Introduction of Laplace Transform and ELzaki Transform with Application (Electrical Circuits)

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Abstract - An introduction of Laplace transform and New ELzaki Transform is the basic topic of this paper. The definition of Laplace, ELzaki transform and its properties have been mentioned. In this paper also include Inverse Laplace and ELzaki transform. Both transform is a power full mathematical tool for the Engineering to solve Engineering problem such as electrical/electronic dynamic system analysis, mechanical and civil engineering.

Keywords: Laplace transform, ELzaki transform, Inverse transform, Differential equation.

1. Introduction: This paper deals with a brief overview of what Laplace & ELzaki transform is and its application in the applied science & Engineering. Laplace & ELzaki transform will be Denoted by $L\{F(t)\}$ & $E\{F(t)\}$ where $F(t)$ is a function of 't' .The analysis of Electronic circuit and solution of linear differential equation (second or higher order) is simplified by use both transform. This paper deals with the solution of ordinary differential equation and system of ODEs that arise in mathematical engineering science.

2. Laplace Transform:

The Laplace Transform expressed by

$$f(s) = L\{F(t)\} = \int_0^\infty e^{-st}F(t)dt \quad \text{.................................} (1)$$

Where, $F(t)$ is piecewise (Sectionally) continuous on a closed interval $a \le t \le b$,

$$t \ge 0 \text{ and } \quad |F(t)| \le Me^{at} \text{ for some constent } a \text{ and } M$$

$$L\{F(t)\} \text{ exists for all } s > a .$$

Above is Existence of Laplace Transform.

3. Properties and Theorems of Laplace Transform: -

3.1 Linear Property:-

$$L\{a_1F_1(t) + a_2F_2(t)\} = a_1L\{F_1(t)\} + a_2L\{F_2(t)\}$$

Where a_1 and a_2 are constant.

3.2 Change of scale property:-

If $L\{F(t)\} = f(s)$ then $L\{F(at)\} = \frac{1}{a}f\left(\frac{s}{a}\right)$

3.3 First Shifting Property:-

If $L\{F(t)\} = f(s)$ then $L\{e^{-at}F(t)\} = f(s+a)$.

3.4 Second Shifting property:-

If $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a), & t > a \\ 0 & t < a \end{cases}$ then

$$L\{G(t)\} = e^{-as} f(s)$$

3.5 Laplace Transform of Derivative of $F(t)$:-

If $F(t)$ be continuous for all $t \geq 0$ and is exponential order a as $t \to \infty$ and if $F'(t)$ is of class A, then,

$$L\{F'(t)\} = sL\{F(t)\} - F(0) \quad \text{also}$$

(i) $L\{F''(t)\} = s^2 L\{F(t)\} - sF(0) - F'(0)$

(ii) $L\{F'''(t)\} = s^3 L\{F(t)\} - s^2 F(0) - sF'(0) - F''(0)$

Laplace transform of derivative is useful to expressed ODEs (Second or Higher order) in algebraic equations.

3.6 Heaviside's Unit Step Function:-

Heaviside unit step function have only two possible values either 0 and 1 .

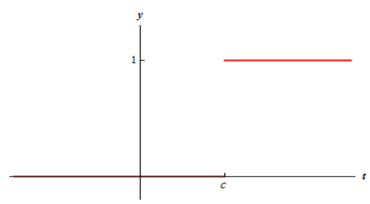
$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

The function takes a jump of unit magnitude at $x = 0$.

$$L\{H(t)\} = \frac{1}{s}$$

Also If the origin is shifted to $t = a$ and a jump to unit magnitude at $t = a$ then function is called Displaced unit step function.

$$H(t - s) = \begin{cases} 0, & t < a \\ 1, & t \geq 1 \end{cases}$$



$$\therefore \ L\{H(t - a)\} = \frac{e^{-as}}{s}$$

3.7 Dirac Delta Function:-

If $L\{F(t)\} = f(s)$ *and* $F_\epsilon(t)$ is define by

$$F_\varepsilon(t) = \begin{cases} \frac{1}{\varepsilon}, & 0 \leq t \leq \varepsilon \\ 0 & t > \varepsilon \end{cases} \quad \text{where } \varepsilon > 0$$

Then $L\{F_\varepsilon(t)\} = \frac{1}{s\varepsilon}[1 - e^{-s\varepsilon}]$

4. ELzaki Transform:

For any function $f(t)$ the sufficient condition for the existence of ELzaki transform are that $f(t)$ for $t \geq 0$ be piecewise continuous and of exponential order.

$$E\{f(t)\} = v \int_0^\infty e^{-\frac{t}{v}} f(t) dt$$

4.1 ELzaki Transform of simple functions :

(i) $Let\ f(t) = 1$ then, $E(1) = v\int_0^\infty 1e^{-\frac{t}{v}}\,dt$

$$= v\left[-ve^{-\frac{t}{v}}\right]_0^\infty = v^2 = T(v)$$

$Inversion\ Formula$: $T^{-1}(V^2) = 1 = f(t)$

(ii) $Let\ f(t) = t\ then,$ $E(t) = v\int_0^\infty te^{-\frac{t}{v}}\,dt$

$after\ intigration\ by\ part,$ $E(t) = v^3 = T(v)$

$Inversion\ Formula$: $T^{-1}(V^3) = t = f(t)$

$also, E(t^n) = n!\,v^{n+2}$

$For\ n = 2,$ $E(t^2) = 2!\,v^4 = T(v)$

$Inversion\ Formula\ ,:$ $T^{-1}(V^4) = \frac{1}{2}t^2 = f(t)$

(iii) $Let\ f(t) = e^{at}\ then,\ E(e^{at}) = v\int_0^\infty e^{at}e^{-\frac{t}{v}}\,dt$

$$= \frac{v^2}{1-av} = T(v)$$

$Inversion\ Formula$: $T^{-1}\left(\frac{v^2}{1+v}\right) = e^{-t} = f(t)$

(iv) $E(sinat) = \frac{av^3}{1+a^2v^2} = T(v)$

$Inversion\ Formula$: $T^{-1}\left(\frac{v^3}{1+v^2}\right) = sint = f(t)$

(v) $E(cosat) = \frac{av^2}{1+a^2v^2} = T(v)$

$Inversion\ Formula$: $T^{-1}\left(\frac{v^2}{1+v^2}\right) = cost = f(t)$

(vi) $E(sinhat) = \frac{av^3}{1-a^2v^2} = T(v)$

$Inversion\ Formula$: $T^{-1}\left(\frac{v^3}{1-v^2}\right) = sinht = f(t)$

(vii) $E(coshat) = \frac{av^2}{1-a^2v^2} = T(v)$

$Inversion\ Formula$: $T^{-1}\left(\frac{v^2}{1-v^2}\right) = cosht = f(t)$

(viii) ELzaki Transform of Derivative:

$T(v)$ be the ELzaki transform of $f(t)$ i.e

$E\{f(t)\} = T(v),\ then$

$E\{f'(t)\} = \frac{1}{v}T(v) - vf(0)$

Similarly $E\{f''(t)\}\frac{1}{v^2}T(v) - f(0) - vf'(0)$

(ix) ELzaki Transform of Integral:

$T(v)$ be the ELzaki transform of $f(t)$ i.e

$E\{f(t)\} = T(v),\ and\ let\ h(t) = \int_0^t f(x)dx$ then,

$E\{h(t)\} = E\left\{\int_0^t f(x)dx\right\} = vT(v)$

(x) Shifting Property:

$If\ \ E\{f(t)\} = T(v),\ then$

$E\{e^{at}f(t)\} = \frac{1}{1-av}T\left\{\frac{v}{1-av}\right\}$

5. APPLICATION OF LAPLACE TRANSFORM:-

This paper describes the application of Laplace & ELzaki Transform in the area of engineering specially Electric analysis. The Laplace & ELzaki transform analyses the time domain in which output and input are function of time to the frequency domain and the transform also use to determine the charge on the capacitors as function of time.

5.1 Electrical Circuits:-

A simple electrical circuit consists of the circuit elements connected in series with a switch (K).

A battery supplying an electromotive force(e.m.f) (E) (in volts). An inductor (L)(in henrys). A Resistance (R) (in ohms). A Capacitor (C) .

When the circuit is completed a charge Q (coulombs) will flow to the capacitor plates. The time rate flow of charge is $\frac{dQ}{dt} = I$

The potential voltage drop across a circuit element are:-

(i) Voltage drop across a resistance $R = RI = R\frac{dQ}{dt}$

(ii) Voltage drop across a capacitor having capacitance $= \frac{Q}{C}$.

(iii) Voltage drop across an inductor having inductance L,

$= L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$

(iv)Voltage drop across a generator, supplying an e.m.f, E

$= voltage\ rise = -E.$

The differential equation for determination of Q is obtained by the use of Kirchhoff's laws.

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E$$

Taking Laplace and ELzaki Transform on both sides with using conditions and get the value $Q\ Or\ I=\frac{dQ}{dt}$.

5.2 Example:

An indicator of 3 henrys, a resistor of 16 ohms and a capacitor of 0.02 farads are connected in series with an e.m.f. of 300 volts. At $=0$, the charge on the capacitor and current in the circuit are zero. Find the charge and current at any time $t>0$.

Solution:

Let Q and I instantaneous charge and current respectively at time t. Then by Kirchboff's laws:

$$L\frac{dI}{dt}+RI+\frac{Q}{C}=E$$

$$2\frac{dI}{dt}+16I+\frac{Q}{0.02}=E$$

$$2\frac{d^2Q}{dt^2}+16\frac{dQ}{dt}+50Q=E\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)\quad \left\{\because I=\frac{dQ}{dt}\right\}$$

With initial condition $Q(0)=0,\quad I(0)=0\,,\quad Q'(0)=0$

If $E=300$ then equation (1),

$$\frac{d^2Q}{dt^2}+8\frac{dQ}{dt}+25Q=150\ldots\ldots\ldots\ldots\ldots(2)$$

Now Apply Laplace Transform at equation (2),

$$L\left\{\frac{d^2Q}{dt^2}\right\}+8L\left\{\frac{dQ}{dt}\right\}+25L\{Q\}=L\{150\}$$

$$\{s^2L(Q)-sQ(0)-Q'(0)\}+8\{sL(Q)-Q(0)\}+25L(Q)=L(150)$$

$$s^2L(Q)-0-0+8sL(Q)-0+25L(Q)=\frac{150}{s}$$

$$(s^2+8s+25)L(Q)=\frac{150}{s}$$

$$L(Q)=\frac{150}{s(s^2+8s+25)}$$

$$Q(t)=L^{-1}\left\{\frac{150}{s(s^2+8s+25)}\right\}$$

By Partial fraction,

$$Q(t)=L^{-1}\left\{\frac{6}{s}-\frac{6s-48}{s^2+8s+25}\right\}$$

$$Q(t)=6L^{-1}\left(\frac{1}{s}\right)-L^{-1}\left\{\frac{6(s+4)}{(s+4)^2+9}\right\}-L^{-1}\left\{\frac{24}{(s+4)^2+9}\right\}$$

Using Shifting property,

$Q(t) = 6 - 6e^{-4t}cos3t - 8e^{-4t}sin3t$ And $I = \frac{dQ}{dt} = 50e^{-4t}sin3t$ **(Required Result)**

Taking ELzaki transform, at equation (2) we have ,

$$E\left\{\frac{d^2Q}{dt^2}\right\} + 8E\left\{\frac{dQ}{dt}\right\} + 25E\{Q\} = E\{150\}$$

$$\frac{1}{u^2}Q(u) - Q(0) - uQ'(0) + 8\left\{\frac{1}{u}Q(u) - uQ(0)\right\} + 25Q(u) = 150u^2$$

$$\frac{1}{u^2}Q(u) - 0 - 0 + \frac{8}{u}Q(u) - 0 + 25Q(u) = 150u^2$$

$$\left\{\frac{1}{u^2} + \frac{8}{u} + 25\right\}Q(u) = 150u^2$$

$$\left\{\frac{25u^2 + 8u + 1}{u^2}\right\}Q(u) = 150u^2$$

$$Q(u) = \frac{150u^4}{25u^2 + 8u + 1}$$

Inversion formula for ELzaki,

$$Q(t) = E^{-1}\left\{\frac{150u^4}{25u^2 + 8u + 1}\right\}$$

After division

$$Q(t) = E^{-1}\left\{6u^2 - \frac{48u^3 - 6u^2}{25u^2 + 8u + 1}\right\}$$

$$= E^{-1}\left\{6u^2 - \frac{6u^2}{(1 + 4u)^2 + 9u^2} - \frac{48u^3}{(1 + 4u)^2 + 9u^2}\right\}$$

$$= E^{-1}\left\{6u^2 - \frac{6u^2 - 24u^3}{(1 + 4u)^2 + 9u^2} - \frac{24u^3}{(1 + 4u)^2 + 9u^2}\right\}$$

$$= E^{-1}\left\{6u^2 - \frac{6u^2(1 + 4u)}{(1 + 4u)^2 + 9u^2} - \frac{24u^3}{(1 + 4u)^2 + 9u^2}\right\}$$

$Q(t) = 6 - 6e^{-4t}cos3t - 8e^{-4t}sin3t$ And $I = \frac{dQ}{dt} = 50e^{-4t}sin3t$ **(Required Result)**

5. Conclusion:-

In this paper I have introduced the Laplace transform and ELzaki Transform with application. The use of those transform of converting the time domain function in to the frequency domain and determination of charge Q of electric circuit. I have discussed the major properties of Laplace and ELzaki Transform and both the Inverse Transform about ODEs initial value problem (I.V.P) with recall the nth derivatives. It goes without saying that Laplace Transform and ELzaki Transform is put to tremendous use in Engineering branches.

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