MRI Reconstruction using Compressive Sensing: A Review Paper

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Abstract - *Compressive sensing is an emerging multi-disciplinary field that focuses on reconstructing an unknown signal from a very limited number of samples. This theory has many applications in signal processing and imaging. This paper provides an overview of image reconstruction process in MRI (Magnetic Resonance Imaging). There are different types of reconstruction algorithms each having their strengths and weaknesses. In this paper, we review some algorithms that reconstructs MRIs using compressive sensing.*

Key Words: Compressive Sensing, Reconstruction

1. INTRODUCTION

The conventional approach of reconstructing signals or images from processed data follows Shannon's celebrated theory "which states that the sampling rate must be at least twice the highest frequency(i.e., fs≥2fm,)". For this process requirement of number of samples is large. Similarly, the fundamental theorem of linear algebra says that only when the number of measurements of a discrete finite-dimensional signal is at least as large as its length, reconstruction can be ensured. The sureness of above two theories depends on the amount of samples i.e. more samples are needed to get more accurate results. Compressive sensing is a method for collecting and reconstructing an audio or image signal with possible advantages in number of applications. It requires less number of samples than Nyquist sampling.

2. COMPRESSIVE SENSING

Theory of compressive sensing cites that recovery of some signals and images from very less number of samples or measurements than conventional methods is possible. For this, CS relies on two theories:

Sparsity

Many signals are sparse, that is, they contain a set of coefficients close to zero or equal to zero if they are represented in the transformed domain. Theory of compressive sensing utilizes the fact that many natural signals are compressible or sparse in a way that they have brief representations when expressed in the proper basis Ψ.

Incoherence

Incoherence grants the duality between frequency and time. It indicates that the object with sparse representation in Ψ must be expanded in the domain in which they are acquired. Incoherence is necessary for good linear assessment in the new space.

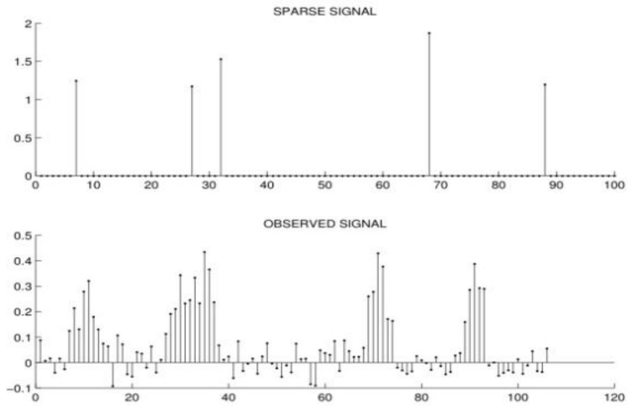

Figure 1: Sparsity[14]

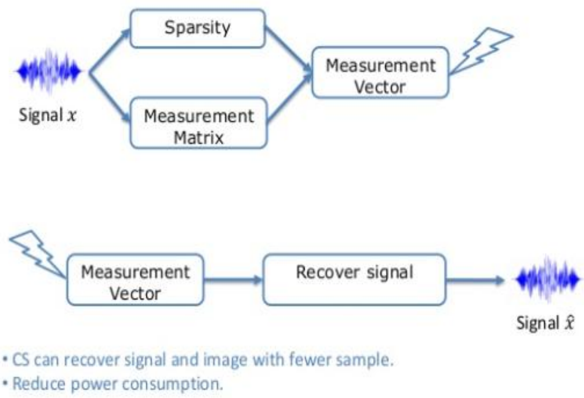

Figure 2: Compressive Sensing[15]

Undersampled measurements

Consider a scenario where a vector x where $x \in R^N$ is reconstructed from measurements y about x of form

$$y_k = \langle \emptyset_k, x \rangle, k = 1, \dots \dots, K$$

$$or$$

$$y = \emptyset x$$

Information about the unknown signal is collected by sensing x against K vectors $\emptyset_k \in R^N$. Area of interest here is the "underdetermined" case K<<N, where number of measurements is vey less than unknown signal values. This type of problems occur countless times. In biomedical imaging for example, number of measurements of image of interest is very fewer compare to number of unknown pixels.0

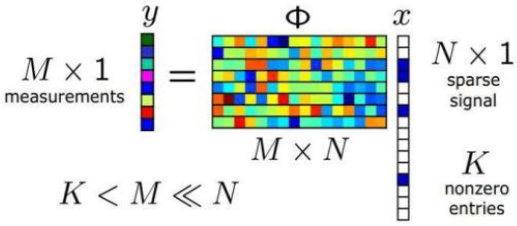

Figure 3: Undersampled Measurements

At first sight, solution to this type of problems seems impossible, as number of examples where it clearly cannot be done is easy to find. But now assume that the signal x is compressible, that it primarily depends on a number of degrees of freedom less than N. Then this presumption thoroughly changes the problem, forming the search for solutions achievable. We can recognize that correct and at times perfect reconstruction is achievable by solving a simple convex optimization problem.

CS has successfully been applied in a wide variety of applications in recent years, including optical system research, shortwave infrared cameras, photography, audio processing , MRI and so on. Rate of MRI collection is bound by requirements of the immense acquisitions in phase-encoding direction. As the theory of compressive sensing MRI with l1-regularization criteria termed CS-MRI emerged it has gained popularity. It capitalizes the signal sparsity itself to reconstruct the MRIs from very less samples than required by traditional methods, which notably reduces the scan time. Though, a more accurate determination of a non-sampling signal using compressive sensing depends on the achievement of two criteria: (a) sufficient sparsity of the MRIs; and (b) a high level of incoherency between the sampling matrix and the sparsity basis.

Compressed Sensing was suggested by Donoho [1] and Candes [2] at the earliest time. The stipulation of conventional theory for frequency of sampling is broken by this theory. By using it quantity of data sampling is reduced and it saves data storage space and computation time. Lustig introduced CS for MRI image collection and reconstruction for first time in [3].

The generic l_1 minimization works very well with the images with sparse features. So MRI images like abdomen images and brain images are first needed to be transformed to a sparse representation so they can be reconstructed successfully. The method used to sparsify the MR image plays an important role in the quality of the resulting CS-MRI reconstruction. In most algorithms transforms like DWT and DCT. Some fast numerical algorithms, based on an operator splitting algorithm (TVCMRI [4]) and variable splitting algorithms (RecPF [5]), are developed for use with TVL1-based CS-MR imaging reconstruction.

To accelerate the reconstruction process SVD as a data-adaptive sparsity basis is proposed by Hong et al. [6] and it is the sparsifying transforms to get a better image quality. Majumdar A. in [7] proposes algorithm that exploits the nuclear norm regularization of the implementation of the CS-MRI reconstruction where nuclear norms have been described as the sum of singular MRI, and the outcomes shows that this reconstruction strategy is much faster than any other method.

3. Literature Review

Output of random under-sampling adds noise-like interference. In the sparse transform domain, there are significant coefficients that exceed the interference. In [3] practical schemes for incoherent under sampling are analyzed and developed by means of their aliasing interference. The reconstruction is constrained by the data fidelity constraint and is done by minimization of l1-norm of a transformed image.

Information, such as organ borders, is pretty rare in many MRIs. With compression sensing, the same MR image can be reconstructed from a very limited set of measurements while dramatically shortening the duration of MRI scans. To handle this in [4], it uses a model that collectively reduces total variation, l_1-norm, and a least squares measure.

The signal is reconstructed to a minimum of the sum of the sum of the three terms of the l1-norm corresponding to the total variation of the fixed transform and least squares data. RecPF([5]) algorithm is very fast because it requires only a small amount of iterations, and each iteration involves two fast Fourier transforms and simple shrinkages. RecPF requires little parameter adjustment, and has a very large regularization/fidelity weight parameter with a dynamic range that consistently performs well.

To achieve the practicality of using data adaptive sparsity in CS-MRI, a method using SVD as data adaptive transform is proposed by Hong et al.[6], which does not need to pre-process the image. This approach may potentially sparsify more types of MRIs than already defined transforms, and still be valid in the CS-MRI framework.

In [7], it is shown that rather than using only the sparseness or ranking deficiency of the image, the superior reconstruction results can be accomplished by combining the sparsity of the transform domain with the rank deficiency. This method presents a comprehensive l1-norm and nuclear norm minimization problem and obtains algorithm of first order to solve it.

Contrary to traditional CS-MRI that only depends on the sparsity of MRIs in gradient or wavelet domain, in [8] it uses the wavelet tree structure to correct CS-MRI. This tree-based CS-MRI problem is divided into three simpler sub problems then each of the sub problems can be efficiently solved by an iterative scheme.

For CS-MRI the way image is sparsified severely affects its reconstruction quality. In [9] a graph-based redundant wavelet transform is showed to sparsely represent MRIs in iterative image reconstructions. For this transformation the image patch is taken as the vertex, with the difference as the edge, the shortest path on the graph minimizes the difference between all image patches.

The group sparse method, which exploits additional sparse representations of the spatial group structure, can increase the degrees of sparsity, as a result better reconstruction performance can be obtained. In [10] , an efficient super-pixel/group assignment method, simple linear iterative clustering (SLIC), is incorporated to CS-MRI studies. Using variable segmentation strategy and classical alternating direct method the problem of group sparseness is solved.

Table -1: Performance Parameters

Paper	Reduce Scan Time	Resolution Improvement	SNR Improvement
[3]	✓	✓	
[4]		✓	✓
[5]	✓		
[6]	✓		
[7]		✓	✓
[8]			✓
[9]	✓		✓
[10]	✓		✓

Table -2: Summary Table

Paper	Summary	Research Gap
Sparse MRI [3]	Focus of this work is only on Cartesian sampling. The l_1 norm of a transformed image is minimized for reconstruction.	Work can be extended for non-Cartesian CS.
Algorithm using total variation and wavelets[4]	Reconstruction algorithm jointly minimizes the l_1 norm, total variation, and a least squares measure.	This algorithm can be speeded up by combining optimization methods such as smoothing and more efficient line search.
TVL1-L2 [5]	Sum of l_1-norm of a certain transform, total variation, and least squares data fitting is minimized for reconstruction. Simple shrinkages and two fast Fourier transforms.	
CS with SVD sparsity basis[6]	SVD as a data-adaptive transform.	Dynamic MRI.
Rank deficiency for MR image [7]	Combined nuclear norm and l1-norm is minimized for reconstruction.	Application to the multicoil parallel MRI problem Three-dimensional (3D) MR volume reconstruction.
Wavelet structure in CS-MRI[8]	Wavelet tree structure for improving CS-MRI. Tree-based CS-MRI : The problem is broken down into three simpler subproblems, each of which can be effectively solved by an iterative scheme	This algorithm can be applied for double-density dual-tree wavelet transform with small changes on group setting.
Graph-based redundant wavelet transform[9]	A graph-based redundant wavelet transform is introduced. The l_1 norm regularized formulation of the problem is solved by an alternating-direction minimization with successive algorithm.	Parallel processes to reduce the computation time caused by redundancy
Super-pixel algorithm and group sparsity[10]	Simple linear iterative clustering (SLIC) and super-pixel/group assignment method is integrated to CS-MRI.	Combing of sparsity regularization with group sparsity regularization methods to further improve the CS-MRI reconstruction quality

4. Discussion

Dynamic MRI reconstruction methods are generally classified in two classes – offline and online. In offline methods image is reconstructed after all the data is collected. In online methods image is reconstructed as each time frame is acquired. Most of the previous studies in dynamic MRI reconstruction were performed using offline methods. These studies exploited the spatio-temporal correlation of the MRI data to express it in a sparse fashion in some transform domain. In online reconstruction each time frame is reconstructed individually, given the reconstructed frames till the previous instant.

Compressive Sensing (CS) based techniques were selected to recover the dynamic MRI videos. For most applications the offline reconstruction is satisfactory, but in cases where the frames need to be visualized in real-time, for example in any tracking application or image guided surgery, offline techniques cannot be employed. Work in [13] follows a prediction-correction framework. Given the previous frames, the current frame is predicted based on a Kalman estimate. This work is inspired from [11,12]. In [11] instead of predicting the frame, it simply uses the past frame as the reference and generates the difference between the previous frame and the current frame presuming that the difference is sparse. In [12] an auto-regressive model predicts the current frame and the difference image is evaluated using CS reconstruction. Work in [13] is different from [11] in two aspects: 1. It uses Kalman prediction for the current frame, and 2. It employs a greedy algorithm for faster recovery.

A reference frame is predicted for the current instant given the reconstructed frames till the previous instant. The difference between the predicted frame and the current frame is corrected using the k-space samples of the current instant.

$$x_t = x_p + \nabla x_t$$

Here x_p denotes the predicted frame. Given the previous frames, we employ a Kalman predictor to estimate x_p. If the difference between the current frame and the predicted frame is normally distributed, a Kalman filter based correction would be optimum. It shows that, such is not the case. The difference between x_p and the actual frame $(x_p - x_t)$ will be sparse and hence a greedy sparse recovery algorithm called Stagewise Orthogonal Matching Pursuit(StOMP) is used for reconstruction. The difference frame, once reconstructed is added to x_p to obtain the final measure of the current frame x_t.

For reconstructing images of size 128 × 128 the Differential CS [12] takes about 0.23 s and with images of size 256 × 256 about 0.50 s. The timing for kalman method (using StOMP) is about 0.12 s for 128 × 128 images and 0.25 s for 256 × 256 images. The improvement in run-time owes to the fact that it uses a greedy algorithm for recovery whereas [12] employs l_1-minimization. In future work advancements in reconstruction times is possible if the algorithm is transferred to C/C++ from Matlab.

We can see that this type of algorithms give much faster results compare to other online reconstruction algorithms. For casual/online MRI reconstruction schematic diagram of proposed method is as shown below.

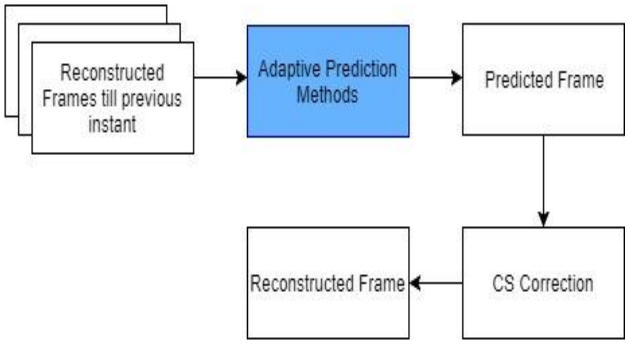

Figure 4: Schematic Diagram of Proposed Approach

5. CONCLUSION

In this research work we reviewed papers on different MRI reconstruction algorithm. Each algorithms exploits different parameters such as rank, transform domain, sparsity degree etc. and takes different approach for reconstruction. After studying these methods, a method based on prediction-correction framework is considered better for online MRI reconstruction as it is much faster. Different prediction methods can be applied to this algorithm to generate better results.

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