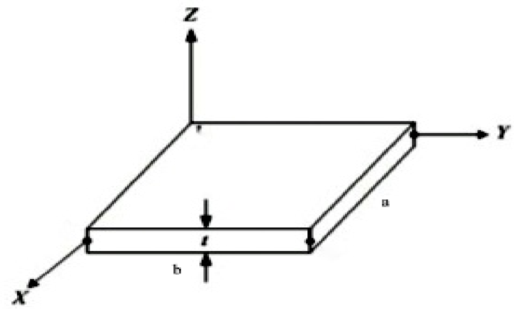

Fig 1: Typical plate diagram

2. Classification of Plates

Plates are categorized based on the thickness as thin plate and thick plate. In case of thin plate, the straight line normal to the middle plane remains normal even after deformation and the thickness of thin plate varies from 0.1 > t > 0.01. In case of thick plate, the straight line at right angle to the middle surface does not remain normal after deformation. The thickness (t) of thick plate is greater than 0.1 that is t > 0.1. The theories applicable for the analysis of plate are Kirchhoff plate theory for thin plate and for thick plate Mindlin theory and Reissner theory is used. For the study purpose Reissner theory is used for thick plate.

Kirchhoff's Thin Plate Theory

Consider a rectangular plate made to undergo to a lateral load 'q' which is acting at right angle to the plane. Let us consider a small element of sides 'dx' and 'dy' of the plate, the force influencing on the plates are shown in the figure.

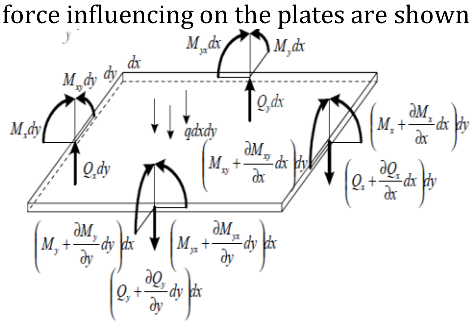

Fig-2 : Laterally loaded rectangular plate

$$\frac{q}{D} = \nabla^4(w)$$

Ressiner's Thick Plate Theory

According to Ressiner, two assumptions were made for thick plate theory. The assumptions are, initially he adopted the displacement fluctuations along thickness of plate be linear. Succeeding he assumed that the line upright to the neutral axis of the plate will change during the plate bends.

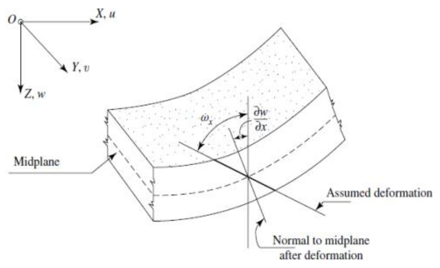

Fig-3 : Ressiner's thick plate cross section

The plate theory gives an accurate result for thin plate in compared with that of thick plate. Its perfection seemed to be reduced with increase in the plate thickness. Reissner improved his theory and in relation to horizontal bending which is a function of x and y, he developed the governing differential equation. Thus the solution obtained from this equation remained most demanding one.

$$\nabla^2 w + \frac{h^2(2-\mu)}{10(1-\mu)}\nabla^3 w = \frac{q}{D}$$

3. Objectives of the work

The objective of this paper is to analyses the moment and shear force variation due to thin plate (Kirchhoff's theory) and thick plate (Riessner's theory) by using Navier's Solution. A detailed program is written in MATLAB to obtain variations of bending moments, twisting moments and shear forces in 3-Dimensional by varying dimensions and in 2-Dimensional by varying thickness of the plate.

4. Convergence Study

A detailed program of Ressiner's and Kirchhoff's theory are written in Matlab7 so as to know the variations of bending moments, twisting moments and shear forces for each term of series. The converging results obtained for a thickness of **125mm** from the program is tabulated as shown below.

Table -1: Bending Moment (Mx)

Number of terms	Kirchhoff's thin plate theory	Ressiner's thick plate theory
1	4,709.1	4,597.3
4	4,580.1	4,477.1
9	4,703.6	4,515.6
16	4,673.1	4,550.9
25	4,695.9	4,505.1
36	4,688.1	4,564.8
49	4,695.1	4,521.3
64	4,692.7	4,697.9
81	4,695.2	4,793.1
100	4,694.6	4,782.1
225	4,695.8	4,836.6
400	4,696.5	4,841.8
625	4,696.4	4,840.4
900	4,696.6	4,842.2
1225	4,696.6	4,842.0
1600	4,696.6	4,842.1
1681	4,696.7	4,842.3

Table -2: Bending Moment (My)

Number of terms	Kirchhoff's thin plate theory	Ressiner's thick plate theory
1	3,289.6	3,149.6
4	2,837.0	2,787.2
9	3,005.7	2,872.2
16	2,944.2	2,870.5
25	2,981.0	2,849.9
36	2,961.9	2,875.6
49	2,975.7	2,845.0
64	2,967.3	2,872.8
81	2,974.0	2,843.0
100	2,969.5	2,869.2
400	2,972.0	2,855.6
900	2,972.4	2,847.7
1600	2,972.5	2,842.3
2500	2,972.5	2,838.3
2809	2,972.6	2,829.0
2971	2,972.5	2,832.8

3193	2,972.5	2,832.3

Table -3: Twisting Moment (Mxy)

Number of terms	Kirchhoff's thin plate theory	Ressiner's thick plate theory
1	2,588.5	2,586.8
4	2,861.8	2,858.9
9	2,922.1	2,918.8
16	2,944.3	2,940.4
25	2,954.7	2,950.4
36	2,960.4	2,955.7
49	2,963.9	2,959.0
64	2,966.2	2,960.5
81	2,967.7	2,962.1
100	2,968.8	2,962.9
121	2,969.7	2,962.6
144	2,970.3	2,963.5
169	2,970.8	2,964.1
196	2,971.2	2,964.6
225	2,971.5	2,965.0
256	2,971.7	2,965.3
289	2,971.9	2,965.5

Table -4: Shear Force (Qx)

Number of terms	Kirchhoff's thin plate theory	Ressiner's thick plate theory
1	7.0592	7.0592
4	7.3447	7.3447
9	7.9525	7.9525
16	7.9932	7.9932
25	8.1832	8.1832
36	8.1989	8.1989
49	8.2903	8.2903
64	8.2986	8.2986
81	8.3521	8.3521
100	8.3573	8.3573
225	8.4415	8.4415
400	8.4717	8.4717
625	8.4968	8.4968
900	8.5090	8.5090
1225	8.5209	8.5209
1521	8.5271	8.5271
1600	8.5274	8.5274

Table -4: Shear Force (Qy)

Number of terms	Kirchhoff's thin plate theory	Ressiner's thick plate theory
1	5.2944	5.2944
4	6.1745	6.1745
9	6.7663	6.7663
16	6.9321	6.9321
25	7.1175	7.1175
36	7.1841	7.1841
49	7.2732	7.2732
64	7.3089	7.3089
81	7.3610	7.3610
100	7.3833	7.3833
400	7.5307	7.5307
900	7.5794	7.5794
1600	7.6037	7.6037
2500	7.6182	7.6182
3600	7.6279	7.6279
4900	7.6348	7.6348
6400	7.6400	7.6400
8100	7.6440	7.6440
9801	7.6470	7.6470
10000	7.6472	7.6472
10201	7.6476	7.6476

5. Comparison of Kirchhoff Theory and Ressiner's Theory

For same material with same dimensions, a = 3000mm, b = 4000mm with varying thickness from zero to 100mm, 125mm, 150mm the variation of Bending moment Twisting moment and Shear forces from both the theories have been plotted at plane passing through maxima. Since the total load contains self-weight also, as the thickness varies, the load also varies slightly. The thin plate theory doesn't contain any thickness term so the stress resultants are independent of thickness. But Ressiner's theory contains the thickness term in the expression, so the stress resultants not only dependent on dimension and applied load, but also on the thickness of the plate. On taking thickness equals to zero, it must cease to Kirchhoff thin plate theory itself, which is a validation for the thick plate Ressiner's theory. From the above said 2D plots, we can compare the difference between Kirchhoff theory (thin plate) results and Ressiner's theory (thick plate) results along with the validation.

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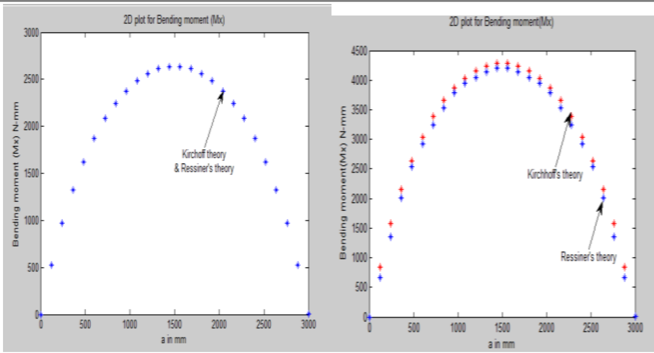

Fig-4: Bending Moment Mx for thickness(h=0,h=100mm)

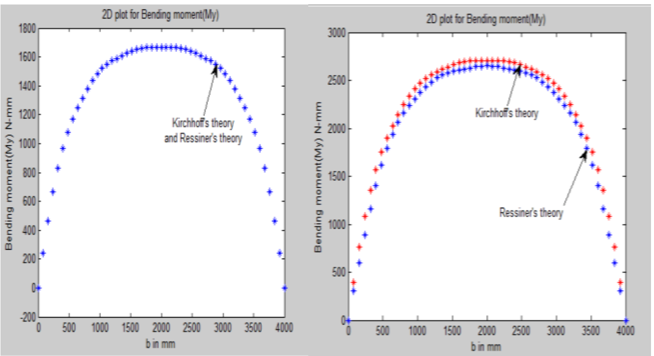

Fig-5: Bending Moment My for thickness (h=0,h=100mm)

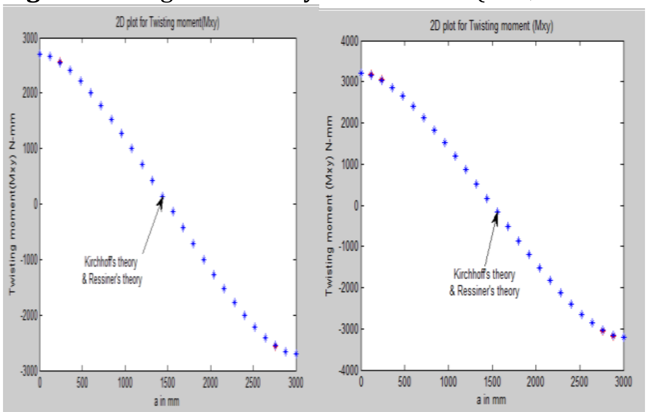

Fig-6 : Twisting moment(Mxy) for(h=100mm,h=500mm)

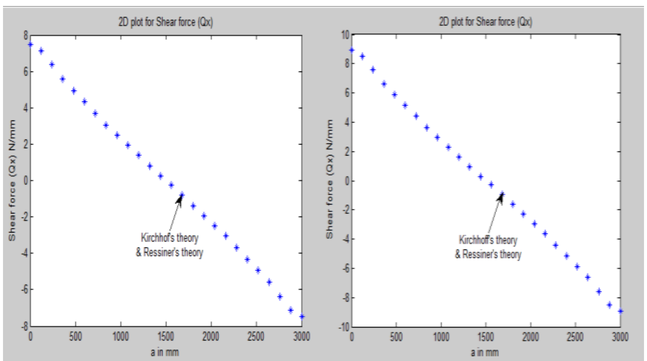

Fig-7: Shear force(Qx) for thickness(h=100mm,h=500mm)

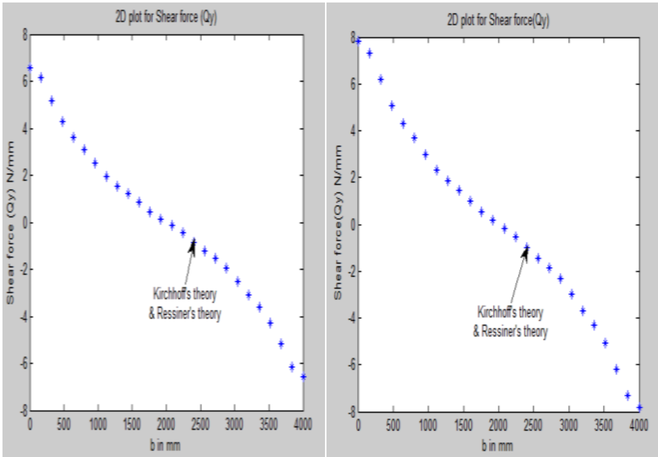

Fig-8: Shear force(Qy) for thickness(h=100mm,h=500mm)

6. Results and Discussions

7.

Detailed program of Ressiner's and Kirchhoff's theory are written in Matlab7. The variations of bending moments, twisting moments and shear forces across the plate are calculated and plotted using the same. The results obtained from the program are discussed through 2-dimensional as well as 3-dimensional plots.

Let us consider a concrete rectangular plate of M20 grade with dimensions a =3000mm, b =4000mm with varying thickness for the study purpose. The modulus of elasticity of the material is given by $5000\sqrt{fck}$ from IS: 456 2000. The total load is the sum of the live load and the self-weight, in which self-weight varies with the considered thickness.

The thickness is kept constant for 3-dimensional plots to study the variation of stress resultants across the plate, whereas thickness is varied for 2-dimensional plots to compare the two theories. The Kirchhoff's theory is independent of thickness term, whereas Ressiner's theory contains the thickness term. As the thickness varies, the load also varies accordingly.

In case of 2D plots, for different thicknesses the variation of stress resultants from both the theory is compared. At the beginning h is taken as zero, on doing so, thick plate results from Ressiner's theory will be same as that of thin plate Kirchhoff's theory, which is a validation for the Ressiner's theory. As the thickness is increases from zero to 100mm, 125mm, 150mm, variations in stress from both the theories have been produced. Which clearly shows how stress resultants very with the thickness.

The stress resultants are plotted for a constant thickness of say 125mm. considering the density of concrete as 25 KN/m^3, the self-weight of the plate is 3.125 KN/m^2. Live load is considered as 3 KN/m^2 from IS:875 part 2. Therefore the total load of the rectangular plate is 7.125 KN/m^2. As a result the variation of Bending moments, Twisting moments and Shear force across the plate are plotted as shown in the following figures below.

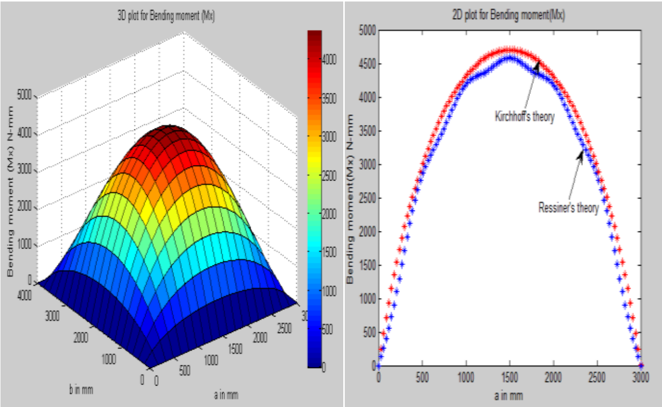

Fig-9 : Short span Bending Moment (Mx)

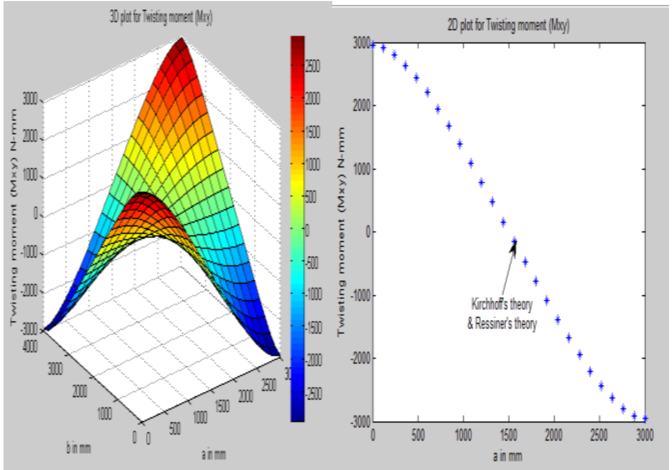

Fig-12 : Twisting moment (Mxy)

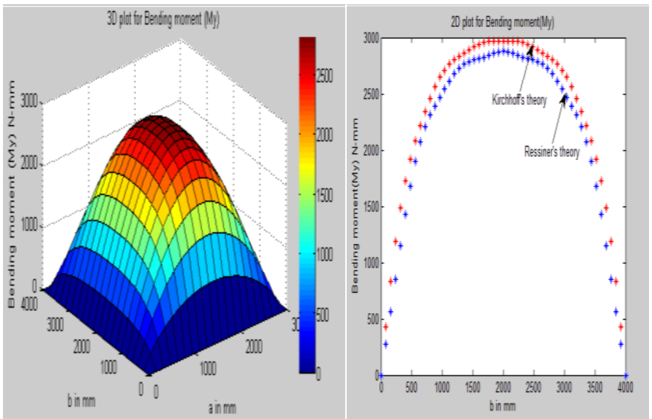

Fig-10 : Long span Bending Moment (My)

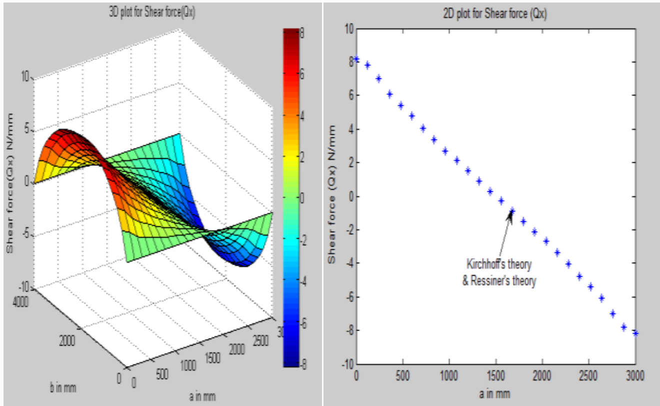

Fig-13 : Short span Shear force (Qx)

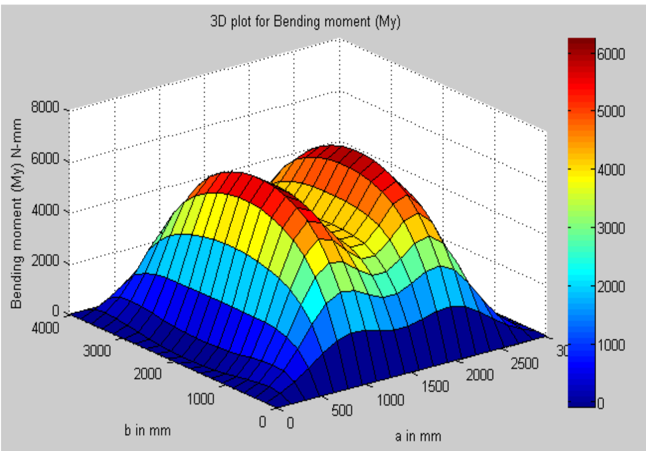

Fig-11 : For higher thickness value 400mm the variation of bending moment (My)

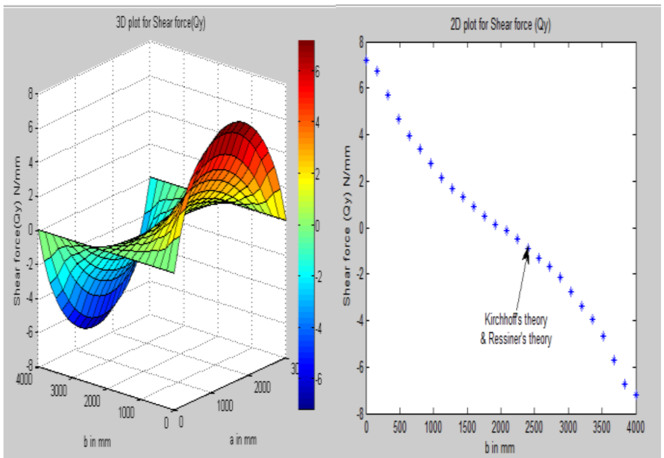

Fig-14 : Long span Shear force(Qy)

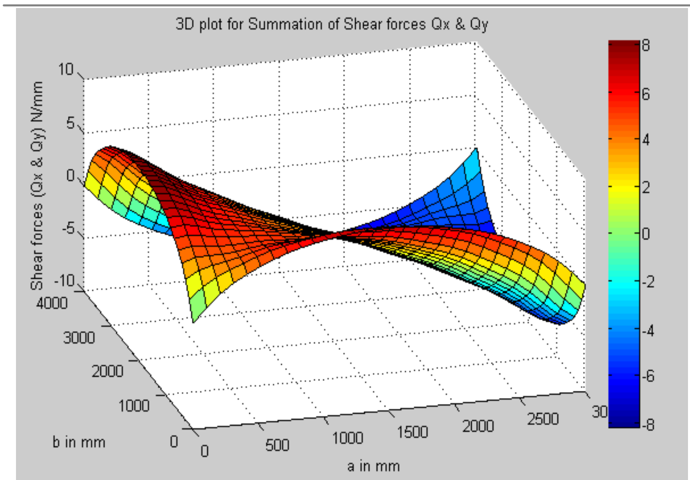

Fig-15 : Summation of Shear forces Qx and Qy

8. Conclusions

By studying both the theories with different thickness of plate, following observations were made:

1. The convergence of bending moment for Kirchhoff's thin plate theory shows the fluctuations initially, but as the number of terms increases the value converge.
2. The convergence of bending moment(Mx) for Ressiner's thick plate theory shows more fluctuation between the range fifty to two hundred and then get converged to a single value. Whereas in case of bending moment (My) the values are fluctuating between the exact values thus the average of two successive terms is considered so as to give a better approximation.
3. The convergence of twisting moment, shear forces are very same for both the theories. In the beginning the value increases suddenly for twisting moment, whereas in case of shear the value shows slight fluctuations and thereafter the values converge so as to give an adequate result.
4. In case of short span bending moment (Mx) as the thickness increases the Ressiner's theory crosses the value of Kirchhoff theory. Whereas for long span bending moment (My) its reverse as the thickness increases the value of Kirchhoff theory is found to be more compared to Ressiner's theory. In both the stress resultants such as Mx and My Kirchhoff theory shows a simple parabolic curve whereas Ressiner's theory shows undulation as thickness varies. Thus by comparing the bending moment along short and long span we observed that the moment along short span (Mx) is more compared to that of moment along long span (My).
5. The Shear force shows cosine variation along respective span and the results of Kirchhoff's thin plate theory and Ressiner's thick plate theory is showing approximate variation. Thus increase of shear force (Qy) along long span with increase in thickness is found to be less when compared to that of shear force (Qx) along short span. Similarly the variation of twisting moment is very same for both theories.

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