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STRONG INVERSE SPLIT AND NON-SPLIT DOMINATION IN JUMP GRAPHS

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Abstract : In this paper strong inverse split , non-split dominating sets are introduced and its properties are studied. Further the notion of strong co-edge split, non-split domination sets are discussed.

Key Words; Strong(weak)inverse split domination set, Strong(weak) inverse non-split domination set Strong (weak) co-edge split domination set Strong (weak) co-edge non-split dominating set.
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1. Introduction:

In 1958, domination as a theoretical area in graph then was formalized by Berge and Ore [2] in 1962. Let G=(V, E) be a graph. A set D ⊂ V is a strong dominating set of G if for every vertex y ∈ V-D there exists x ∈ D with xy ∈ E of larger or equal degree that is deg (X, G) ≤ d(x,G). The strong dominate number $\surd_{st}$(G) is defined as the minimum cardinality of a strong dominating set and was introduced by Sampathkumar and Puspalata[1] in 1996.Kulli V.R and Janakiram B [6, 5] was introduced split dominating and non-split domination number was introduced . In 2010 K.Ameenal Bibi and R. Selvakumar [4] was introduced the inverse split and non-split dominating sets are introduced and properties are discussed. We also introduced the notation of Co-edge split and non-split domination sets in jump graphs and study its properties.

2.Preliminaries:

Definition 2.1 [3] A graph G is an ordered triple (V,E, Ψ)consisting of a nonempty set V(G) of vertices a set E(G) of edges, disjoint from V(G) of edges and incident function Ψ that associates with each edge of G an unordered pair of vertices of G. If e is an edge and u and v are vertices such that Ψ(e)=uv there is said to join u and v. The vertices u and v are called ends of e.

Definition 2.2[2] A vertex v in a graph G is said to be dominate itself and each of its neighbors that is v dominates the vertices in its closed neighborhood N[v]. a set S of vertices of G is a dominating set of G if every vertex of G dominated by atleast one vertex of S. Equivalently a set S of vertices of G is a dominating set of every vertex in V-S is adjacent to at least one vertex in S. The minimum cardinality among the dominating setoff g is called dominating number and is denoted by √(G). a dominating set of cardinality of √(G) is then referred to as minimum dominating set.

Definition 2.3[1] A set D ⊆ V is a dominating set (strong dominating set sd-set), weak dominating set (wd-set of G if every v ∈ V-D is dominated (strongly dominating , weakly dominating respectively) by some vertex u ∈ D. The domination number sd, wd number) √ = √(G) ($\surd_s$= $\surd_s$(G), $\surd_w$ = $\surd_w$(G)) of G is the minimum cardinality of a dominating set (sd-set, wd-set) of G.

Definition 2.4 A set D ⊆ V is a dominating set (strong dominating set sd-set), weak dominating set (wd-set of jump gaph J(G) if every v ∈ V-D is dominated (strongly dominating , weakly dominating respectively) by some vertex u ∈ D. The domination number sd, wd number) √ = √(J(G)) ($\surd_s$= $\surd_s$(J(G)), $\surd_w$ = $\surd_w$(J(G)) of jump graph J(G) is the minimum cardinality of a dominating set (sd-set, wd-set) of J(G).

Definition 2.5[9, 10] A dominating set D of a .jump graph J(G)=(V,E) is a split(non-split) dominating set if the induced sub graph<V(JG))-D > is disconnected (connected). The split (non-split) domination number
$\surd_s$(J(G)) ($\surd$ns(J(G)) is the minimum cardinality of a split (non-split) dominating set.

Definition 2.6[11] The domination number $\surd$(J(G)) is the minimum cardinality taken over all the minimum dominating sets of J(G). Let D be the minimum dominating set of J(G). If V(J(G))-D contains dominating set D' then D' is called inverse dominating set of J(G) with respect to D.

Definition 2.7 A co-edge split dominating set (CESD-set) of a jump graph J(G) is a co-edge split dominating set X of a jump graph J(G) such that the edge induced sub graph <E(J(G))-X> is disconnected and the co-edge split domination number $\surd$cs(J(G)) is the minimum cardinality of the minimal co-edge split dominating set of J(G).

Definition 2.8. A co-edge split dominating set X of a jump graph J(G) is a co-edge non-split dominating set (CENSD-set) if the edge induced sub graph <E(J(G))-X) is connect. The co-edge non-split domination number $\surd'_{cens}$(J(G)) is the minimum cardinality of the minimal co-edge non-split dominating set of J(G) Non-split dominating set

Theorem 3.1; For any jump graph J(G)
$\surd'(J(G)) \leq \surd'_{ss}(J(G))$
$\surd'(J(G)) \leq \surd_{sns}(J(G))$

Proof; Since every strong inverse split dominating set of J(G) is an inverse dominating set of J(G) we have $\surd'(J(G)) \leq \surd'_{ss}(J(G))$. Similarly every strong inverse non-split dominating set of J(G) is an inverse dominating set of J(G) we have $\surd'(J(G)) \leq \surd'_{sns}(J(G))$

Theorem3.2; For any jump graph J(G)

$$\surd'(J(G)) \leq \min \{ \surd'_{ss}(J(G)), \surd'_{sns}(J(G)) \}$$

Proof; Since every strong inverse split dominating set and every inverse non-split dominating set of J(G) are the inverse dominating set of J(G), we have .
$\surd'(J(G)) \leq \surd'_{ss}(J(G))$ and $\surd'(J(G)) \leq \surd'_{sns}(J(G))$ and Hence

$$\surd'(J(G)) \leq \min \{ \surd'_{ss}(J(G)), \surd'_{sns}(J(G)) \}$$

Theorem 3.3; Let T be a tree such that any two adjacent cut vertices u and v with at least one of u and v is adjacent to an end vertex then,
$\surd'(J(T)) = \surd'_{ss}(J(T))$

Proof; Let D' be a $\surd'$-set of J(T) then we consider the following two cases.
Case(i): Suppose that at least one of u, v .∈ D' then <V(J(T))-D' > is disconnected with at least one vertex. Hence D' is a $\surd'_{ss}$ -set of J(T) Thus the theorem is true.
Case9ii); Suppose u, v ∈ (V(J(T))-D') Since there exists an end vertex is adjacent to either u or v say u, it implies that w ∈ D'. Thus it follows that
D'' = D' – {w} ∪ {u} is a $\surd'$-set of J(T).
Hence by case (i) the theorem is true.

Theorem3.4; Let J(G) be a jump graph which is no a cycle with atleast 5 vertices. Let J(H) be connected spanning sub graph of J(G) then

(i) $\surd'_{ss}(J(G)) \leq \surd'_{ss}(J(H))$
(ii) $\surd'_{sns}(J(G)) \leq \surd'_{sns}(J(H))$

Proof; Since J(H) is connected then any spanning tree of jump graph J(G0 is minimally connected sub graph of J(G) such that
$\surd'_{ss}(J(G)) \leq \surd'_{ss}(J(T)) \leq \surd'_{ss}(J(H))$.
In similar way

$\surd'_{sns}(J(G)) \le \surd'_{sns}(J(T)) \le \surd'_{sns}(J(H))$.

4.STRONG CO-EDGE SPLIT AND NON-SPLITDOMINATING SETS

Definition 4.1 A strong co-edge split dominating set (SCESD-SET) of a jump graph J(G) is a strong co-edge dominating set X of a gaph J(G) such that the edge induced sub graph < E(J(G)) – X > is disconnected and strong co-edge split domination number $\surd'_{scs}$ (JG)) is the minimum cardinality of the minimal strong co-edge split dominating set of J(G).

Definition 4.2 a strong co-edge dominating set X of a jump graph is a co-edge non-split dominating set (SCENSD-SET) If the ege induced sub graph < E(J(G))-X >is connected . the strong co-edge non-split domination number is denoted by
$\surd'_{scens}(J(G))$ and it is the minimum cardinality of the minimal strong co-edge non-split dominating set of J(G).

Theorem 4.3; For any jump graph J(G)
$\surd'(J(G)) \le \surd'_{scens}(J(G))$
Proof; Since every SCENSD-set of J(G) is and Ed-set of J(G) and hence the result.

Theorem 4.4; For any jump graph J(G).
$\surd'(J(G)) \le \min\{\surd'_{sces}(J(G)), \surd'_{scens}(J(G))\}$
Proof; Since every SCESD-set and SCENSD-set of J9G)are ED-set of J(G) which gives
$\surd'(J(G)) \le \surd'_{secs}(J(G))$ and $\surd'(J(G)) \le \surd'_{scens}(J(G))$ and Hence
$\surd'(J(G)) \le \min\{\surd'_{sces}(J(G)), \surd'_{scens}(J(G))\}$.
Theorem 4.5; A SCENSD-set of X of J(G) is minimal if and only if for each edge e ∈ X one of the following condition is satisfied**.**

i) There exists an edge x ∈ E- X such that N(x) ∩ X = {e}
ii) E is an isolated edge in < x > and
iii) N(e) ∩ (E – X) = ϕ

Proof; Let X be a SCENSD-set of J9G0.Assume that X is minimal then X – [g} is not a SCENSD-set for any g ∈ X such that e does not satisfied any of the given condition then X' = X – {e} is ED-set of J(G0 Also N(e) ∩ (E – X) ≠ ϕ given
< E – X' > is connected. This implies that X' is a SCENSD-set of J(G)Which contradict the minimally of X. This proves the necessity.
Conversely, for connected J(G) if any one of the givn three conditions is satisfied gives sufficiency. Next, we obtained a relationship between $\surd'_{scen\,s}(J(H))$. and $\surd'_{scens}(J(G))$, where H is any spanning connected sub graph of G.

Theorem 4.6; For the jump graph J(G) which is not a cycle graph with at least 5 vertices, then $\surd'_{scens}(J(G))$, $\le \surd'_{scen\,s}(J(H))$. Where h is a spanning connected sub graph of J(G).

Proof; Since J(G) is connected then any spanning tree J(T) of J(G) is the minimal connected sub graph of J(G) such that
$\surd'_{scens}(J(G)) \le \surd'_{scens}(J(T)) \le \surd(J(H))$
Hence the result

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