

Numerical Simulation of Stresses acting on Spur Gears Using Simulink

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Abstract: In this project work, static stress analysis is simulated on gear in order to study the behavior of the gear tooth bending stress and the surface contact stress. Static stress analysis is carried out using Simulink software in Matlab. By using Simulink software the calculated analytical data's are validated. Thus validate the correctness and effectiveness of proposed various stress model acting on gear.

This present work carried out serves a novel approach in gear design simulation, and the study of stress behavior in gear needed in the workshop of small scale industries.

Keywords: Spur gears, Stress analysis, Bending Stress, Surface Contact Stress & Surface deformed distance, Matlab and Simulink.

1. STATIC STRESS ANALYSIS

Static stress analysis includes two types of stress acting on the various gear arrangement they are gear tooth bending stress and gear surface contact stress that are caused on two contacting pairs of gear tooth in contact during mating. These stress analysis is carried on all the three gear trains arrangements i.e. for without idler, one idler, two idler and stress distribution between these arrangements is studied under this category which will be analyzed for both metallic and composite spur gears.

1.1 Gear tooth bending stress

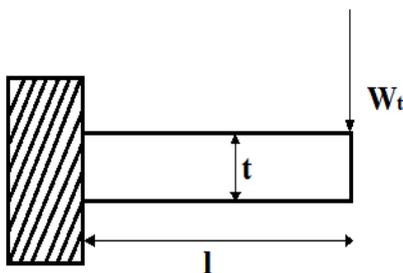


Figure 1.1: Tooth considered as cantilevered beam by Lewis.

Bending stress analytically in gear design is based on Lewis equation. This equation is applied with stress concentration factor of k which defines the stress geometry factor as J for standard close gears. The first equation is used to determine bending in the root of gear tooth which was

derived by Lewis Wilfred. The gear tooth equation is as cantilever simple beam as shown in figure 1.1.

Lewis equation is generally stated as below:

$$\sigma_t = \frac{W_t P_d}{b Y} \tag{1.1}$$

where P_d is diametral pitch, b is face width of the gear and Y is the Lewis form factor given by,

$$Y = \frac{2xP_d}{3} \tag{1.2}$$

and x dimension is determined by,

$$x = \frac{t^2}{4l} \tag{1.3}$$

In order to analyse the load condition in worst state on gear tooth, Lewis originally proposed the tip condition on the gear tooth. In almost all designs in gear, contact ratio is well high enough for the second pair of tooth to be contact in the tip load condition of the member. Thus, considering for the worst load condition the Lewis form factor for 20 teeth, full depth profile and a 20° pressure angle (ϕ) is approximately 0.33. Figure 1.2 shows the clearly the applied load to the tooth gear at the pitch circle diameter for tooth surface in determining the tooth bending stress.

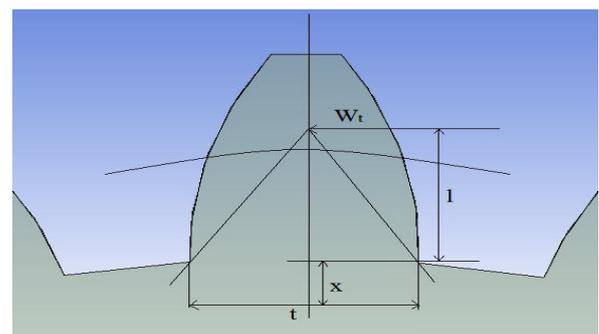


Figure 1.2: Dimensions used to determine tooth bending stress.

The Lewis equation has following assumptions:

1. The effect due to radial load is ignored.
2. The effect due to stress concentration in the root fillet is ignored.

3. It is presumed that in any time only a pair of teeth is in contact.
4. It doesn't take into account the dynamics during mating of the teeth which considers only static loading.
5. Form factors by Lewis equations for various numbers of teeth are for only pressure angle of 20° and full depth involute.

1.1.1 Analytical

Lewis Bending Stress:

$$\sigma_t = \frac{W_t P_d}{b Y}$$

$$Y = \frac{2x P_d}{3}$$

$$x = \frac{t^2}{4l}$$

Where $W_t = \frac{M}{z} = 5000 \text{ N}$

$$M = 200 \text{ Nm}$$

$$l = 6.8 \text{ mm}$$

$$P_d = 250 \text{ mm}^{-1}$$

$$t = 6.5 \text{ mm}$$

$$b_w = 40 \text{ mm}$$

$$x = \frac{6.5^2}{4 \times 6.8}$$

$$x = 1.55 \text{ mm}$$

$$Y = \frac{2 \times 1.55 \times 250}{3}$$

$$Y = 258.33$$

$$\sigma_t = \frac{5000 \times 250}{40 \times 258.33}$$

$$\sigma_t = 120.713 \text{ MPa}$$

∴ Bending Stress (σ_t) = 120.713 MPa

1.2 Gear tooth surface contact stress

While considering the critical bending stresses it also important rather it can be said as equally to determine the surface contact stress due to reasons like scoring, pitting and scuffing of surfaces.

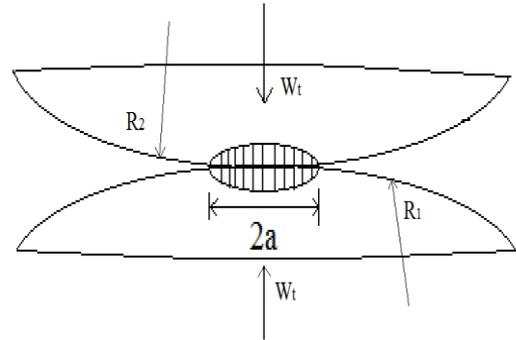


Figure 1.3: The model for Hertzian concept for 2 cylinders in contact under normal load

The surface contact stress is calculated by using Hertzian contact stress analysis. This Hertzian stress analysis of gear teeth depends on the analysis where radial load is acting on two cylinders. The two involute tooth curve mating gear teeth is considered as the radii of cylinders which are radii of curvature the band of contact as shown in figure 1.3. The band of contact between the two curves or the cylinders is obtained to be '2a' where a is the deformed distance given by,

$$a = 2 \sqrt{\frac{W_t \left[\frac{(1-\mu_1^2)}{E_1} + \frac{(1-\mu_2^2)}{E_2} \right]}{b \pi \left[\frac{1}{R_1} + \frac{1}{R_2} \right]}} \quad (1.4)$$

The Hertzian contact stress theory assumes as elliptic stress distribution, as shown in figure 4.10 where the maximum stress level will be in the middle part and is given by the equation,

$$\sigma_0 = \sqrt{\frac{W \left[\frac{1}{R_1} + \frac{1}{R_2} \right]}{b \pi \left[\frac{(1-\mu_1^2)}{E_1} + \frac{(1-\mu_2^2)}{E_2} \right]}} \quad (1.5)$$

where W is normal load, E_1 and E_2 are young's modulus of elasticity for the pinion and the gear respectively, μ_1, μ_2 are the Poisson's ratio of these two gears and b is face width, R_1 and R_2 are the radii of involute curve at point contact as shown in figure 1.4,

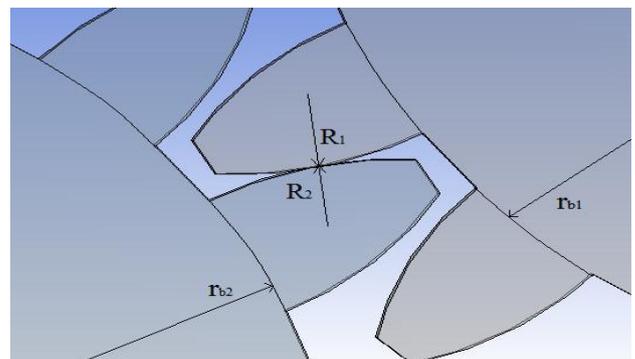


Figure 1.4: Contact of two involute teeth

The pitch radius for pinion and gear is denoted as r_{b1} and r_{b2} which relate to the involute teeth radii as $R_1 = r_{b1} \sin \phi$ and $R_2 = r_{b2} \sin \phi$. Thus, Hertzian surface contact stress is given by,

$$\sigma_0 = \sqrt{\frac{W \left[1 + \frac{r_{b1}}{r_{b2}} \right]}{r_{b1} b \pi \left[\frac{(1-\mu_1^2)}{E_1} + \frac{(1-\mu_2^2)}{E_2} \right] \sin \phi}} \quad (1.6)$$

In this contact stress equations, few assumptions considered such as elliptical distribution at contact of the tooth, friction in the contacting gear, bending of beam is not accounted in this equation of stress evaluation.

Some size and shape of bodies are taken into account themselves as the way they are supported. Mostly in practical cases calculations are complex to perform which results in some various approximate formulae for calculating the bodies of the elastic compression for roller and gear teeth of line contact.

1.2.1 Analytical

$$a = 2 \sqrt{\frac{\frac{W_t (1 - \mu_1^2)}{E_1} + \frac{(1 - \mu_2^2)}{E_2}}{b \pi \left[\frac{1}{R_1} + \frac{1}{R_2} \right]}}$$

$$\sigma_0 = \sqrt{\frac{W \left[1 + \frac{r_{b1}}{r_{b2}} \right]}{r_{b1} b \pi \left[\frac{(1 - \mu_1^2)}{E_1} + \frac{(1 - \mu_2^2)}{E_2} \right] \sin \phi}}$$

Where,

$$\begin{aligned} \mu_1 = \mu_2 &= 0.3 \\ R_1 = R_2 &= 16 \text{ mm} \\ E_1 = E_2 &= 200 \text{ Gpa} \\ r_{b1} = r_{b2} &= R_1 / \sin \phi \\ W_t &= 5000 \text{ N} \end{aligned}$$

$$\begin{aligned} r_{b1} &= \frac{R_1}{\sin \phi} \\ r_{b1} &= \frac{16}{\sin 20} \end{aligned}$$

$$r_{b1} = 46.78 \text{ mm}$$

$$\begin{aligned} r_{b2} &= \frac{R_2}{\sin \phi} \\ r_{b2} &= \frac{16}{\sin 20} \\ r_{b2} &= 46.78 \text{ mm} \end{aligned}$$

$$r_{b1} = r_{b2} = 46.78 \text{ mm}$$

$$a = 2 \sqrt{\frac{\frac{5000 \times (1 - 0.3^2)}{200000} + \frac{(1 - 0.3^2)}{200000}}{40 \times \pi \times \left[\frac{1}{16} + \frac{1}{16} \right]}}$$

$$a = 0.038 \text{ mm}$$

$$\sigma_0 = \sqrt{\frac{5000 \times \left[1 + \frac{46.78}{46.78} \right]}{46.78 \times 40 \times \pi \left[\frac{(1 - 0.3^2)}{200000} + \frac{(1 - 0.3^2)}{200000} \right] \sin 20}}$$

$$\sigma_0 = 739.29 \text{ MPa}$$

∴ Deformed Distance (a) = 0.038 mm
Surface Contact Stress (σ_0) = 739.29 MPa

2. MATLAB AND SIMULINK

Matlab is a technical language for computing various problems. It integrates computation, visualization and programming in easy environment where solutions can be obtained through mathematical notations. Simulink is integrated in MATLAB which is a software package for simulating and modelling various dynamic systems.

The use of Simulink in this case of project is to obtain the various results of analysis performed.

It necessary because as time is the main concern in any field of engineering design where complex number of problems are involved which can be solved using Simulink in MATLAB simulating software.

The various simulation carried out under this project is in calculating the values such as:

1. Bending Stress using Lewis equation.
2. Surface contact Stress using Hertzian equation
3. Surface deformed distance

Depending on the analytical formulas the Simulink model are created using various mathematical notations in order to obtain the required results. Figure 6.1, 6.2 and 6.3 shows the different Simulink model to evaluate the values of Bending stress, Surface contact stress and Surface deformed distance.

2.1 Simulink Program:

2.1.1 Bending Stress:

$$\sigma_t = \frac{W_t P_d}{b_w Y} \quad Y = \frac{2xP_d}{3} \quad x = \frac{t^2}{4l}$$

$$W_t = \frac{M}{D/2}$$

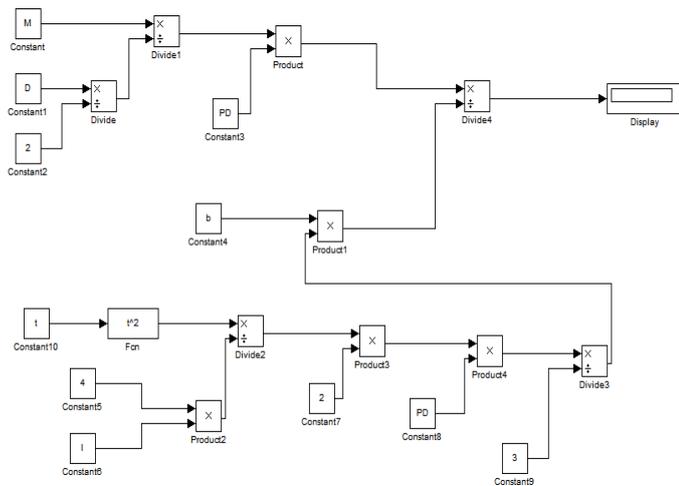


Figure 2.1: Simulink model for bending stress

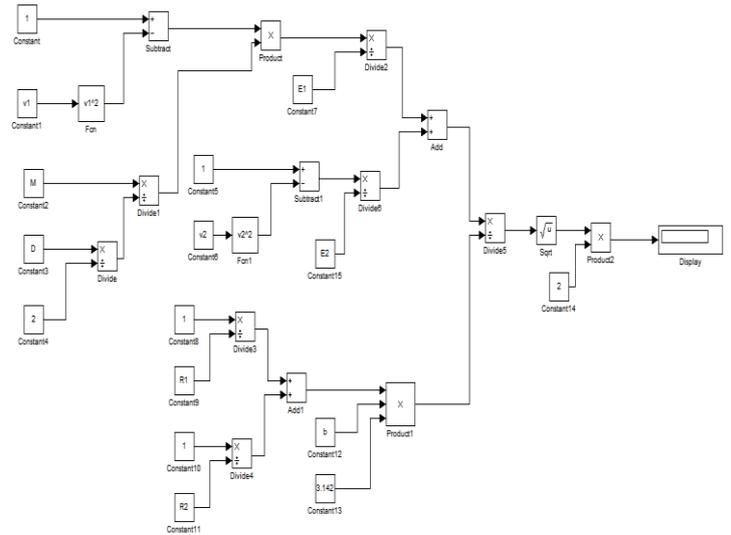


Figure 2.3: Simulink model for calculating deformed distance

2.1.2 Surface Contact Stress:

$$\sigma_0 = \sqrt{\frac{W_t \left[1 + \frac{r_{b1}}{r_{b2}} \right]}{r_{b1} b \pi \left[\frac{(1 - \mu_1^2)}{E_1} + \frac{(1 - \mu_2^2)}{E_2} \right]} \sin \phi}$$

$$r_{b1} = \frac{R_1}{\sin \phi} \quad r_{b2} = \frac{R_2}{\sin \phi} \quad W_t = \frac{M}{D}$$

Table 2.1: Simulink results for Bending Stress, Surface Contact Stress and Deformed Distance

Moment (Nm)	Bending Stress (MPa)		Surface Contact Stress (MPa)		Deformed Distance (mm)	
	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
100	60.356	60.356	522.76	522.76	0.053	0.053
200	120.71	120.71	739.29	739.29	0.076	0.076
300	181.06	181.06	905.45	905.45	0.093	0.093
400	241.42	241.42	1045.52	1045.52	0.107	0.107
500	301.78	301.78	1168.92	1168.92	0.120	0.120
600	362.13	362.13	1280.49	1280.49	0.131	0.131
700	422.49	422.49	1383.09	1383.09	0.142	0.142
800	485.85	485.85	1478.59	1478.59	0.152	0.152

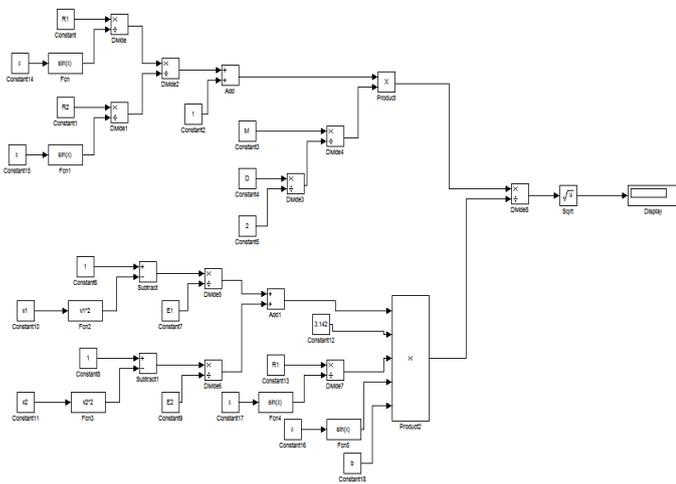


Figure 2.2: Simulink model for calculating surface contact stress

2.1.3 Deformed Distance:

$$a = 2 \sqrt{\frac{W_t (1 - \mu_1^2) + (1 - \mu_2^2)}{E_1 + E_2}}{b \pi \left[\frac{1}{R_1} + \frac{1}{R_2} \right]}$$

The table 2.1 shows the values of simulated data using Simulink and analytical values. From the table it is clear the simulated data coincides exactly with analytical. Hence by Simulink technique a lot of time involved in calculating manually can be reduced using this kind of Simulink technique. Any kind of problems that are complex and tedious in manually calculating the results then Simulink would be a better technique.

3. CONCLUSION

The stresses occurring in Spur gear as well as the deformed distance are validated using Simulink software in MATLAB. The results of the simulated data show that they coincide exactly with the analytical results. Hence Simulink

technique would be a better idea of using it since whenever a problem that are complex where manually calculating consumes time then Simulink would be of better idea.

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