Comparative study of states under the effect of amplitude damping and protecting entanglement by using weak measurement and quantum measurement reversal

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Abstract - *In this paper we have studied the effect of one and two sided amplitude damping decoherence on the entanglement of Bell's state by measurening their concurrence and compare them .We found that in the case of two sided amplitude damping state α│01⟩+β│10⟩ shows more protection of entanglement compare to the state α│00⟩+β│11⟩ to amplitude damping i.e first bell's state perform better than second one. While in the case of one sided amplitude damping both state perform equally. We have also shown that entanglement can be protected by using weak measurement and Quantum measurement reversal in both cases . We found that in the case of two sided amplitude damping after applying weak measurement and Quantum measurement reversal ,state α│01⟩+β│10⟩ is protected more compare to the state α│00⟩+β│11⟩ and similar results are obtained in the case of one sided amplitude damping. We have also studied the evolutions of concurrence with initial parameter and find that except state α│00⟩+β│11⟩ in the case of two sided amplitude damping, other are showing maximum entanglement when they are initially maximally entangled.*

Key Words: Decoherence, Amplitude damping, Entanglement, concurrence, Weak measurement, Quantum measurement reversal

1. INTRODUCTION

Quantum computation and quantum communication are two most important discovery in the field of information technology as they have great superiorities over classical computation. Quantum entanglement is the essential resource for these superiorities of Quantum computation and quantum communication, as it is the vital for quantum computing [1,2], quantum Cryptography [3], quantum teleportation [4-6] and quantum metrology [7] etc. But quantum entanglement is fragile and can be easily damaged by decoherence, which occur due to the interaction of quantum system with the surrounding [8]. There are many channel which are responsible for decoherence, eg- amplitude damping, phase damping, depolarization channel etc[9,10] . In this paper we have used two sided and one sided amplitude damping. Normally a quantum channel is defined by trace preserving completely positive linear map on density matrix. Amplitude damping is one of its simplest example. It modeled the dissipative interaction of qubits with its zero temperature environment . It is described by following quantum map

$$|0\rangle_S|0\rangle_E \rightarrow |0\rangle_S|0\rangle_E$$
$$|0\rangle_S|0\rangle_E \rightarrow D^{1/2}|1\rangle_S|0\rangle_E + (1-D)^{1/2}|0\rangle_S|1\rangle_E$$

The action of the above map can be described by the set of operator known as Kraus operator, which is given by

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-D} \end{bmatrix} \quad E_1 = \begin{bmatrix} 0 & \sqrt{D} \\ 0 & 0 \end{bmatrix}$$

Where D is decoherence strength parameter and it varies from 0 to 1.

Our main task is to tackle with the decoherence and suppresses it . There are number of way of doing this eg: quantum error correction[11],Decoherence free sub space[12], dynamical decoupling[13] etc. As from the postulates of quantum mechanics we know that projective Measurement irreversibly collapse the states into one of the Eigen state of the operator .But in the case of non projective measurement we have different situation .It is known as weak measurement In compare to projective measurement it is more gentle ie it can extract the information from the system without collapsing it to one of its Eigen state[14] .So a suitable can revive the state with certain probabilities. In this paper we are using weak measurement and quantum measurement reversal.

This paper is organize as follow in section 2 we have applied the one and two sided amplitude damping to state Iψ› = α│00⟩+β│11⟩ and measure its concurrence as it is the measure of entanglement. Then after we apply the weak measurement and Quantum measurement reversal in both cases. In section 3 we repeat the same process with state Iφ›= α│01⟩+β│10⟩ further we study the evolution of concurrence with initial parameter and in section 4 we give a discussion between the difference between two cases two cases and section 5 we concluded the paper.

2 for state │ψ⟩ = α│00⟩+β│11⟩ (STATE 1)

Let consider the two qubit quantum state in maximal entangle state ,which is given by

$$|\psi\rangle = \alpha|00\rangle + \beta|11\rangle \qquad (1)$$

Its density matrix can be written as

$$\rho = \begin{bmatrix} \alpha^2 & 0 & 0 & \alpha\beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta\alpha & 0 & 0 & \beta^2 \end{bmatrix} \quad (2)$$

Where $\alpha^2 + \beta^2 = 1$

The state after applying decoherence is given by

$$\varepsilon_{AD}(\rho) = A_0\rho A_0^* + A_1\rho A_1^* + A_2\rho A_2^* + A_3\rho A_3^* \quad (3)$$

Where A_0, A_1, A_2, A_3 are Kraus operator for two sided amplitude damping noise and written as

$$A_0 = E_0 \otimes E_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-D} & 0 & 0 \\ 0 & 0 & \sqrt{1-D} & 0 \\ 0 & 0 & 0 & 1-D \end{bmatrix},$$

$$A_1 = E_0 \otimes E_1 = \begin{bmatrix} 0 & \sqrt{D} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{D(1-D)} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = E_1 \otimes E_0 = \begin{bmatrix} 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D(1-D)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_3 = E_1 \otimes E_1 = \begin{bmatrix} 0 & 0 & 0 & D \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

For simplicity we have assumed that decoherence strength are same in both side ie $D_1=D_2=D$

Now the resultant density matrix is given by

$$\rho = \begin{bmatrix} \alpha^2+\beta^2D^2 & 0 & 0 & \alpha\beta(1-D) \\ 0 & \beta^2D(1-D) & 0 & 0 \\ 0 & 0 & \beta^2D(1-D) & 0 \\ \beta\alpha(1-D) & 0 & 0 & \beta^2(1-D)^2 \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2(1-D)\beta(\alpha - D\beta) \quad (5)$$

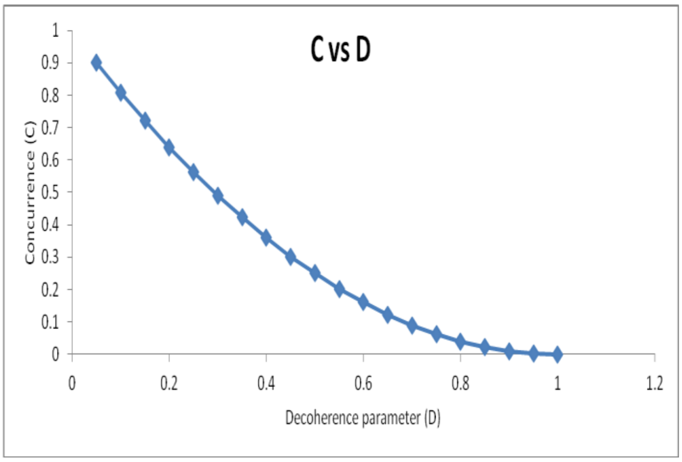

Figure 1 C Vs D

Let us now protect the entanglement by applying weak measurement and Quantum measurement reversal. we apply weak measurement before system undergoes amplitude damping decoherence. which partially collapse state towards $|0\rangle$ which is more protected for amplitude damping channel.

The two qubit weak measurement operator can be written as

$$M_W = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-M_1} \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-M_2} \end{bmatrix}$$

for simplicity we assume that $M_1=M_2=M$,Then we have

$$M_W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-M} & 0 & 0 \\ 0 & 0 & \sqrt{1-M} & 0 \\ 0 & 0 & 0 & 1-M \end{bmatrix}$$

AS system qubit does not interact with environment as given in eq (4).so system qubit is more protected to decoherence. After applying amplitude decoherence we apply Quantum measurement reversal .The two qubit reversing measurement operator is given by

$$M_{rev} = \begin{bmatrix} \sqrt{1-M_{r1}} & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} \sqrt{1-M_{r2}} & 0 \\ 0 & 1 \end{bmatrix}$$

for simplicity we asuume that $M_{r1}=M_{r2}=M_r$,then we have

$$M_{rev} = \begin{bmatrix} 1-M_r & 0 & 0 & 0 \\ 0 & \sqrt{1-M_r} & 0 & 0 \\ 0 & 0 & \sqrt{1-M_r} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where M_r=M-(1-M)D.

Assuming that reversing measurement is optimal ,the two qubit state after the sequence of weak measurement, amplitude damping and Quantum measurement reversal the two qubit state is given by

$$\rho' = \frac{1}{A}\begin{bmatrix} \alpha^2+\beta^2 D^2(1-M)^2 & 0 & 0 & \alpha\beta \\ 0 & \beta^2 D(1-M) & 0 & 0 \\ 0 & 0 & \beta^2 D(1-M) & 0 \\ \beta\alpha & 0 & 0 & \beta^2 \end{bmatrix}$$

WhereA=$1+\beta^2[D(1-M)\{1+D(1-M)\}+D(1-M)]$

Now again entanglement can be measure by measuring the concurrence, which is calculated be

$$C = \frac{2\left[\alpha\beta - D(1-M)\beta^2\right]}{1+[D(1-M)\{1+D(1-M)\}+D(1-M)]} \quad (6)$$

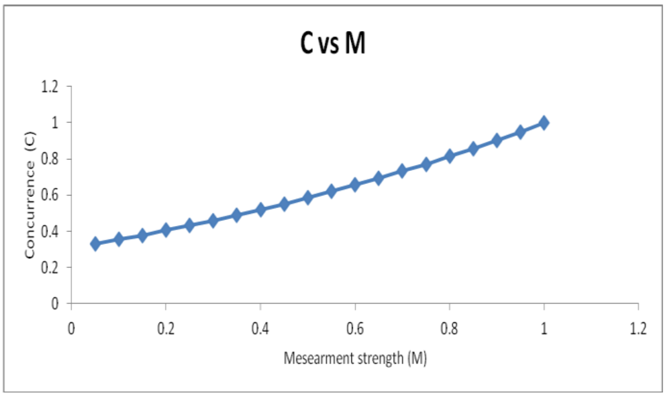

Figure 2 C Vs M (D=0.5)

Now for one sided amplitude damping , The state after applying decoherence is given by

$$\varepsilon_{AD}(\rho') = A_0\rho A_0^* + A_1\rho A_1^*$$

Where A_0, A_1 are Kraus operator for one sided amplitude damping noise and written as

$$A_0 = E_0 \otimes I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-D} & 0 \\ 0 & 0 & 0 & 1-D \end{bmatrix},$$

$$A_1 = E_1 \otimes I = \begin{bmatrix} 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

Now the resultant density matrix is given by

$$\rho = \begin{bmatrix} \alpha^2 & 0 & 0 & \varepsilon\beta\sqrt{1-D} \\ 0 & \beta^2 D & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta\alpha\sqrt{1-D} & 0 & 0 & \beta^2(1-D)^2 \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2\alpha\beta\sqrt{1-D} \quad (7)$$

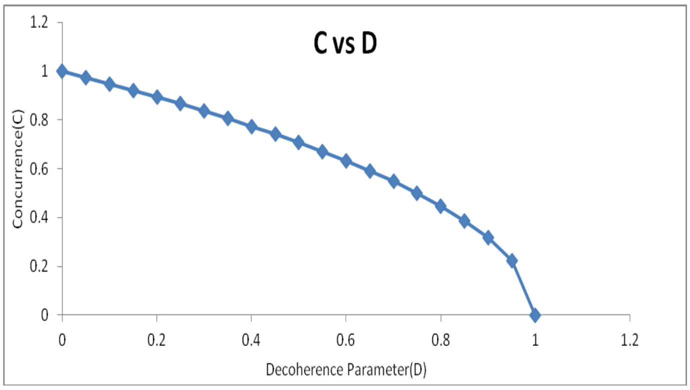

Figure 3 C Vs D

Repeating the all process as in the above , the resultant density matrix is given by

$$\rho = \begin{bmatrix} \alpha^2(1-M_r) & 0 & 0 & \alpha\beta\sqrt{1-D}(1-M)(1-M_r) \\ 0 & \beta^2 D(1-M)^2(1-M_r) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta\alpha\sqrt{1-D}(1-M)(1-M_r) & 0 & 0 & \beta^2(1-M)^2(1-D) \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = \frac{2\alpha\beta\sqrt{1-D}}{\alpha^2(1-D)+\beta^2(1+D(1-M))} \quad (8)$$

3 for state |φ⟩ =α|01⟩+β|10⟩ (STATE 2)

Let consider the two qubit quantum state which is given by

$|\varphi\rangle = \alpha|01\rangle + \beta|10\rangle$

Its density matrix can be written as

$$\rho = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta & 0 \\ 0 & \beta\alpha & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

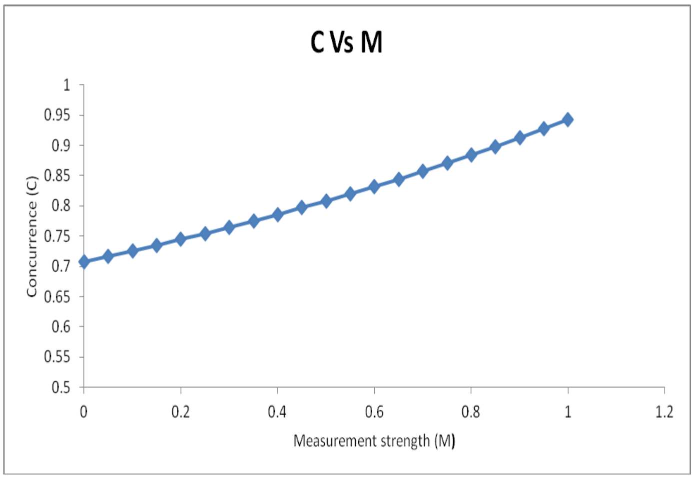

Figure 4 C Vs M (D=0.5)

The state after applying decoherence is given by

$$\varepsilon_{AD}(\rho) = A_0\rho A_0^* + A_1\rho A_1^* + A_2\rho A_2^* + A_3\rho A_3^* \quad (3)$$

Where A_0, A_1, A_2, A_3 are Kraus operator for two sided amplitude damping noise

$$\rho = \begin{bmatrix} \alpha^2 D + \beta^2 D & 0 & 0 & 0 \\ 0 & \alpha^2(1-D) & \alpha\beta(1-D) & 0 \\ 0 & \beta\alpha(1-D) & \beta^2(1-D) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $\alpha^2 + \beta^2 = 1$

. Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2\alpha\beta(1-D) \quad (9)$$

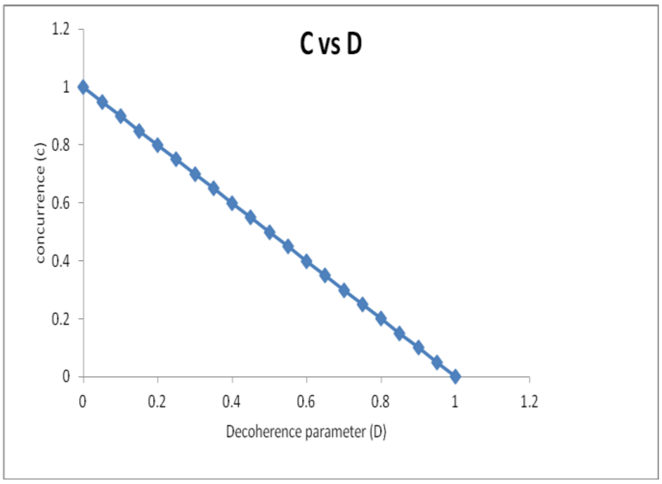

Figure 5 C Vs D

Now as earlier we apply sequence of weak measurement, amplitude damping and Quantum measurement reversal and the two qubit state is given by

$$\rho' = \frac{1}{A'} \begin{bmatrix} D(1-M) & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta & 0 \\ 0 & \beta\alpha & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $A' = D(1-M) + \alpha^2 + \beta^2$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = \frac{2\alpha\beta}{1 + D(1-M)} \quad (10)$$

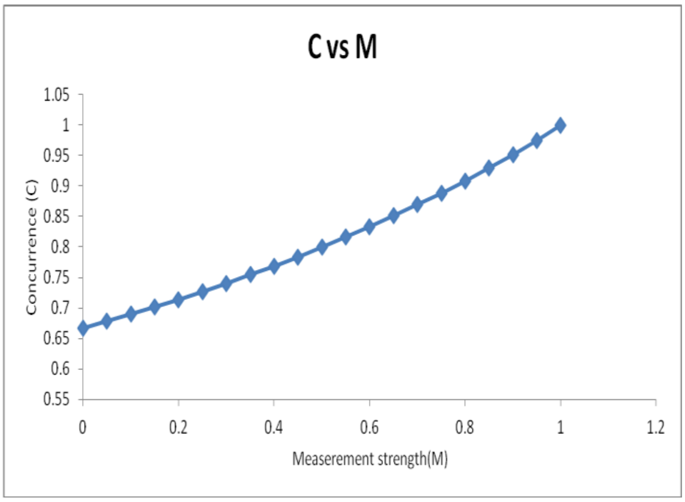

Figure 6 C Vs M (D=0.5)

Now for one sided amplitude damping , repeating all the process as earlier the resultant density matrix is given by

$$\rho = \begin{bmatrix} \beta^2 D & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta\sqrt{1-D} & 0 \\ 0 & \beta\alpha\sqrt{1-D} & \beta^2(1-D) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = 2\alpha\beta\sqrt{1-D} \qquad (11)$$

Now as earlier we apply sequence of weak measurement, amplitude damping and Quantum measurement reversal and the two qubit state is given by

$$\rho' = \frac{1}{A'}\begin{bmatrix} \beta^2 D(1-M)(1-D) & 0 & 0 & 0 \\ 0 & \alpha^2 & \alpha\beta\sqrt{1-D} & 0 \\ 0 & \beta\alpha\sqrt{1-D} & \beta^2(1-D) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $A' = \beta^2 D(1-M)(1-D) + \beta^2(1-D) + \alpha^2$

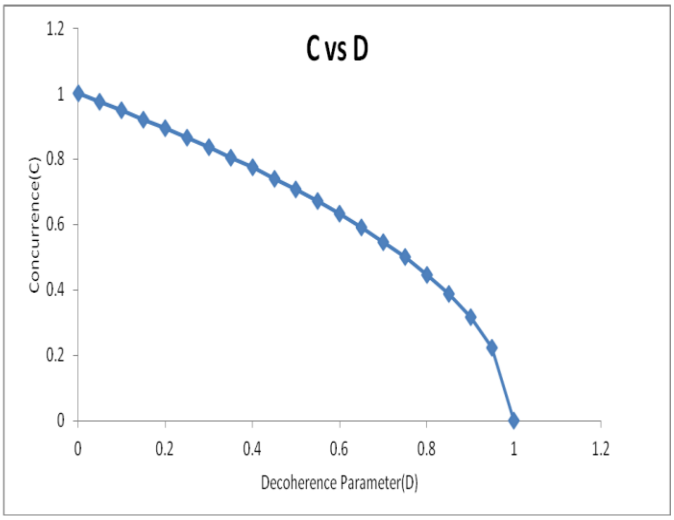

Figure 7 C Vs D

Now entanglement can be measure by measuring the concurrence, which is calculated be

$$C = \frac{2\alpha\beta\sqrt{1-D}}{\beta^2 D(1-M)(1-D) + \alpha^2 + \beta^2(1-D)} \qquad (12)$$

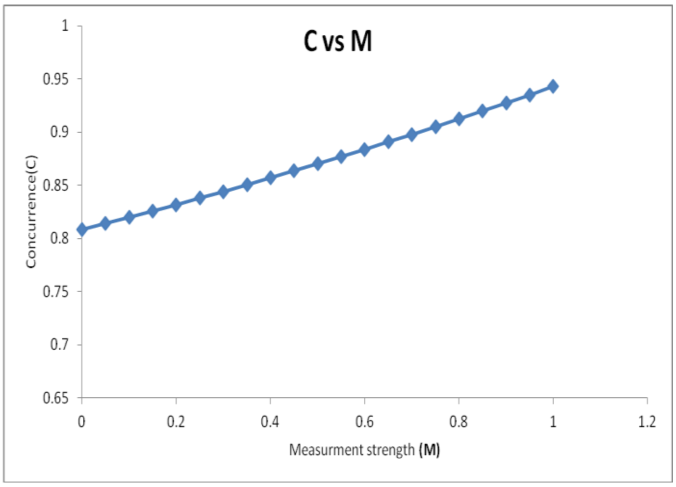

Figure 8 C Vs M (D=0.5)

Now we are interested to finding out the variation of concurrence with initial parameter alpha, concurrence in term of initial parameter alpha for state α|00⟩+β|11⟩ for two sided amplitude damping is given by

$$C = 2(1-D)\sqrt{1-\alpha^2}(\alpha - D\beta) \qquad (13)$$

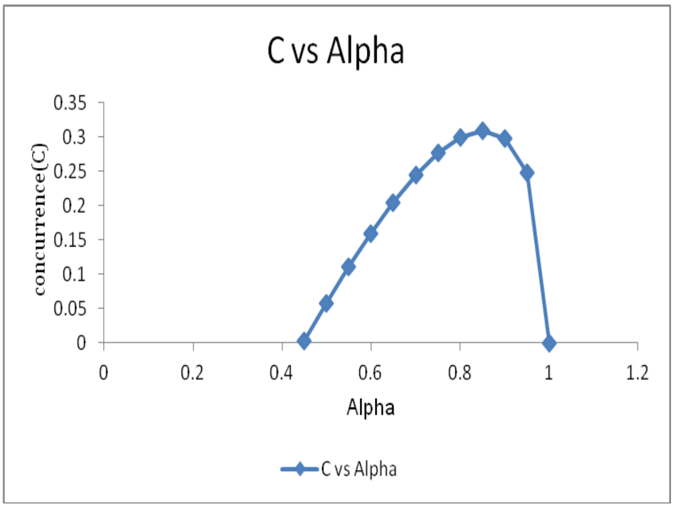

Figure 9 C Vs Alpha (D=0.5)

Here $\alpha_{\max} = \sqrt{\frac{1}{2} - \frac{D}{2\sqrt{1-D^2}}} \qquad (14)$

Similarly, concurrence in term of initial parameter alpha for state α|00⟩+β|11⟩ for one sided amplitude damping is given by

$$C = 2\alpha\sqrt{1-\alpha^2}\sqrt{1-D} \qquad (15)$$

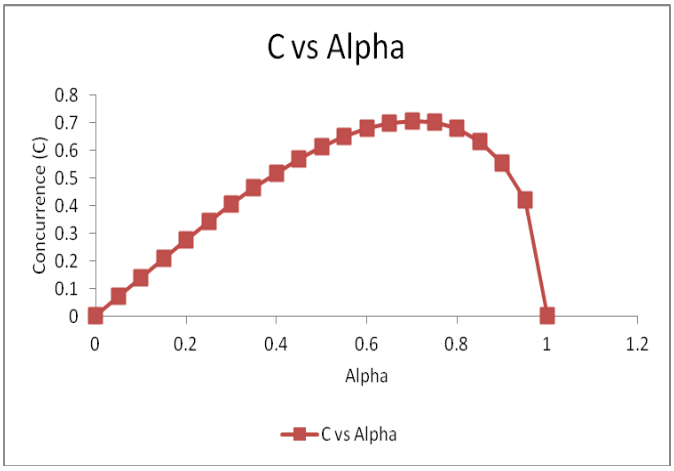

FIGURE 10 C Vs Alpha (D=0.5)

Here $\alpha_{\max} = \frac{1}{\sqrt{2}}$ (16)

Now ,concurrence in term of initial parameter alpha for state α|01⟩+β|10⟩ for two sided amplitude damping is given by

$$C = 2\alpha\sqrt{1-\alpha^2}(1-D) \tag{17}$$

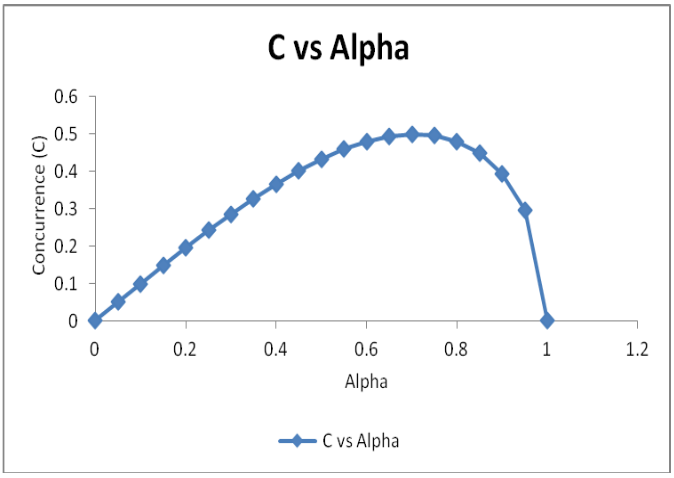

Figure 11 C Vs Alpha (D=0.5)

Here $\alpha_{\max} = \frac{1}{\sqrt{2}}$ (18)

Now ,concurrence in term of initial parameter alpha for state α|01⟩+β|10⟩ for one sided amplitude damping is given by

$$C = 2\alpha\sqrt{1-\alpha^2}\sqrt{1-D} \tag{19}$$

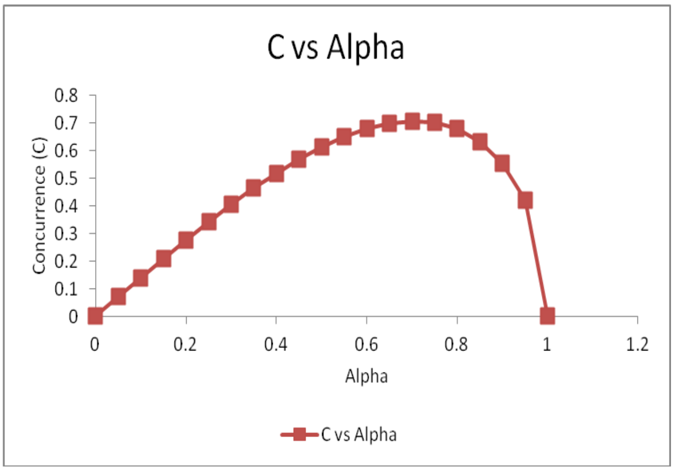

Figure 12 C Vs Alpha (D=0.5)

Here $\alpha_{\max} = \frac{1}{\sqrt{2}}$ (20)

4. DISCUSSION

First we explain the results obtained from two sided amplitude damping. from figure 1 and figure 5 we find that on applying same decoherence on state α|00⟩+β|11⟩ and α|01⟩+β|10⟩ may contain unequal entanglement their comparison is given in figure below.

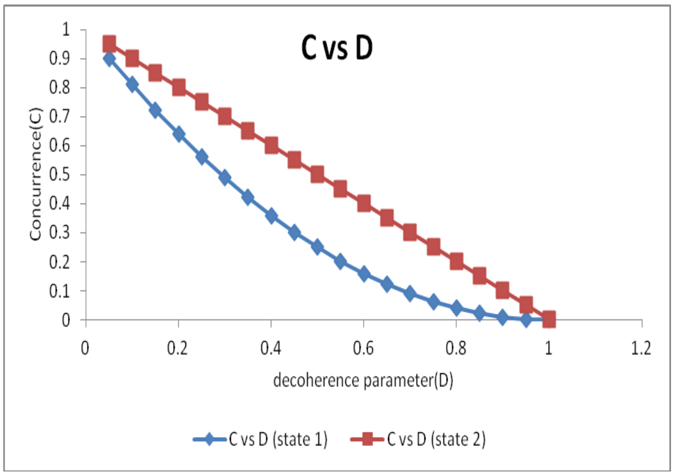

Figure 13 C Vs D

It is clear that state α|01⟩+β|10⟩ shows more protection of entanglement compare to the state α|00⟩+β|11⟩ to amplitude damping . As from figure 2 and figure 6 we find that same decoherence and entanglement protection protocol of two LO equivalent state α|00⟩+β|11⟩ and α|01⟩+β|10⟩ may contain unequal entanglement their comparison is given in figure below.

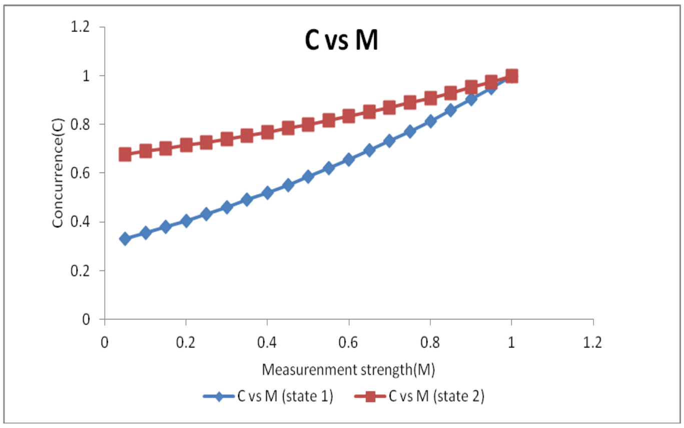

Figure 14 C Vs M (D=0.5)

It is clear that state α|01⟩+β|10⟩ shows more protection of entanglement compare to the state α|00⟩+β|11⟩ to two sided amplitude damping.

Now we explain the results obtained from one sided amplitude damping. from figure 3 and figure 7 it is clear that state α|01⟩+β|10⟩and α|00⟩+β|11⟩ shows exactly same level of protection of entanglement to one sided amplitude damping as shown in figure.

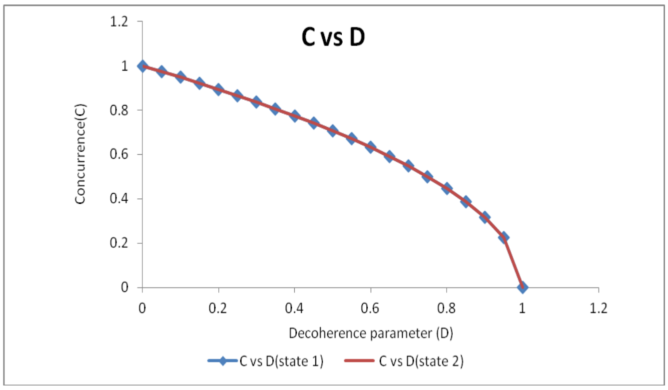

Figure 15 C Vs D

As from figure 4 and figure 8 we find that same decoherence and entanglement protection protocol of two LO equivalent state α|00⟩+β|11⟩ and α|01⟩+β|10⟩ may contain unequal entanglement their comparison is given in figure below.

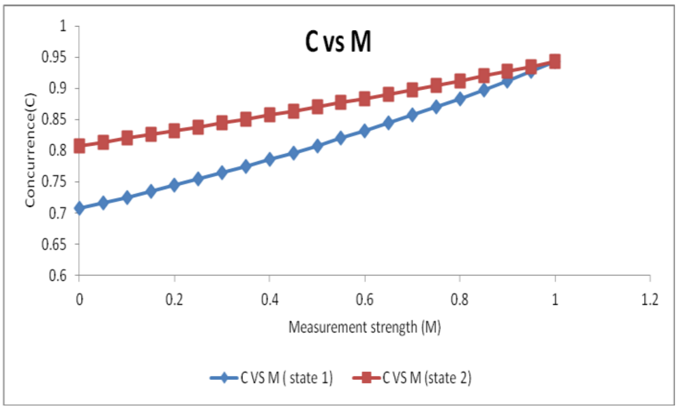

Figure 16 C Vs M (D=0.5)

It is clear that state α|01⟩+β|10⟩ shows more protection of entanglement compare to the state α|00⟩+β|11⟩ to one sided amplitude damping. Further from figure 9,10,11,12 it can be seen that state α|00⟩+β|11⟩ in the case of two sided amplitude damping are showing maximum entanglement, when it is not initially maximally entangled which show that non maximally entangled state can be used for better entanglement distribution [15]

5. CONCLUSION

In short we have studied the one and two sided amplitude damping decoherence suppuration for state α|00⟩+β|00⟩ and α|01⟩+|10⟩ via weak measurement and quantum measurement reversal. In the case of two sided amplitude damping channel we find that state α|01⟩+|10⟩ protect entanglement better than α|00⟩+β|00⟩ before and after applying the weak measurement and quantum measurement reversal. while in the case of one sided amplitude damping before applying the weak measurement and quantum measurement reversal both state protected same amount of entanglement while after applying them α|01⟩+|10⟩ behave better than first state.

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