COMPARATIVE ANALYSIS OF NATURAL FREQUENCY FOR CANTILEVER BEAM THROUGH ANALYTICAL AND SOFTWARE APPROACH

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Abstract— Beam is a inclined or horizontal structural member casing a distance among one or additional supports, and carrying vertical loads across (transverse to) its longitudinal axis, as a purling, girder or rafter. Flexible structures usually have low flexible rigidity and small material damping ratio. A little excitation may lead to destructive large amplitude vibration and long settling time. These can result in fatigue, instability and poor operation of the structures. Vibration control of flexible structures is an important issue in many engineering applications, especially for the precise operation performances in aerospace systems, satellites, flexible manipulators, etc. Beams and beam like elements are main constituent of structures and widely used in aerospace, high speed machinery, light weight structure, etc and experience a wide variety of static and dynamic loads of certain frequency of vibration which leads to its failure due to resonance. Vibration testing has become a standard procedure in design and development of most engineering systems. The system under free vibration will vibrate at one or more of its natural frequencies, which is the characteristic of the dynamical nature of system. The natural frequency is independent of damping force because the effect of damping on natural frequency is very small.

Index Terms : Natural Frequency, Beam, Analytical

1. INTRODUCTION

Beam is a inclined or horizontal structural member casing a distance among one or additional supports, and carrying vertical loads across (transverse to) its longitudinal axis, as a purling, girder or rafter. Three basic types of beams are:

1. Simple span, supported at both end
2. Continuous, supported at more than two points
3. Cantilever, supported at one end with the other end overhanging and free.

Flexible structures usually have low flexible rigidity and small material damping ratio. A little excitation may lead to destructive large amplitude vibration and long settling time. These can result in fatigue, instability and poor operation of the structures. Vibration control of flexible structures is an important issue in many engineering applications, especially for the precise operation performances in aerospace systems, satellites, flexible manipulators, etc. Beams and beam like elements are main constituent of structures and widely used in aerospace, high speed machinery, light weight structure, etc and experience a wide variety of static and dynamic loads of certain frequency of vibration which leads to its failure due to resonance.

Vibration testing has become a standard procedure in design and development of most engineering systems. The system under free vibration will vibrate at one or more of its natural frequencies, which is the characteristic of the dynamical nature of system. The natural frequency is independent of damping force because the effect of damping on natural frequency is very small. s. The Euler-Bernoulli beam theory is the most commonly used because it is simple and provides realistic engineering approximations for many problems.

2. ANALYTICAL METHOD

Dynamic systems can be characterized in terms of one or more natural frequencies. The natural frequency is the frequency at which the system would vibrate if it were given an initial disturbance and then allowed to vibrate freely.

There are many available methods for determining the natural frequency. Some methods are listed below:

1. Newton's Law of Motion
2. Rayleigh's Method
3. Energy Method
4. Lagrange's Equation
5. Eulers Method

Not that the Rayleigh, Energy, and Lagrange methods are closely related.
Some of these methods directly yield the natural frequency. Others yield a governing equation of motion, from which the natural frequency may be determined.

Normally Euler-Bernoulli's equation is used for calculation. By the theory of Euler-Bernoulli's beam it is assumed that

1. Cross-sectional plane perpendicular to the axis of the beam remain plane after deformation.
2. The deformed cross-sectional plane is still perpendicular to the axis after deformation.
3. The theory of beam neglects the transverse shearing deformation and the transverse shear is determined by the equation of equilibrium.

1. Newton's law of Motion.

In vibration, the motion along any degree of freedom is also subject to a law, which may be expressed as Newton's second law of motion. We were able to write an equation of motion for a single degree of freedom system by summing all the actions (forces or moments) along the direction of motion in which the system is free to move, and equating the sum of the actions to the product of acceleration and an inertial resistance factor. A discrete system with n degrees of freedom will have n equations of motion. This means more work for us.

These equations may be obtained by applying Newton's second law of motion to the discrete masses along each of the n independent translational/rotational co-ordinates. This is not the only way to obtain the equations of motion.

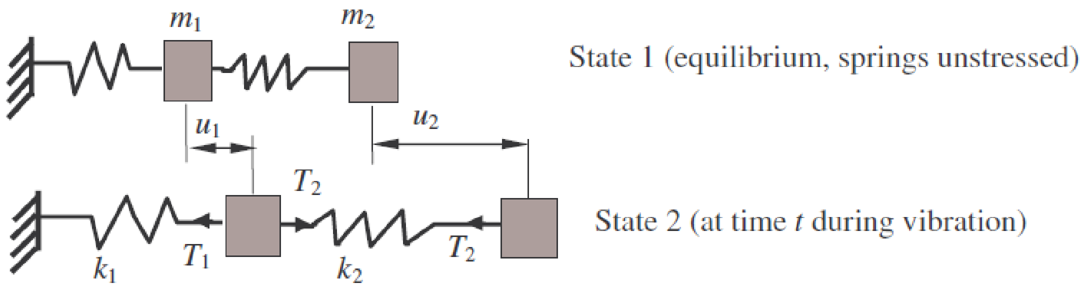


Figure 3.1.1 A 2 DOF system

Let the dynamic displacement of the masses be $u1$, $u2$ and their amplitudes be $\hat{u}1$, $\hat{u}2$. Since x,y are used for representing the static co-ordinates of continuous systems

Natural frequency

$$k_2 - k_2 \frac{k_2}{(k_1 + k_2 - m_1\omega^2)} - m_2\omega^2 = 0$$

This is the frequency equation. The roots of this frequency equation are the two natural frequencies.

2. Rayleigh's Method

Rayleigh method gives a fast and rather accurate computation of the fundamental frequency of the system. It applies for both discrete and continuous systems. This method gives the upper bound approximation of the fundamental frequency of the system.

Rayleigh's principle can be stated as in a conservative system the frequency of vibration has a stationary value in neighborhood of a natural mode.

This method is used to find the bending frequency of cantilever beam

For Ex. A steam turbine blade of length l, can be considered as a uniform cantilever beam, mass *m* per unit length with a tip mass *M*. The flexural rigidity of the blades is EI then bending frequency will be

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

$$\Rightarrow \omega_n^{\,2} = \frac{g}{\Delta} = \frac{g.256EI}{3mgl^3} = \frac{256EI}{3mgl^3}$$

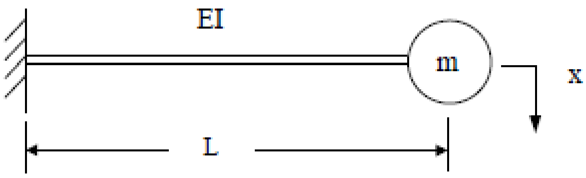
3. Energy Method :

The total energy of a conservative system is constant. Thus,

$$\frac{d}{dt}(KE + PE) = 0$$

where
KE = kinetic energy
PE = potential energy
Kinetic energy is the energy of motion, as calculated from the velocity.
Potential energy has several forms. One is strain energy. Another is the work done against a gravity field.

For Ex.
Cantilever Beam with End Mass
Consider a mass mounted on the end of a cantilever beam, as shown in Figure B-1.
Assume that the end-mass is much greater than the mass of the beam.



Where,

E is the modulus of elasticity.
I is the area moment of inertia.
L is the length.
g is gravity.
m is the mass,
x is the displacement.

The static stiffness at the end of the beam is

$$k = \frac{3EI}{L^3}$$

The potential energy is

$$PE = \frac{1}{2}\left[\frac{3EI}{L^3}\right]x^2$$

The kinetic energy is

$$KE = \frac{1}{2}m\dot{x}^2$$

The natural frequency of the end mass supported by the cantilever beam is thus

$$\omega_n^2 = \left[\frac{3EI}{mL^3}\right]$$

$$\omega_n = \sqrt{\frac{3EI}{mL^3}}$$

4. Euler–Lagrange's Method:

The Euler–Lagrange equation was developed in the 1750s by Euler and Lagrange in connection with their studies of the tautochrone problem. This is the problem of determining a curve on which a weighted particle will fall to a fixed point in a fixed amount of time, independent of the starting point.

Lagrange solved this problem in 1755 and sent the solution to Euler. Both further developed Lagrange's method and applied it to mechanics, which led to the formulation of Lagrangian mechanics. Their correspondence ultimately led to the calculus of variations, a term coined by Euler himself in 1766

The Euler–Lagrange equation, then, is given by

$$L_x(t, q(t), q'(t)) - \frac{\mathrm{d}}{\mathrm{d}t} L_v(t, q(t), q'(t)) = 0.$$

where L_x and L_v denote the partial derivatives of L with respect to the second and third arguments, respectively.If the dimension of the space X is greater than 1, this is a system of differential equations, one for each component:

$$\frac{\partial L}{\partial q_i}(t, \boldsymbol{q}(t), \boldsymbol{q}'(t)) - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i}(t, \boldsymbol{q}(t), \boldsymbol{q}'(t)) = 0 \quad \text{for } i = 1, \ldots, n.$$

Euler's Formula:

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{mL^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^3 \times d}{\rho \times b \times d \times 12 \times L^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^2}{\rho \times 12 \times L^4}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

Where

ω_n = Angular frequency of beam

α_n = constant of end condition

E = young's modulus

I = moment of inertia

m = mass of the beam

L = length of the beam

Here the α_n is the constant which depends upon the supports which are provided to the beam. It is different for different types of beam.

3. ANALYTICAL RESULTS

Specimens

Material	Density (Kg/m3)	Young's modulus (GPa)	Poisson's ratio
Aluminum	2700	70	0.32
Mild steel	7850	200	0.29
Copper	8940	117	0.33

1. Aluminium:

Properties:
Young Modulus (N/mm2) = E = 70.
Density (Kg/m3) = ρ = 2700.
Dimension:
Length (mm) = L = 150.
Width (mm) = b = 25.
Thickness (mm) =d= t = 3

Calculation:

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{mL^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^3 \times d}{\rho \times b \times d \times 12 \times L^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^2}{\rho \times 12 \times L^4}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

$$\omega_n = (1.875)^2 \times \sqrt{\frac{0.69 \times 10^{11} \times 0.003^2}{12 \times 2700 \times 0.15^4}}$$

$$= 684.05 \, {}^{rad}/_{\sec}$$

$$= 684.05 \times 0.1592 Hz$$

$$= 108.90 Hz$$

$$f_n = 17.33 Hz$$

2. Mild Steel:

Properities:
Youngs Modulus (N/mm2) = E =200.
Density (Kg/m3) = ρ = 7850.
Dimensions:
Length (mm) = L =150
Width (mm) = b = 30.
Thickness (mm) = t = 25

Calculation:

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{mL^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^3 \times d}{\rho \times b \times d \times 12 \times L^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^2}{\rho \times 12 \times L^4}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

$$\omega_n = (1.875)^2 \times \sqrt{\frac{2.1 \times 10^{11} \times 0.003^2}{12 \times 7850 \times 0.15^4}}$$

$$= 699.88 \, {rad}/{\sec}$$

$$= 699.88 \times 0.1592 Hz$$

$$= 111.42 Hz$$

$$f_n = 17.73 Hz$$

3) Copper :

Properities:
YoungsModulugs (N/mm2) = E = 117.
Density (Kg/m3) = ρ = 3940.
Dimensions:
Length (mm) = L = 150.
Width (mm) = b = 25
Thickness (mm) = t = 3

Calculation-

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{mL^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^3 \times d}{\rho \times b \times d \times 12 \times L^4}}$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \times b^2}{\rho \times 12 \times L^4}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

$$\omega_n = (1.875)^2 \times \sqrt{\frac{1.2\times10^{11}\times0.003^2}{8933\times12\times0.15^4}}$$

$$= 495.95\, {rad}/{\sec}$$

$$= 413.296\times0.1592$$

$$= 78.95Hz$$

$$f_n = 12.56Hz$$

Natural frequency (f_n) in Hz	Materials		
	Aluminum	Mild Steel	Copper
	17.33	17.73	12.56

5. SOFTWARE APPROACH BY ANSYS

The goal of modal analysis in structural mechanics is to determine the natural mode shapes and frequencies of an object or structure during free vibration. It is common to use the finite element method (FEM) to perform this analysis because, like other calculations using the FEM, the object being analyzed can have arbitrary shape and the results of the calculations are acceptable. The types of equations which arise from modal analysis are those seen in eigen systems. The physical interpretation other Eigen and eigenvectors which come from solving the system are that they represent the frequencies and corresponding mode shapes. Sometimes, the only desired modes are the lowest frequencies because they can be the most prominent modes at which the object will vibrate, dominating all the higher frequency modes.

It is also possible to test a physical object to determine its natural frequencies and mode shapes. This is called an Experimental Modal Analysis. The results of the physical test can be used to calibrate a finite element model to determine if the underlying assumptions made were correct (for example, correct material properties and boundary conditions were used.

In structural engineering, modal analysis uses the overall mass and stiffness of a structure to find the various periods at which it will naturally resonate. These periods of vibration are very important to note in earthquake engineering, as it is imperative that a building's natural frequency does not match the frequency of expected earthquakes in the region in which the building is to be constructed. If a structure's natural frequency matches an earthquake's frequency [citation needed], the structure may continue to resonate and experience structural damage. Modal analysis is also important in structures such as bridges where the engineer should attempt to keep the natural frequencies away from the frequencies of people walking on the bridge. This may not be possible and for this reasons when groups of people are to walk along a bridge, for example a group of soldiers, the recommendation is that they break their step to avoid possibly significant excitation frequencies. Other natural excitation frequencies may exist and may excite a bridges natural modes. Engineers tend to learn from such examples (at least in the short term) and more modern suspension bridges take account of the potential influence of wind through the shape of the deck, which might be designed in aerodynamic terms to pull the deck down against the support of the structure rather than allow it to lift. Other aerodynamic loading issues are dealt with but minimizing the area of the structure projected to the oncoming wind and to reduce wind generated oscillations of, for example, the hangers in suspension bridges.

Although modal analysis is usually carried out by computers, it is possible to hand-calculate the period of vibration of any high-rise building through idealization as a fixed-ended cantilever with lumped masses. For a more detailed explanation, see "Structural Analysis" by Ghazi, Neville, and Brown, as it provides an easy-to-follow approach to idealizing and solving complex structures by hand.'·

ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of mechanical problems. These problems include: static/dynamic structural analysis (both linear and non-linear), heat transfer and fluid problems, as well as acoustic and electro-magnetic problems.

In general, a finite element solution may be broken into the following three stages. This is a general guideline that can be used for setting up any finite element analysis.

- Preprocessing: defining the problem; the major steps in preprocessing are given below:
- Define key points/lines/areas/volumes

- Define element type and material/geometric properties
- Mesh lines/areas/volumes as required

The amount of detail required will depend on the dimensionality of the analysis (i.e. 1D, 2D, axis-symmetric, 3D).

- Solution: assigning loads, constraints and solving; here we specify the loads (point or pressure), constraints (translational and rotational) and finally solve the resulting set of equations.
- Post processing: further processing and viewing of the results; in this stage one may wish to see:
  - Lists of nodal displacements
  - Element forces and moments
  - Deflection plots
  - Stress contour diagrams

Preprocessing:

Defining the Problem The simple cantilever beam is used in all of the Dynamic Analysis Tutorials. If you haven't created the model in ANSYS, please use the links below. Both the command line codes and the GUI commands are shown in the respective links.

Solution: Assigning Loads and Solving

1. Define Analysis Type

Solution > Analysis Type > New Analysis > Modal ANTYPE,2

2. Set options for analysis type:

Select: Solution > Analysis Type > Analysis Options..

The following window will appear

{ As shown, select the Subspace method and enter 5 in the 'No. of modes to extract' { Check the box beside 'Expand mode shapes' and enter 5 in the 'No. of modes to expand' { Click 'OK' Note that the default mode extraction method chosen is the Reduced Method. This is the fastest method as it reduces the system matrices to only consider the Master Degrees of Freedom (see below). The Subspace Method extracts modes for all DOF's. It is therefore more exact but, it also takes longer to compute (especially when the complex geometries). { The following window will then appear

For a better understanding of these options see the Commands manual. { For this problem, we will use the default options so click on OK. 3. Apply Constraints Solution > Define Loads > Apply > Structural > Displacement > On Key points Fix Key point 1 (ie all DOFs constrained). 4. Solve the System Solution > Solve > Current LS SOLVE

Post processing:

Viewing the Results

1.Verify extracted modes against theoretical predictions

Select: General Postproc> Results Summary...

The following window will appear

2. View Mode Shapes

Select: General Postproc> Read Results > First Set

This selects the results for the first mode shape

Select General Postproc> Plot Results > Deformed shape . Select 'Def + undef edge'

The first mode shape will now appear in the graphics window.

{ To view the next mode shape, select General Postproc> Read Results > Next Set . As above choose General Postproc> Plot Results > Deformed shape . Select 'Def + undef edge'.

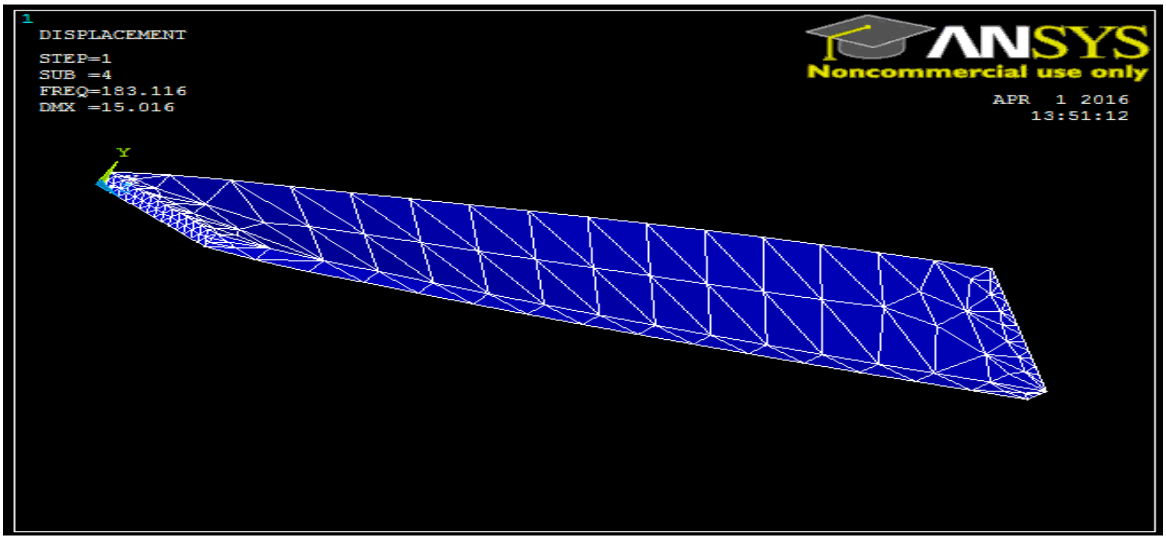
The first four mode shapes should look like the following:

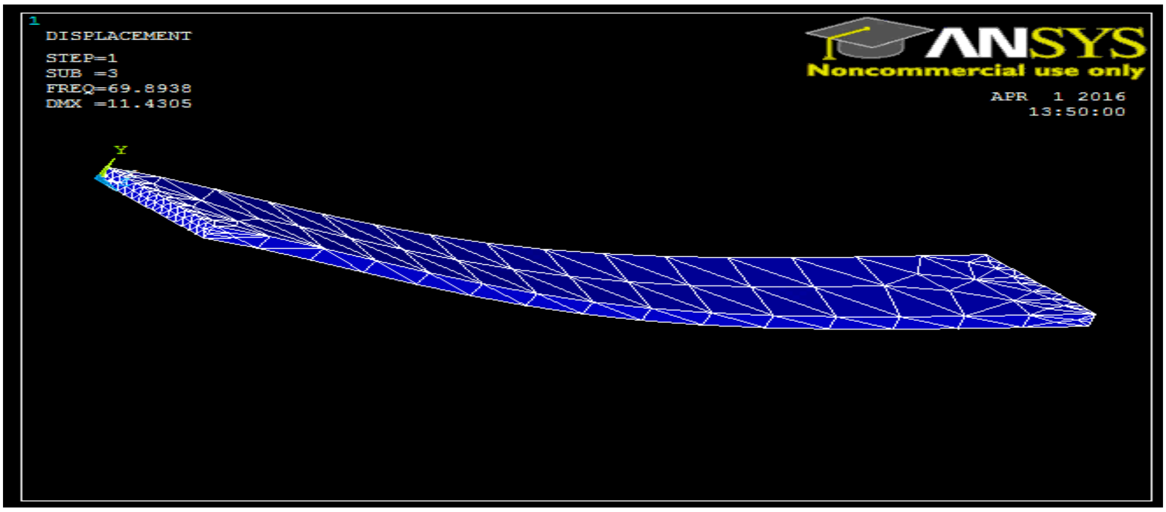
3. Animate Mode Shapes

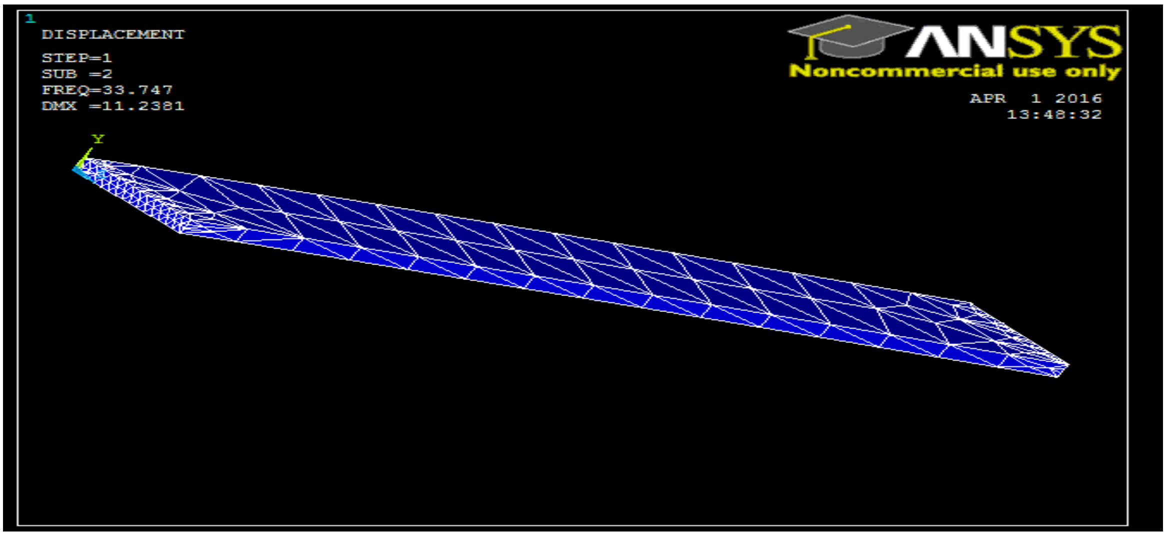
Select Utility Menu (Menu at the top) > Plot Ctrls> Animate > Mode Shap

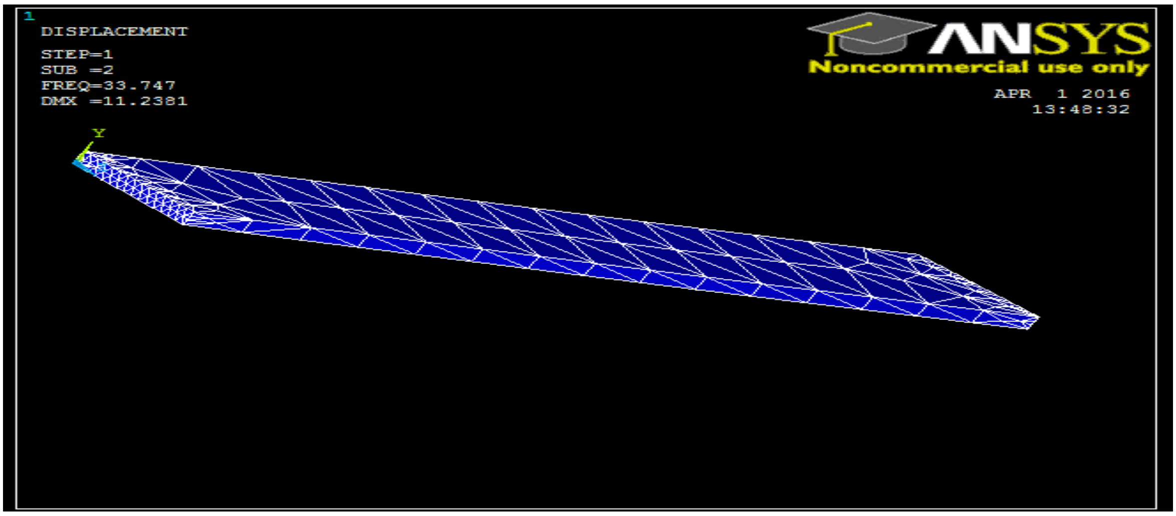
Keep the default setting and click 'OK'
The animated mode shapes are shown below.

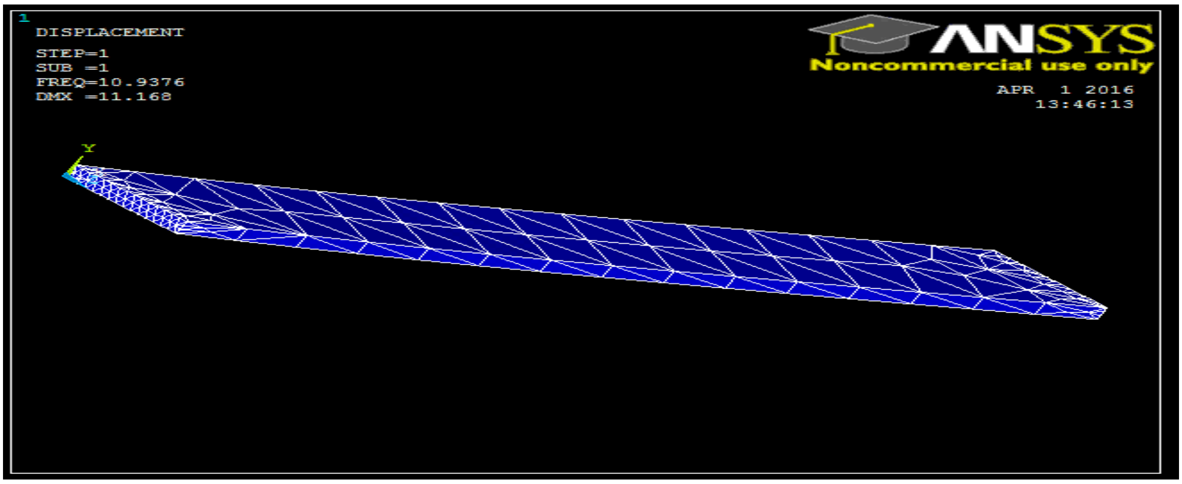
ALUMINIUM-

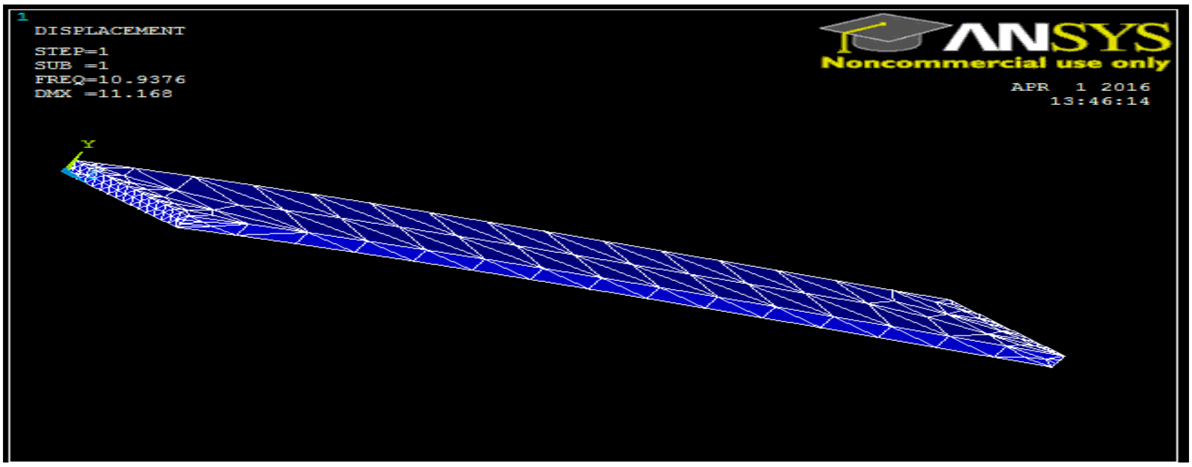












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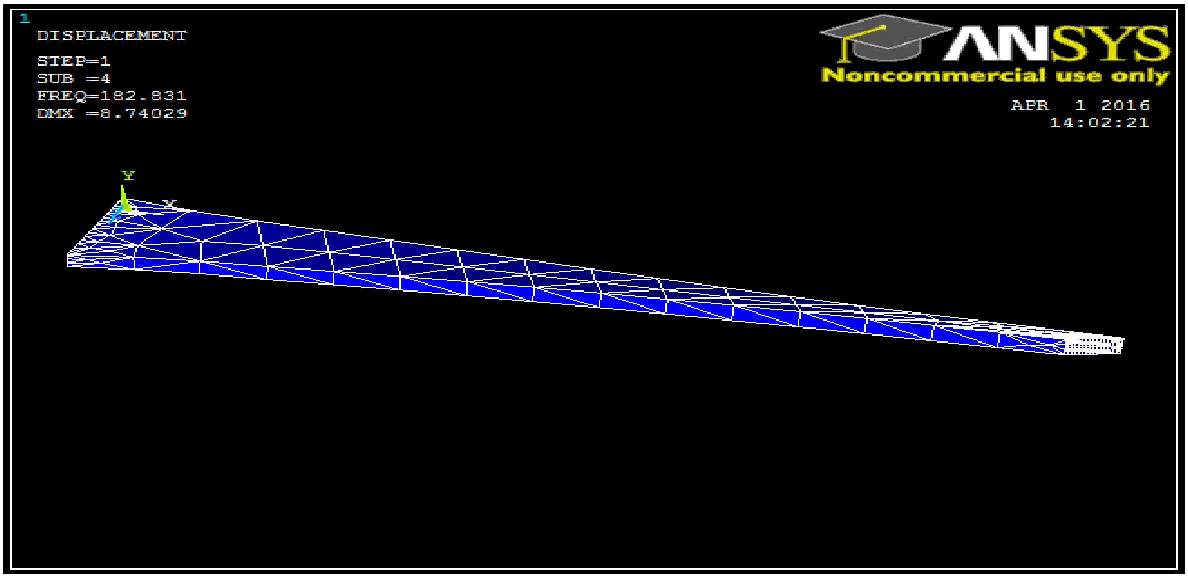
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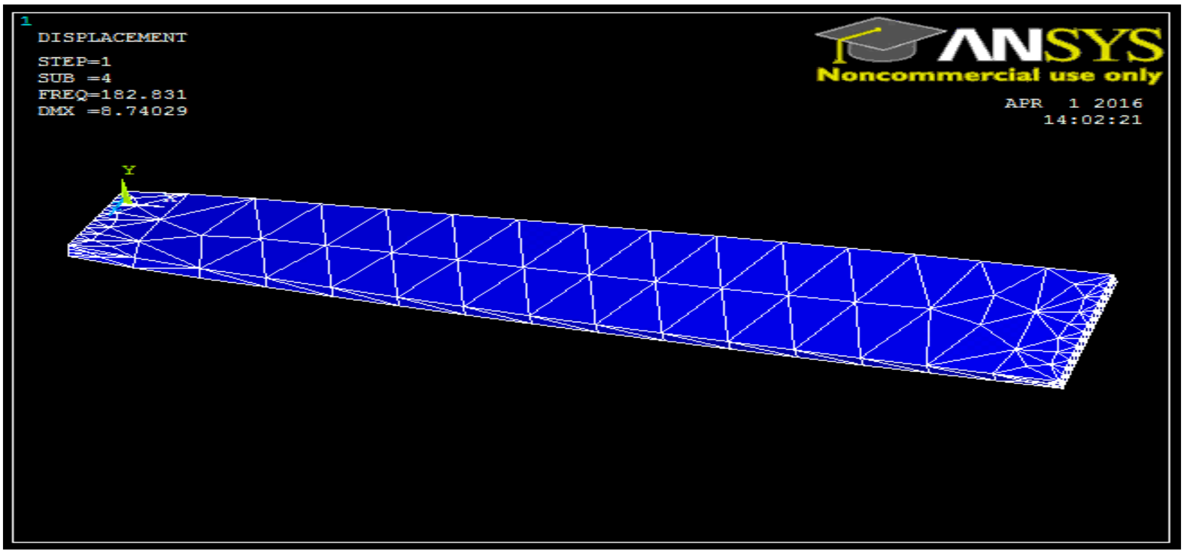
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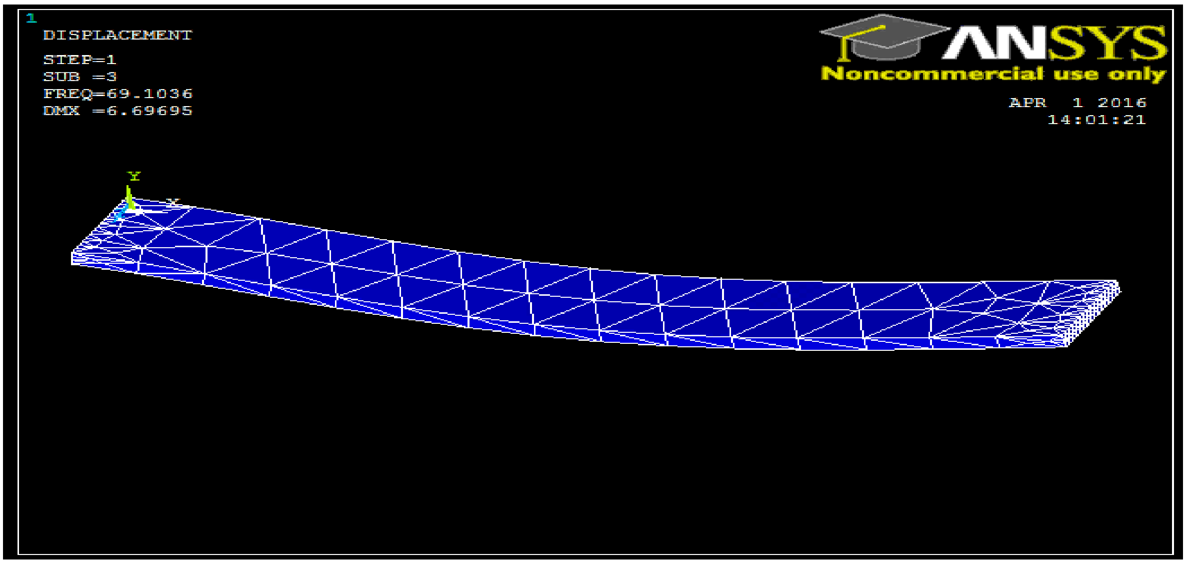
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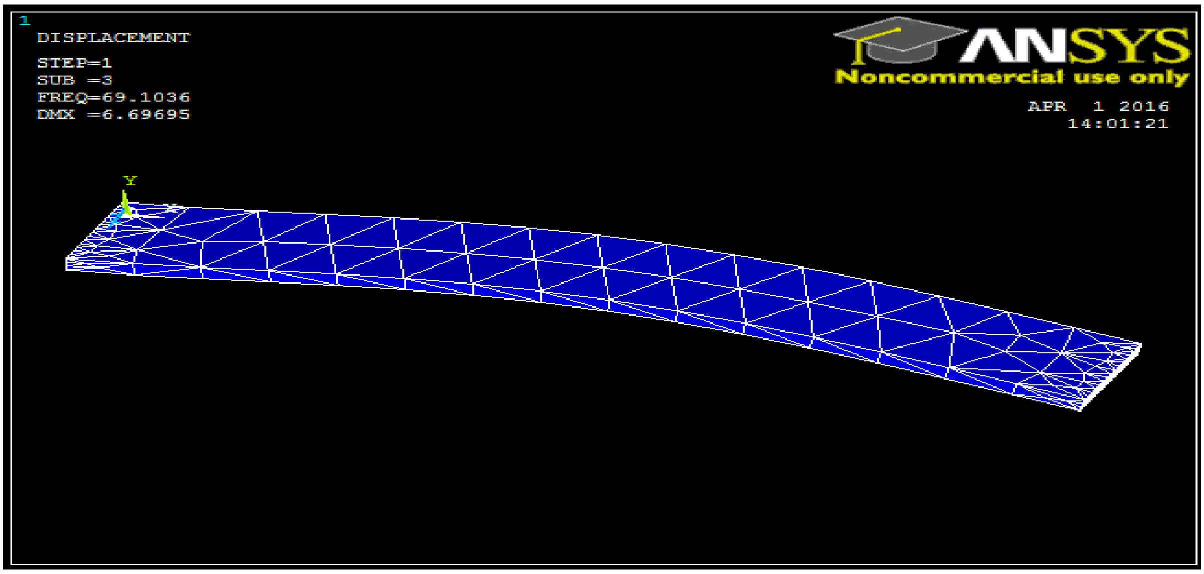
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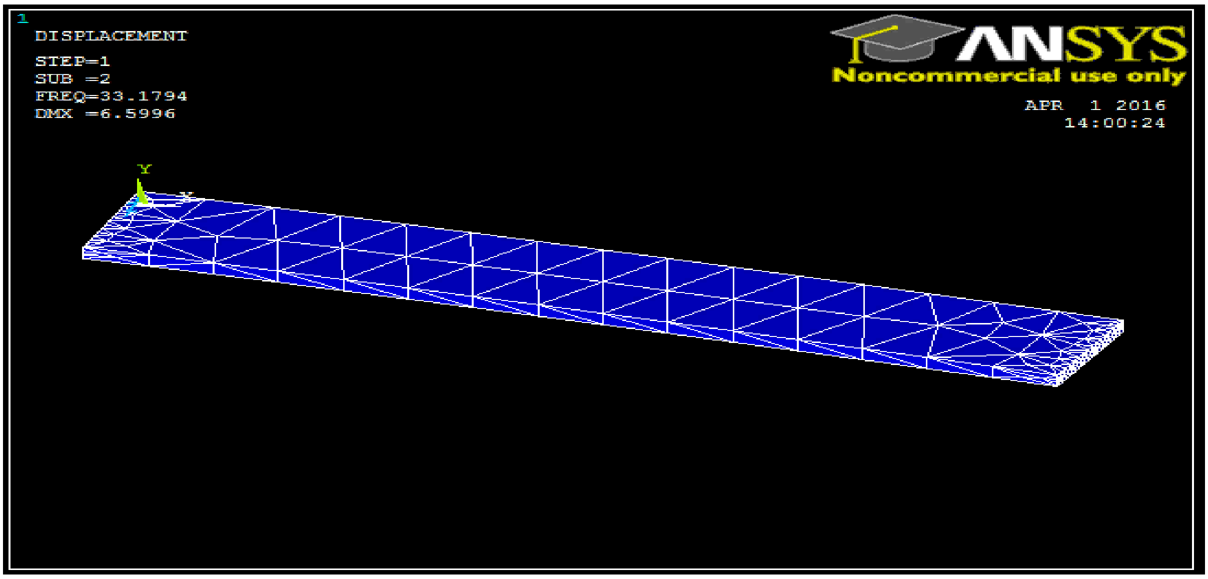
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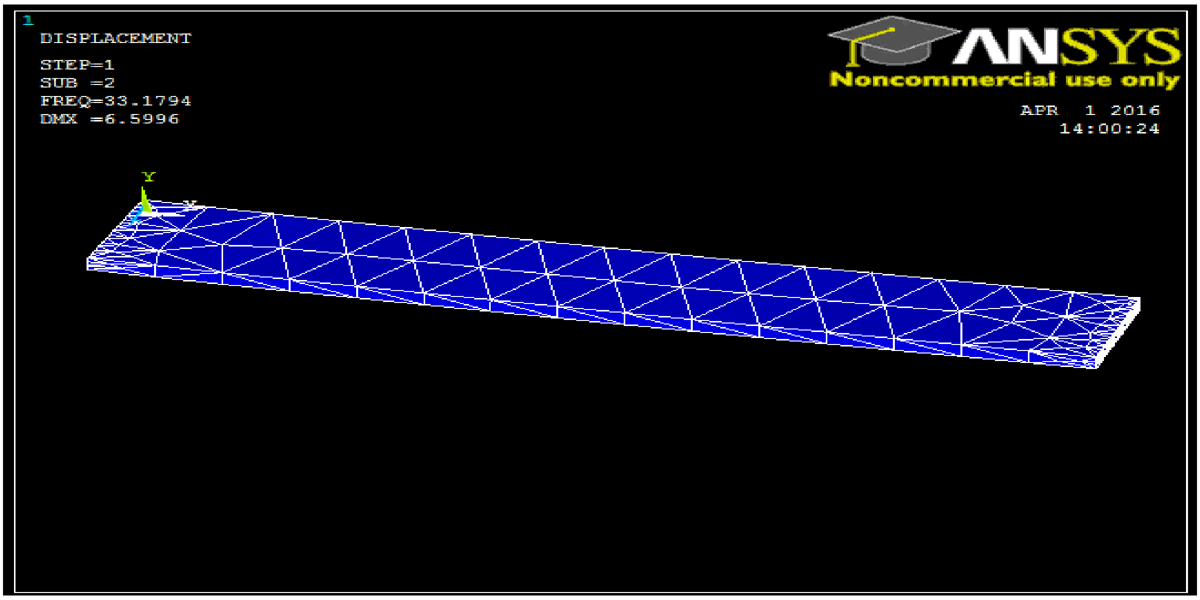
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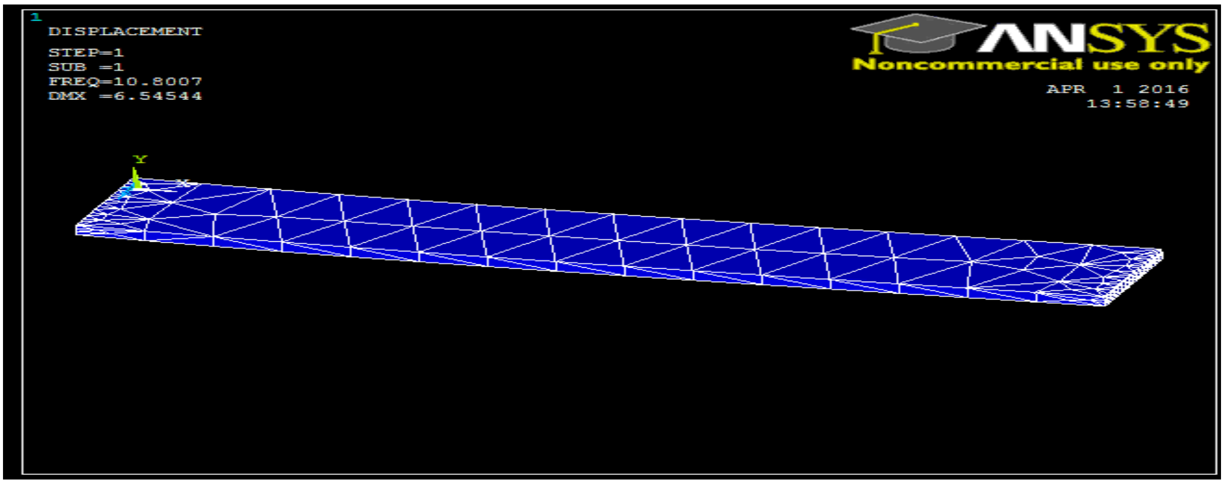

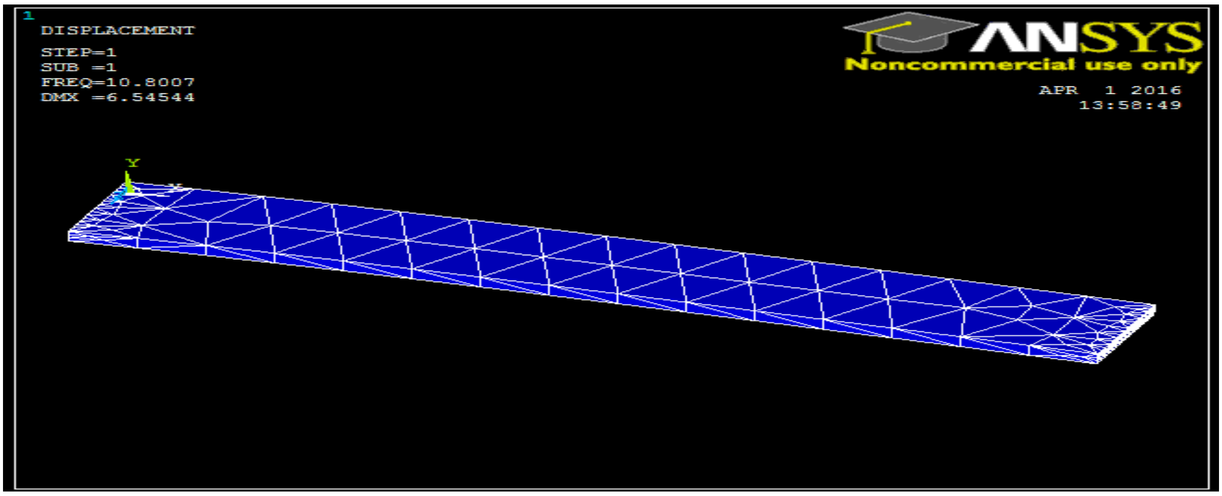

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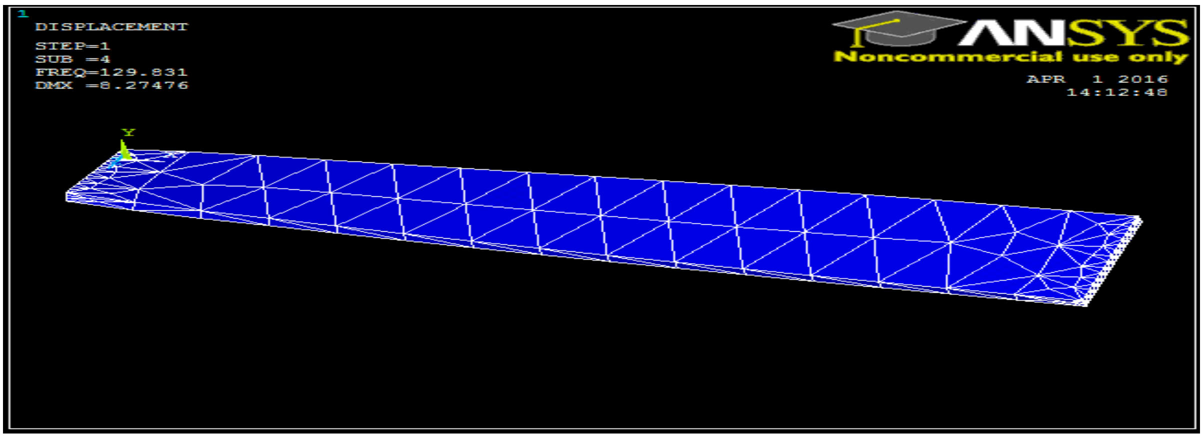

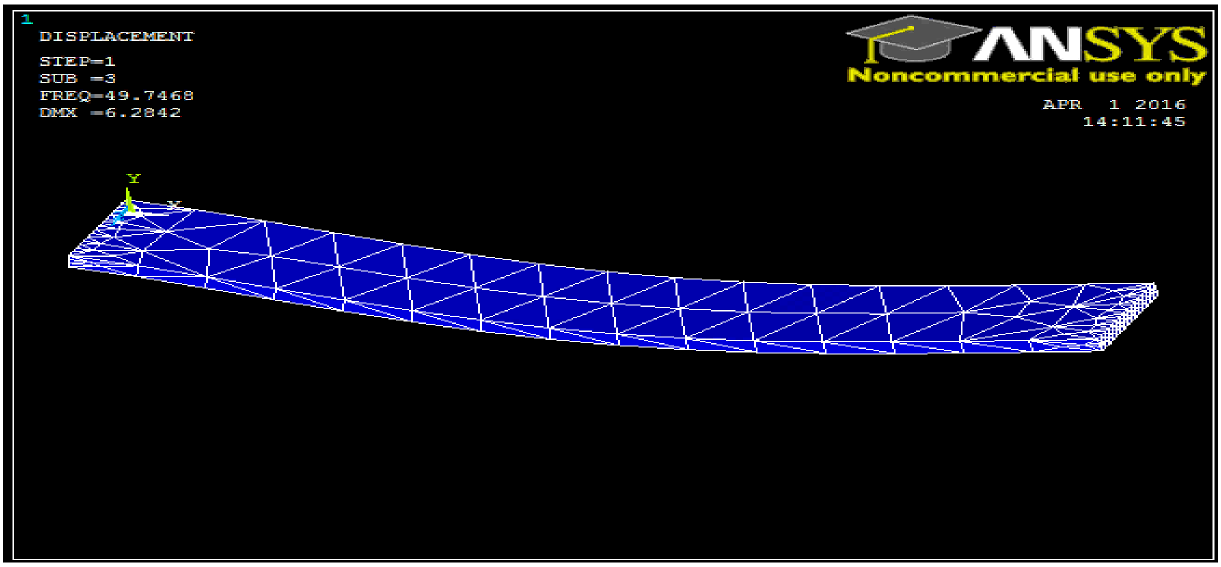
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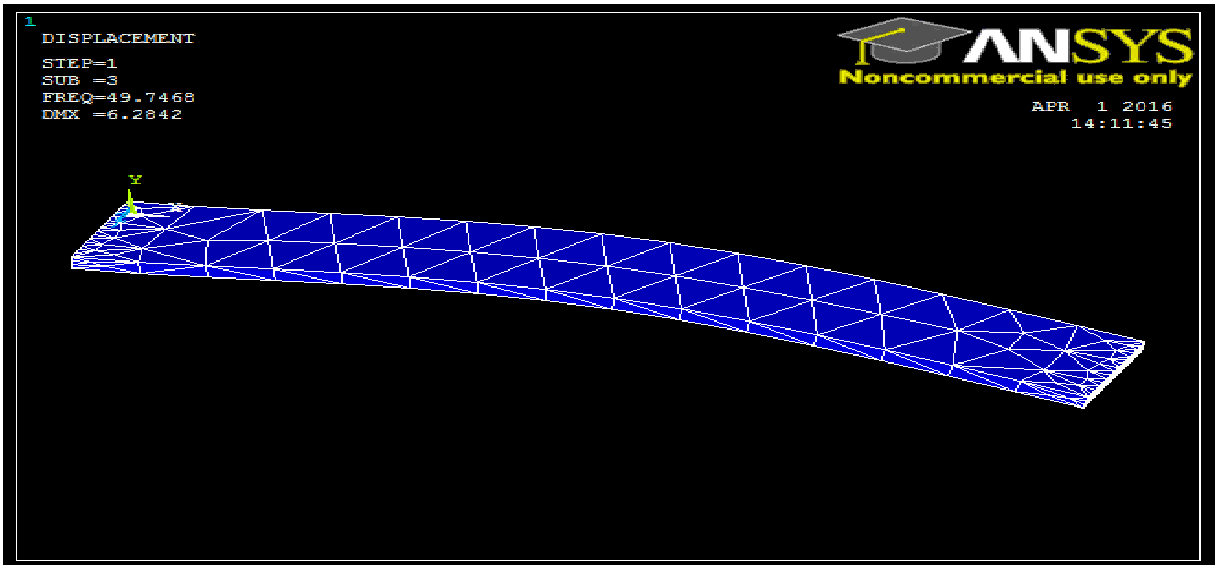
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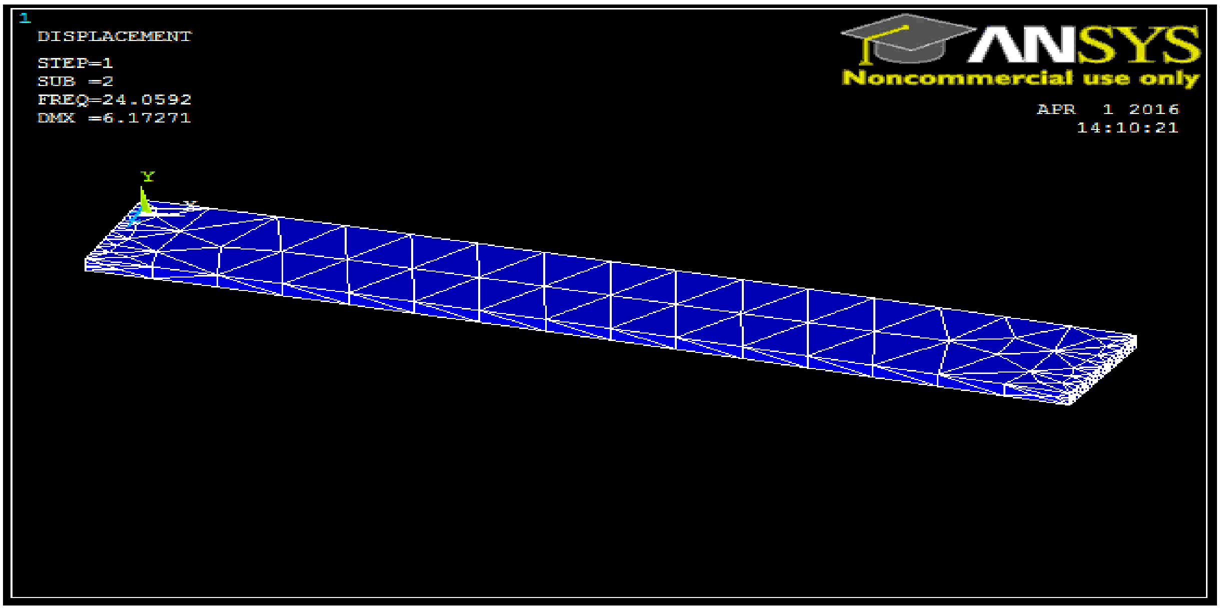
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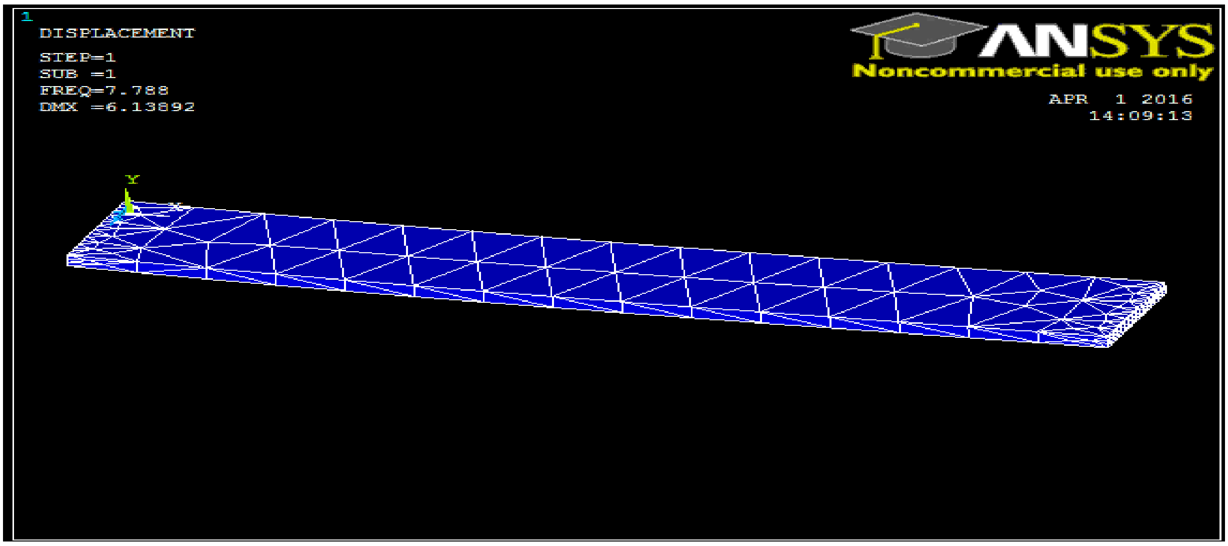

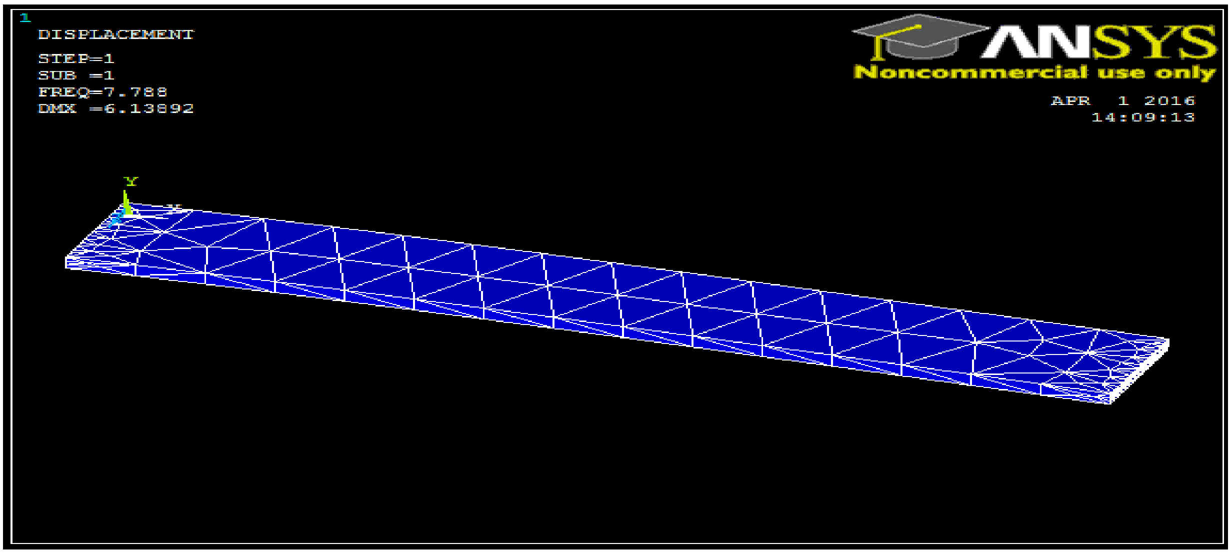

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4	129.83	1	4	4

Natural frequency (f_n)	Materials		
	Aluminium	Mild Steel	Copper
	13.5	15	12.2

6. CONCLUSION

Materials	Analytical	Experimental
Aluminum	17.33	13.5
Mild Steel	17.73	15
Copper	12.56	12.5

1. The resonant frequency obtained experimentally at the first mode of vibration of all three specimens can be compared with theoretical result.

2. There is good agreement of the theoretical calculated natural frequency with the experimental one.

3. From the experiment it is found that damping is higher for copper as compared with aluminum and steel and damping of aluminum was found to be lowest.

4. The material damping decreases with increase in natural frequency of cantilever specimen for each material.

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