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INVENTORY MODEL UNDER TRADE CREDIT WHEN PAYMENT TIME IS PROBABILISTIC FOR VARIOUS CASES

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Abstract - In practice, the supplier may concurrently tender the retailer a permitted delay in payments to motivate fresh customers and increases his/her sales and also a cash discount to encourage immediate payment and reduce credit expenses. However, not all the time retailer is able to pay within the fixed period. Here we take into account the chances of all situations like making the payment before and after the trade credit limit. This is incorporated in the model through probability distribution functions. Since all cash outflows related to inventory control that happen at different points of time have different values, we use discount cash flow concept to set up an optimal ordering policies to the problem. The model is examined through various numerical examples.

Key Words: (Inventory model, trade credit , discounted cash flow, probability distribution

1. INTRODUCTION

The standard economic order quantity (EOQ) model assumes that the retailer has to for the items as soon as the items were accepted. However, in real-life positions, the supplier hopes to motivate his products, hence he will offer a delay period to retailer, which is the trade credit period, in paying for the amount of purchasing cost. In addition, the supplier offers a cash discount to encourage the retailer to pay for his purchases as early as possible. The retailer can obtain the cash discount when the payment is made before the cash discount period presented by the supplier. Or else the retailer will make full payment within the trade credit period. Thus the supplier often consider this trade credit policy to promote his /her items, also supplier uses the cash discount policy to invite retailer to pay the full payment of the amount of purchasing cost to cut down the collection period. The credit term that contains cash discount is very practical in real life business situations as an incentive for an earlier payment.

Several papers discussing this topic have shown in the literature that look into inventory problems under varying conditions. Some of the prominent papers are discussed here. Goyal (1985) derived an Economic order quantity model under the condition of permissible delay in payments. However, in real situations “time” is a important key concept and plays an important role in inventory models. Certain types of commodities deteriorate in the course of time and hence are unstable. To provide more practical features of the real inventory systems, Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) extended Goyal’s (1985) model for deteriorating items. Jamal et al. (1997), Sarker et al.(2000) and Chang et.al.(2002) further extended Aggarwal and Jaggi (1995) model to allow for shortages and makes it more appropriate in the real world. Liao et al.(2000) expanded an inventory model for stock dependent expenditure rate when delay in payment is permited. Huang and Chung (2003) developed Goyal’s (1995) model to cash discount period suggested by the supplier. Cash discount rate is decided by the behavioral and operating Characteristics of suppliers and retailers. Buyers with lesser credit quality are offered superior cash discounts. superior cash discounts are also connected with suppliers who typically pay cash and who b inveild up inventory to take advantage of the higher cash discount. Many related articles regarding inventory models with cash discount and trade credits are found in Jinn – Tsair, Teng , (2006), Yung-Fu Huang , Kuang- Hua Hsu, (2008) , Kun-Jen Chung, Jui-Jung Liao (2009) and their references.

Most of the earlier inventory management studies have not considered the discounted cash flow concept and hence the importance of money value is not considered. One of the inventory models that is discussed in the modern literature of inventory model is considering the time value of money. As the transactions that happen at different points of time will have different values and that cannot be compared with one another, so the face value of amounts paid at different time points cannot be considered as such. Certain authors discussed inventory models taking DCF concept. K.H Chung (1989) modeled the discounted cash flows concept for the analysis of an optimal inventory policy by considering trade credit. Kim & Chung (1990) identified the need to discover the inventory problems using the net present value aconcept or discounted cash flow concept. (DCF).

In the literature of inventory model, so far we discussed trade credit policy by considering cash discount and delayed payment. Thus to make the payments at the most two intervals are considered. However, in certain situations the retailer is not able to make the payment within the trade credit period. Though it may not happen quite commonly but it is not improbable. Such situations need to be captured in the model. Hence in this present paper, we suggest one more payment interval with a penalty rate which happens for a retailer with a certain probability.

Mainly in this paper an attempt is made to develop the model that includes the possibility that the retailer may not be able to pay within the trade credit period, but he can pay later with interest as a form of penalty. Model is proposed by including the possibility of later payment which happens with a certain probability and the delay duration for the payment after trade credit could be assumed to follow an appropriate probability density function. Here the general tendency of making payment will be usually last day of the trade credit period rather than any point of time within the trade credit period, because of value of money. Under these conditions, we try to model retailers inventory model as a cost minimization problem to obtain the retailers optimal order quantity and also optimum inventory cycle. Numerical examples and sensitivity analysis are presented to illustrate the proposed model. Finally, summary and conclusion are made.

2. NOTATIONS

D = Demand rate per year.

C = Purchasing cost per item

A = Ordering cost per replenishment

h = Unit stock holding cost per item per year .excluding interest charges

∂ = Cash discount rate ($0 < \partial < 1$)

M = Trade credit period

T = Cycle time in years

r_1= Discounted interest rate for payment made earlier to M but not at t =0.

r = Interest rate for net present value

- r_p = Penalty rate, where $r_p > r$
- T* = The optimal cycle time
- Q* = The optimal order quantity = DT*

3. ASSUMPTIONS

- Demand rate "D" is known and constant.
- Shortages are not allowed.
- Planning horizon is infinite.
- Replenishment happens instantaneously on ordering, which means, lead time is zero.

4. METHODOLOGY

Suppliers offer cash discount if retailer makes payment at the beginning . If he makes payment at trade credit period M, then regular price is applied, whereas, if he makes the payment after the trade period M, then supplier charges the penalty rate r_p for the amount of purchasing cost.

Generally, retailer may not be able to follow the consistent pattern of payment, that is, same pattern of payment schedule is not possible because of uncertainty of cash in hand. However, the retailer's payment pattern can be modeled through a probability distribution though the payments made are at different time points. According to the previous payment habit we assume that he makes payment in the beginning of the trade credit period and be eligible for the discount price with probability p_1, the probability that he makes payment at trade credit by paying regular price is p_2 and he makes payment after the trade credit period with probability p_3, where $p_3=1-p_1-p_2$. Let g(.) denote the conditional density function of the random duration of the payment which is made after the trade credit period.

Hence the cumulative probability function of the payment made is obtained by,

$$F(t) = \begin{cases} 0 & for \quad -\infty < t < 0 \\ p_1 & for \quad 0 \le t < M \\ p_1 + p_2 + p_3 \int_M^t g(y-M)dy & for \quad M \le t < \infty \end{cases}$$

Where $p_1 + p_2 + p_3 = 1$

The present value of the total cost is based on the following elements:

- The present value of the ordering cost
- The pesent value of the inventory carrying Cost
- The present value of the purchasing cost.

PV_1 (T) = Present value of all future cash flow when payment made within trade credit period

PV_2 (T) = Present value of all future cash flow when payment made at trade credit period "M"

PV_3 (T) = Present value of all future cash flow when payment made after M with penalty rate r_p.

The present value of the ordering cost:

$$A + Ae^{-rT} + Ae^{-2rT} + \dots\dots\dots$$

$$A \left[1 + e^{-rT} + e^{-2rT} + \dots\dots\right]$$

$$= \frac{A}{(1-e^{-rT})}$$

The present value of the inventory carrying cost:

$$= hC \left[\int_0^T D(T-t) e^{-rT} dt + \int_T^{2T} D(2T-t) e^{-rt} dt + .. \right]$$

$$= hC \left[\int_0^T D(T-t) e^{-rT} dt + \int_0^T D(T-t) e^{-r(T+t)} + .. \right]$$

$$= \frac{hCDT}{r\left(1-e^{-rT}\right)} - \frac{hCD}{r^2}$$

The present value of the purchasing cost can be discussed in three different cases as follows:

Case (I) When payment is made without any delay.

$$CD(1-\partial)T + CD(1-\partial)e^{-rT} + CD(1-\partial)e^{-2rT} + \ldots.$$

$$= CDT(1-\partial)(1+e^{-rT} + e^{-2rT} + \ldots.)$$

$$= \frac{CD(1-\partial)T}{(1-e^{-rT})}$$

Case(II): When payment is made at Trade credit period

$$CDTe^{-rM} + CDTe^{-2rM} + CDTe^{-3rM} \ldots\ldots.$$

$$= CDTe^{-rM}\left[1+e^{-rT} + e^{-2rT} + \ldots\ldots.\right]$$

$$= \frac{CDTe^{-rM}}{(1-e^{-rT})}$$

Case(III): When payment is made after Trade credit period "M "with penalty rate r_p.

Payment towards the purchasing cost if payment is made at time t after the trade credit period M, with penalty rate r_p is,

$CDTe^{r_p(t-M)}$. Hence conditional expectation of the this payment is

$$\int_M^{\infty} CDTe^{r_p(t-M)} g(t-M) dt$$

Net present value of the above cost with interest rate r is,

$$DT \int_M^{\infty} Ce^{r_p(t-M)} e^{-rt} g(t-M)\, dt$$

Take y = t-M

$$= CDT \int_0^{\infty} e^{r_p y} e^{-r(y+M)} g(y)\, dy$$

$$= CDT \int_0^{\infty} e^{y(r_p - r)} \; e^{-rM} g(y)\, dy$$

$$= CDTe^{-rM} \int_0^{\infty} e^{(r_p - r)y} \; g(y)\, dy$$

$$= CDTe^{-rM} M_x(r_p - r)$$

Where $M_x(r_p - r)$ represents Moment generating function of distribution function X.

Any distribution which has limit zero to infinity can be considered to derive the cost function.

One of the suitable distribution for the delay in payment beyond trade credit period is gamma distribution. By considering this, we derive the cost function.

$$= CDT\; e^{-rM} \frac{1}{\Gamma\alpha} \int_0^{\infty} e^{(r_p - r)y} \; e^{-y} \; \theta^{\alpha} \; (r_P - r)^{\alpha-1} dy$$

Where α and θ are parameters of gamma distribution.

$$= \frac{CDT\theta^{\alpha}}{\Gamma\alpha} e^{-rM} \int_0^{\infty} \mathrm{e}^{-(1-(r_p - r))y} \; (r_p - r)^{\alpha-1} dy$$

$$= \frac{CDT\theta^{\alpha} e^{-rM}}{\Gamma\alpha} \frac{\Gamma\alpha}{(\theta + r - r_p)^{\alpha}}$$

$$= \frac{CDT\theta^{\alpha} e^{-rM}\, \Gamma\alpha}{\Gamma\alpha(\theta + r - r_p)^{\alpha}} \text{ , (by definition of gamma function)}$$

$$= \left(\frac{\mathrm{CDT}\,\theta^{\alpha} \mathrm{e}^{-\mathrm{rM}}}{(\theta + \mathrm{r} - \mathrm{r_p})^{\alpha}} \right)$$

Continuously discounted present value of the purchasing cost is

$$\left(\frac{\mathrm{CDT}\,\theta^{\alpha} \mathrm{e}^{-\mathrm{rM}}}{(\theta + \mathrm{r} - \mathrm{r_p})^{\alpha}} \quad \frac{1}{1-e^{-rT}} \right).$$

4.1 THE PRESENT VALUE OF TOTAL COST

The present value of all future cash flow when payment made within trade credit period with probability p_1 + The present value of all future cash flow when payment made at trade credit period "M" with probability p_2 + The present value of all future cash flow when payment made after M with penalty rate r_p with probability p_3 is

$$PV(T) = \quad p_1 PV_1(T) + p_2 PV_2(T) + p_3 PV_3$$

(1)

$$= p_1\left[\frac{A}{(1-e^{-rT})} + \frac{CD\,(1-\partial)T}{(1-e^{-rT})} + \frac{h\,CDT}{r(1-e^{-rT})} - \frac{hCD}{r^2}\right]$$
$$+ p_2\left[\frac{A}{(1-e^{-rT})} + \frac{C\,DTe^{-rM}}{(1-e^{-rT})} + \frac{h\,CDT}{r(1-e^{-rT})} - \frac{hCD}{r^2}\right] +$$
$$p_3\left[\frac{A}{(1-e^{-rT})} + \left\{\frac{\mathrm{C\,DT}\theta^{\alpha}\mathrm{e}^{-\mathrm{rM}}}{\left(\theta+\mathrm{r}-\mathrm{r_p}\right)^{\alpha}} \quad \frac{1}{(1-e^{-rT})}\right\} + \frac{hCDT}{r\,(1-e^{-rT})} - \frac{hCD}{r^2}\right]$$

$$= \frac{A}{1-e^{-rT}} - \frac{h\ \ C\ \ D}{r^2}$$
$$+ \frac{CDT}{1-e^{-rT}}\left[\frac{h}{r} + p_1\,(1-\partial) \quad + \quad p_2\,e^{-rM} + p_3\,\frac{\theta^{\alpha}\,\mathrm{e}^{-\mathrm{rM}}}{\left(\theta+\mathrm{r}-\mathrm{r_p}\right)^{\alpha}}\right] \quad (2)$$

To obtain optimal time T* we need to minimize PV (T) with respect to 'T' and we get,

$$PV(T) = \frac{A}{1-e^{-rT}} - \frac{hCD}{r^2} + \frac{CDT}{1-e^{-rT}}(k) \qquad (3)$$

Where

$$k = \left[\frac{h}{r} + p_1(1-\partial) + p_2 e^{-rM} + p_3\,\frac{\theta^{\alpha}\ \mathrm{e}^{-\mathrm{rM}}}{\left(\theta+\mathrm{r}-\mathrm{r_p}\right)^{\alpha}}\right]$$

$$\frac{dPV}{dT} = \frac{d}{dT}\left\{\frac{A}{1-e^{-rT}} - \frac{hCD}{r^2} + \frac{CDTK}{1-e^{-rT}}\right\}$$

$$\frac{dPV}{dT} = 0 \quad which \quad implies,$$

$$rAe^{-rT^*} = CDK\,(1-e^{-rT^*} - rT^* e^{-rT^*})$$

(4

$$rA = CDK\,(e^{rT^*} - 1 - rT^*) \qquad (5)$$

Note that e^{rT} can be approximated as

$$e^{rT} \approx 1 + rT + \frac{(rT)^2}{2}$$

Then equation (5) can be written as

$$rA = \quad CDK \quad \frac{(rT^*)^2}{2}$$

After simplification, we get $T^* = \sqrt{\dfrac{2A}{CDrK}}$ (6)

Using T* optimal cycle time the optimal order quantity Q* is obtained as Q*= DT*

We get $Q^* = \sqrt{\dfrac{2DA}{crK}}$ (7)

Hence, if there is no trade credit period, the DCF approach gives an identical solution to that of the traditional inventory analysis.

5. Numerical examples

To illustrate and verify the above theoretical results, we consider few examples here.

The sensitivity analysis on various payment time with different probability values , Purchase values and trade credit period is shown in Table 1-3, respectively

Table 1:

Effects of changing payment time with different probability values on the optimal solution

Demand rate per year D =1000 units; r=0.06; M=0.1year; h=$0.2/unit/year; A=$100/order ; C=50 ∂=0.1; r_p=0.2; α =0.1; θ =2;

Probabilities	**Q***	**T***	**PV(T)**
p_1=0.8, p_2=0.1, p_3=0.1	125	0.1250	792960
p_1=0.1, p_2=0.1, p_3=0.8	124	0.1242	852250
p_1=0.1, p_2=0.8, p_3=0.1	124	0.1242	848010
p_1=0.2, p_2=0.4, p_3=0.4	124	0.1241	841960
p_1=1, p_2=0, p_3=0	125	0.1253	776630
p_1=0, p_2=1, p_3=0	124	0.1239	855270

It is observed from above table that

(i) Higher the value of p1 compared to p2 and p3 will results the lower values of total relevant cost

(ii) Higher the value of p2 compared to p1 and p3 will results higher the values of total relevant cost compare to case (i)

(iii) Higher the value of p3 compared to p2 and p3 will results higher the values of total relevant cost compare to case (i) and case (ii)

Table 2:

Effects of changing purchase cost C, on the optimal solution

Demand rate per year D =1000 units; r=0.06; M=0.1year; h=$0.2/unit/year; A=$100/order ∂=0.1; r_p=0.2; α =0.1; θ =2; p1=0.8, p2=0.1, p3=0.1

Purchase cost	Q*	T*	PV(T)
30	161	0.1614	480450
50	125	0.1250	792960
80	99	0.0989	1259800
120	81	0.0808	1881400
150	72	0.0723	2345000
200	63	0.0626	3118400

It is oserved from the Table 2. that there is a significant decrease in value of optimal quantity as well as the value of an optimal cycle time as purchase cost increases, But total relevant cost shows significant increase as purchase cost increases.

Table 3:

Effects of changing trade credit period M, on the optimal solution

Demand rate per year D =1000 units; r=0.06; h=$0.2/unit/year; A=$100/order; C=50 ;∂=0.1; r_p=0.2; α =0.1; θ =2; p1=0.8, p2=0.1, p3=0.1

Trade credit period	Q*	T*	PV(T)
0.1	125	0.1252	792960
0.15	125	0.1252	792460
0.20	125	0.1252	791960
0.25	125	0.1252	791470
0.30	125	0.1252	790970
0.35	125	0.1252	790480

It is observed from the Table 3. that as trade credit period M increases ,there is no change in optimal order quantity as well as in the value of optimal cycle time. But there is marginal decrease in total relevant cost. Which implies that credit period offered to retailers has positive impact

From the above numerical examples it is clear that when paying habits changes, especially after the trade credit period there is significant difference between the total optimal costs. Hence when a retailer is not making payment before the trade ctredit then actual cost will be much different.

6. COCLUSIONS

Most of the inventory models with trade credit assumed that retailer pays either before the trade credit period to offer cash discount or at the time of credit period every time. Thus existing models allow making the payment every time at one of these two possible points. However, in the real marketplace it is common that the retailer is not able to make the payment consistently at the similar point of time. Sometimes the retailer pays before the trade credit and sometimes at the trade credit period. In extreme cases he/she makes payment after trade credit period. In order to model this and possibly not very punctual payment habit, the model incorporates possibility of payment even after the trade credit period of course with a penalty rate that will happen with certain probability and retailer's payment time is also considered as a chance point which is modeled through a probability distribution.

Further under the condition of trade credit it is favorable to pay only at the trade credit point rather than before the trade credit point due to time value money . But retailer may find it suitable to make the payment whenever cash is available and hence further situations occur. From this study it can be seen that if retailer is not able to stick to same payment pattern then the total cost differs very much. Hence, assuming models without considering various probabilities will not only mislead the total cost but also the solutions obtained are suboptimal. By observing the above tables we conclude that the total costs varies when probabilities are different. All models discussed earlier can be taken care as special cases by assuming appropriate probabilities as zero in the present model. Hence the present model is a generalization by taking various possibilities into the present model. In addition, the calculation results on the model discussed in the paper disclose that a smaller value of purchasing cost results in larger values for the optimal replenishment cycle time T* and also the optimal order quantity Q* and vice versa. The present value of total cost is also calculated for different time points.

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