Design and Implementation of Butterworth, Chebyshev-I Filters for Digital Signal Analysis

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Abstract - Filters have become a very essential part in the digital field emerging exponentially in today's time. Filters are responsible for removing the undesirable components from a specific signal and help in extracting the required frequency range for the respective output. Filters have many applications in the signal processing and communication systems, like, noise reduction, channel equalization, circuit analysis, audio-video signal processing, radar field, data analysis. Also, the biomedical fields have the use of filters for filtering the noisy signals of the ECG, EEG and EMG. This paper explains the basic functions of filters, their types, i.e. low pass, high pass, band pass, band stop, and all pass filters. The IIR filters have been studied theoretically and also designed using Python software. The filter responses of the Butterworth, and Chebyshev-I filters have been observed and compared successfully.

***Key Words*: Filters, Digital Filters, Impulse Response, Butterworth Filter, Chebyshev-I Filter**

1.INTRODUCTION

The field of signal processing includes filtering as a basic and very essential process. It is a linear system which is used for the removal of noise and all other unwanted components from the signal and gets the desired signal in the output. Using the filters, the desired amplitude phase and frequency of a signal can be obtained from the original signal. Both the digital and analog filters are a part of filtering. The digital filters are more preferable as compared to the analog filters in many fields as it is efficient for detecting and filtering the noise signals.[1]

For the digital filtering the input analog signal is converted into digital signal using sampling and then it is processed and converted back to the analog form and received as the output. The digital filters are classified into many various types based on several factors and characteristics that have been discussed in the further section. The IIR filters and the FIR filters have been explained in detail with the transfer functions and henceforth the Chebyshev-I and Butterworth filters are explained in detail. Also, they have been designed using Python Software for the comparison of their respective response.

2. TYPES OF FILTERS

The digital filters function differently on the basis of the requirement of the user. There are different characteristics exhibited by the digital filters. On the basis of their different characteristics, they can be classified into various different types.

The two main classifications of the digital filters based on their functioning and the response. The very first basic classification is done based on their magnitude characteristics. They are:

1) **Low pass filter**: - As the name suggests, the low pass filters allow only the low frequencies required up to the cut-off frequency to pass through and the other frequencies are attenuated.

2) **High pass filter**: - The high pass filter attenuates the frequencies lower than the cut-off frequencies and allows the higher frequencies to pass through.

3) **Band pass filter**: - This filter allows the selected band of frequency lying between the lower and higher cut-off frequencies to pass through and attenuates the rest of the frequencies.

4) **Band stop filter**: - This filter attenuates the frequency band between the lower and higher cut-off frequencies and lets all the other frequencies pass through.

5) **All pass filter**: - This filter passes all the frequencies of equal gain. [2]

The other classification of filters is based on the time domain. They are:

1) Infinite Impulse Response (IIR) Filters
2) Finite Impulse Response (FIR) Filters

2.1 IIR Filters

IIR Filters are the digital filters that have an infinite impulse response. They are also known as recursive filters as they have a feedback and hence they produce a better frequency response. The IIR filters do not possess linear phase characteristics. This being their limitation, they cannot be preferred for a linear phase system. The IIR filters acquire less memory and also include fewer calculations. The transfer function of the IIR filters is as shown in equation 1 below.

$$H(z) = \frac{\sum_{k=o}^{M} b_k z^{-k}}{1+\sum_{k=a}^{N} a_k z^{-k}} \qquad (1)$$

The IIR filters can be realized by Butterworth, Chebyshev-I and Elliptic filters, of which the Butterworth and Chebyshev-I filters are explained in the further sections. [3]

2.1 FIR Filters

The design methods of FIR Filters are based on ideal filter approximation. Using this approximation, the filter designed is of a higher order, due to which it becomes complex to implement. The transfer function of FIR filters is as shown in equation 2 below.

$$H(z) = \frac{Y(Z)}{X(z)} = \sum_{k=0}^{N} b_k z^{-k} \qquad (2)$$

The designing process takes into consideration the required characteristics and specifications. The FIR filters are designed using various windowing techniques as shown in the figure below.

Name of Window function	Transition Width (Hz) (Normalize-zed)	Pass band ripple (dB)	Main Lobe Relative to Side Lobe (dB)	Stopband attenuation (dB) (Maximum)	Window Function w(n), |n|≤(N-1)/2
Rectangular	0.9/N	0.7416	13	21	1
Hanning	3.1/N	0.0546	31	44	$0.5 + 0.5cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	$0.54 + 0.46cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	75	$0.42 + 0.5cos\left(\frac{2\pi n}{N-1}\right) + 0.8cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274	-	50	$\frac{I_o\{\beta(1-\left[\frac{2n}{N-1}\right]^2)^{\frac{1}{2}}\}}{I_o(\beta)}$
	4.32/N (β=6.76)	0.00275		70	
	5.71/N (β=8.96)	0.000275		90	

Fig -1: Windowing Techniques

3. BUTTERWORTH FILTER

The square magnitude function of a N order Butterworth filter is given by,

$$|H_a(j\Omega)|^2 = \frac{1}{1+\varepsilon^2\Omega^{2N}} \qquad (3)$$

Unity value is achieved by this function at Ω = 0 that is considered a flat pass band. Zero value is achieved at infinity and is considered to be a flat stop band. Figure 2 shows the normalized specifications and the response of Butterworth filter. The following figure gives the normalized specifications, where $\Omega_p = 1$, $|H_a(j\Omega_p)|^2 = \frac{1}{1+\varepsilon^2}$ and it is required that $|H_a(j\Omega_s)|^2 \le \frac{1}{A^2}$.

Where,

$$\varepsilon^2 = 10^{\frac{A_p}{10}} - 1$$

$$A^2 = 10^{\frac{A_s}{10}}$$

A_p = maximum pass-band variation

A_s = minimum stop band attenuation

N = Order of filter, that means the no. of stages used in the design of analog filter. [4]

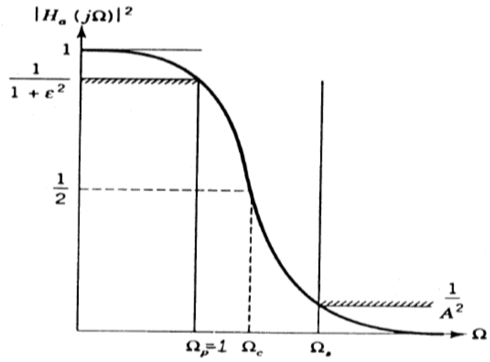

Fig-2: Response of Butterworth Filter

4. CHEBYSHEV-I FILTER

The square magnitude function of a N order Chebyshev-I filter is as shown.

$$|H_a(j\Omega^2)| = \frac{1}{1+\varepsilon^2 T_N^2(\Omega)} \qquad (4)$$

Where,

$$T_N(\Omega) = \begin{Bmatrix} \cos[Ncos^{-1}(\Omega), & |\Omega| \le 1 \\ \cos[Ncosh^{-1}(\Omega), & |\Omega| \ge 1 \end{Bmatrix}$$

In the normalized pass band 0 ≤ Ω ≤ Ω_p = 1, this function alternatingly achieves the values of 1 and $\frac{1}{1+\varepsilon^2}$ at N + 1 points such that $|H_a(j\Omega_p)|^2 = \frac{1}{1+\varepsilon^2}$. For N even, $|H_a(j0)|^2 = \frac{1}{1+\varepsilon^2}$ and for N odd, $|H_a(j0)|^2 = 1$ (equi-ripple pass band). At infinity, the value of $|H_a(j\Omega_p)|^2$ is zero and the first 2N-1 derivatives are zero (maximally flat stop band), as shown in the figure below [5]. The Figure below shows the response of the Chebyshev-I filter.

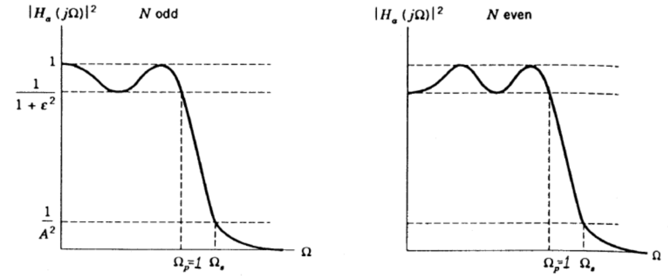

Fig-3: Response of Chebyshev-I Filter

5. SIMULATION RESULTS

Considering the following signal for filtering,

$$X = 2 + \sin(2\pi \times 10^6 \times t) + \sin(2\pi \times 3.5 \times 10^6 \times t) + \sin(2\pi \times 20 \times 10^6 \times t) + \sin(2\pi \times 10^3 \times t)$$

and the desired signal to be

$$X1 = \sin(2\pi \times 10^6 \times t)$$

Cutoff frequency = 1MHz

Sampling frequency = 10MHz

The following graphs demonstrate low pass filter design using different windows and also compare the different filter response along with filtered output.

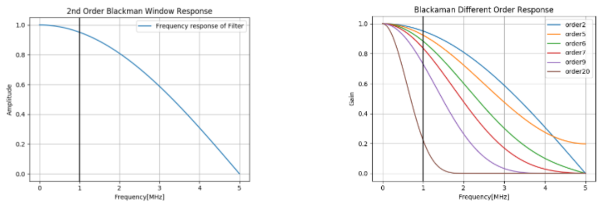

Fig-4: Responses of Blackman Windowing Technique.

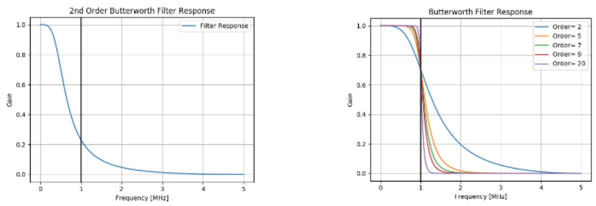

Fig-5: Responses of Butterworth Filter

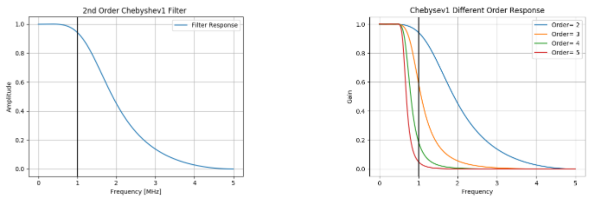

Fig-6: Responses of Chebyshev-I Filter

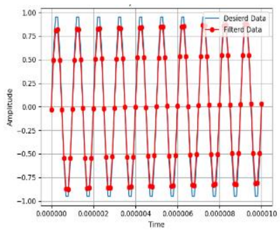

Fig-7: Desired Output and Filtered Output

6. CONCLUSION

In this paper, we have successfully studied the different types of digital filters and their functions. Also, we have understood in detail, the Butterworth and Chebyshev-I filters, their response, and characteristics. The simulations have been successfully performed and the comparison has been concluded that using Butterworth filter, the best output is obtained in reference to the attenuation and phase response. It has a flat pass band and the stop band without any ripples. The results obtained using the FIR filter is very complex and costly as the order of the filter required is high. This leads us to conclude that the use of IIR Filters is better. Among the IIR filters, the Chebyshev-I Filter gives a sharp response.

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