International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 05 Issue: 12 | Dec 2018 www.irjet.net p-ISSN: 2395-0072

Sensitivity Analysis Study of CVT Parameters using Mathematical Model

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Abstract – *The paper presented here discusses one of the most valuable invention in the field of automobile engineering, the **Continuously Variable Transmission** or the **CVT.** Although the invention of CVT dates back to 1490s there is a lot more to learn about this technology. This paper will present the current problem of manufacturing and assembly of the CVT setup of a two wheeler, and also discuss how a small variation in the CVT parameter such as length of V-belt, mass of the rollers, spacer length, angle between the pulley, center distance between the pulley, even within the tolerance range can cause drastic changes in the performance of the vehicle. This paper will also study how and what kind of different combinations of above mentioned CVT parameters(keeping one or the other parameters constants) will cause different effects on vehicle performance parameters such as fuel efficiency, acceleration time , reliability etc. All of this will be studied using a mathematical model, such that various outputs can be generated for various inputs at least possible time, and plotting these results as a graphical representation.*

Key Words: **CVT performance**, **Mathematical Model**, **Microsoft Excel**, **Roller Mass**, **Spacer Length**, **Pulley**, **V-Belts.**

1. INTRODUCTION

Continuously Variable Transmission or simply CVT is the most common power transmission system in the field of automobile. In general, there are two pulleys that are divided perpendicular to their axis of rotation, and with a V-belt running in between them. The gear ratio can be automatically changed by moving the two sections of one pulley closer together and the two sections of the other pulley little bit apart. Due to the V-shaped cross section of the belt, this causes the belt to ride higher on one pulley and lower on the other. Because of this there are changes in the effective diameters of the pulleys, which changes the overall gear ratio. The distance between the two pulleys cannot change, and neither does the length of the V- belt changes , so changing the gear ratio means both pulleys must undergo adjustment (one can bigger, the other smaller, or both can be of same size) simultaneously to maintain the proper amount of tension on the belt. The V-belt needs has to be very stiff in the pulley's axial direction in order to make only small radial movements while moving in and out of the pulleys.

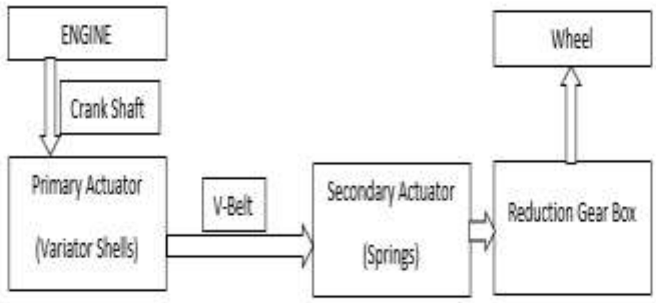


Fig -1: General Layout of the CVT.

COMPONENTS OF A CVT SYSTEM

- A V type rubber belt
- A variable-input "driving" pulley
- An output "driven" pulley

Fig -2: Components of CVT

The V- belt and the variable-diameter pulleys are the main part of the CVT system. Each pulley is made of two 23-degree cones facing each other. A belt drives in the seams between of the two cones of the pulleys. V-belts are preferred if the belt is made of rubber. When the two cones of the pulley are far apart (i.e, the diameter increases), the belt drives lower in the seams, and the radius of the belt loop going around the pulley gets smaller. When the cones are close together (i.e., the diameter decreases), the belt drives higher in the seams, and the radius of the belt loop going around the pulley gets larger. CVTs may use hydraulic pressure, centrifugal force or spring tension to create the force necessary to adjust the pulley halves. Variable-diameter pulleys must always come in pairs. One of the pulleys, known as the drive pulley (or driving pulley), is connected to the crankshaft of the engine. The driving pulley is also called the input pulley because it's where the energy from the engine

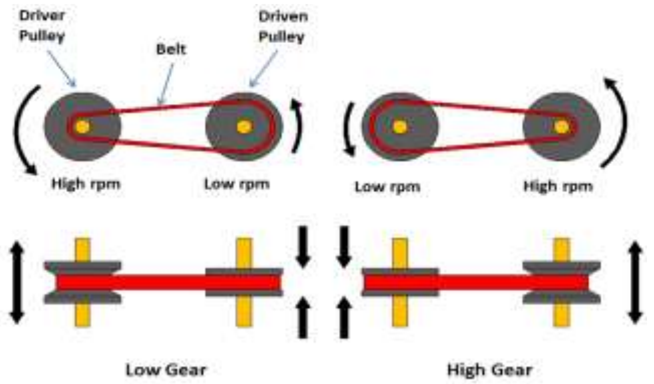


Fig -3: Working Principle of CVT

Enters the transmission. The second pulley is called the driven pulley because the first pulley is turning it. As an output pulley, the driven pulley transfers energy to the driveshaft. When one pulley increases its radius, the other decreases its radius to keep the belt tight. As the two pulleys change their radii relative to one another, they create an infinite number of gear ratios -- from low to high and everything in between. When the pitch radius is small on the driving pulley and large on the driven pulley, the rotational speed of the driven pulley decreases resulting in a lower gear ratio. When the pitch radius is large on the driving pulley and small on the driven pulley, then the rotational speed of the driven pulley increases, resulting in a higher gear ratio. Thus, in theory, a CVT has an infinite number of "gears" that it can run through at any time, at any engine or vehicle speed. The simplicity and steeples nature of CVTs make them an ideal transmission for a variety of machines and devices, not just automobiles. CVTs have been used for years in power tools and drill presses. They've also been used in a variety of vehicles, including tractors, snowmobiles and motor scooters. In all of these applications, the transmissions have relied on high-density rubber belts, which can slip and stretch, thereby reducing their efficiency. The distance between the centers of the pulleys to where the belt makes contact in the seams is known as the pitch radius. When the pulleys are far apart, the belt rides lower and the pitch radius decreases. When the pulleys are close together, the belt rides higher and the pitch radius increases.

1.1 LITERATURE REVIEW

1.1.1 The Kim Kwangwon & Hyunsoo Kim Model

According to this method of modelling, the meandering radius on both the actuators must undergo the changes because of the belt force differentiation. The experimental research results, described in the work [4], indicate that such belt behaviour occurs only for the driven wheel. For the driving wheel, the meandering radius remains same on the complete wrap angle. This behaviour can be explained with the help of self-locking phenomenon occurrence.

As a result, it is presumed that the force exerted by the belt is constant. Thus, the circumferential friction component is neglected. For the primary (driving) actuator, only the radial static friction is assumed. Thus, single contact area is considered. The total axial force for the primary actuator is expressed by equation (3) and for the secondary actuator the two contact areas are assumed.

In the passive contact area (at the entrance), the belt tension is constant. In this area the belt element forces distribution is familiar with the belt element forces distribution on the primary actuator. The kinetic force in radial and circumferential direction is considered in the active contact area. The total axial force of the secondary actuator is expressed by equation (4).

1.1.2 The Cammalleri model

The main difference between the Kim-Kim described models and the Cammalleri model [1] lies in the consideration of the belt flexibility and the changes of the slip angle along the pulley wrap angle.

1.2 OBJECTIVES

1. Create a Mathematical Model of the sensitivity analysis of the CVT parameter.
2. Verification of the Mathematical Model with the practical values.
3. Plot graphs for different parameters and analysis of how the changes in the CVT parameters effects the vehicle performance.

2. NOMENCLATURES

1. F_R Axial Force Produced by Rollers
2. M Mass of Rollers
3. y_g Distance of Roller Mass center G from pulley axis
4. β Angle Formed by the Pulley Axis and the Tangent To the Curved Profile at Contact Point
5. Υ Angle of the Back Plate with Roller
6. φ_{sx} Angle of Sliding Friction between Roller and Ramp
7. φ_{dx} Angle of Friction of the Guide
8. F_0 Pre Load on the Secondary Actuator Spring
9. K Stiffness of the spring
10. D Mean Diameter of Spring
11. d Mean Diameter of the Helical Guide
12. ζ Angle of Helical Guide Slope
13. V Poisson's Ratio
14. Ca Torque at Primary Actuator
13. C_b Torque at Secondary Actuator
14. F_r Axial Force of the Driver (Primary) Actuator
15. F_n Axial Force of the Driven (Secondary) Actuator
16. T_1 Tension on Tight Side of the Belt
17. T_2 Tension on Slack Side of the Belt
18. θ_a Active Wrap Angle
19. θ Length of Arc of Contact
20. α V-Belt Wedge Angle
21. μ Co-efficient of Friction between Belt and Actuator
22. R_a Active Radius of Driver (Primary) Actuator
23. R_b Active Radius of Driven (Secondary) Actuator

3. METHODOLOGY

The tests was carried out on a Dynamometer test rig.
The test gives us the following outputs
i. The engine RPM with respect to the vehicle speeds at different loading conditions.
ii. The wheel force.
Using these values as input we determine the various parameters that affects the CVT performance.

I. The Primary Actuator.

The primary actuator or the primary pulley is directly connected to the crank shaft of the engine and thereby it is assumed that the speed of the primary actuator (n_a) is equal to the engine speed in rotations per minute (RPM).

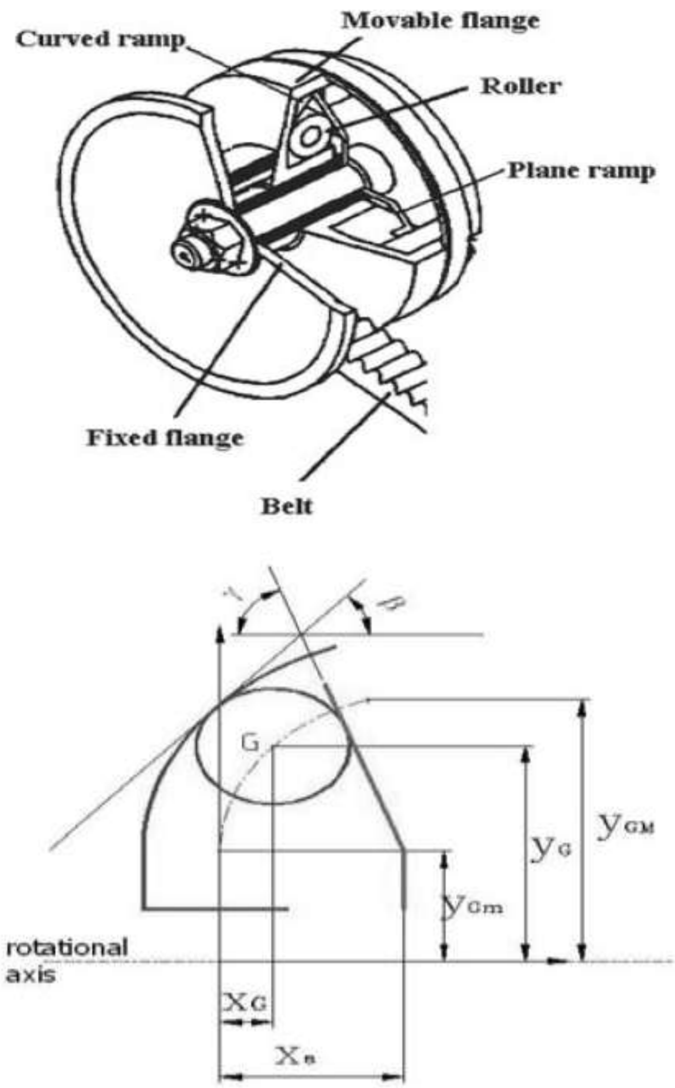


Fig -4: Primary Actuator

This actuator consists of number of centrifugal mass rollers which are placed on a curved profile called ramp, so as the speed of the engine increases which indeed increases the speed of the primary actuators and vice versa, there is a centrifugal force which is acting on these rollers which creates an axial force and acts on the back plate, thereby pushing the back plate forward or backward depending on the speed of the actuators and thereby varying the radius of the belt. The free body diagram of the roller is given in the figure below.

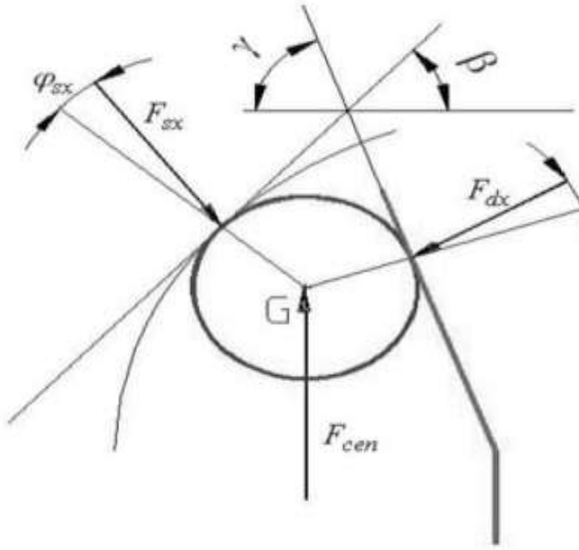


Fig -5: Free Body Diagram

The *figure 2* shows FBD for the case upshift i.e, increase in the speed ratio. Where F_{cen} is the centrifugal force acting on the rollers. Imposing radial, axial, and rotational equilibrium, it is possible to write:-

$$\uparrow \quad F_{cen} = F_{dx}\cos(\gamma-\varphi_{dx}) + F_{sx}\cos(\beta-\varphi_{sx}) \qquad \text{\_\_\_(1)}$$

$$\leftarrow \quad F_{dx}\sin(\gamma-\varphi_{dx}) = F_{sx}\sin(\beta-\varphi_{sx}) \qquad \text{\_\_\_(2)}$$

$$\circlearrowleft \quad F_{dx}\sin(\varphi_{dx}) = F_{sx}\sin(\varphi_{sx}) \qquad \text{\_\_\_(3)}$$

As $F_{cen}=Mn^2_a y_g$ and $F_R= F_{dx}\sin(\beta-\varphi_{sx})$, the equations 1 and 2 gives the axial forced produced by the centrifugal rollers and is given by the below equation

$$F_R = \frac{M \cdot n_a^2 \cdot y_g}{COT(\gamma-\varphi_{dx}) + COT(\beta-\varphi_{sx})} \qquad \text{\_\_\_(4)}$$

From equations 2 and 3 the roller motion is given by

$$\frac{Sin(\gamma-\varphi_{dx})}{Sin(\varphi_{dx})} = \frac{Sin(\beta-\varphi_{sx})}{Sin(\varphi_{sx})} \qquad \text{\_\_\_(5)}$$

It is possible to come to a conclusion that,
for $\beta > \gamma$,

$\varphi_{sx} = \varphi_a > \varphi_{dx}$

[φ_{dx} from equation (5)] and that the roller rolls on the back plate and slips on the left ramp.
While, for $\beta < \gamma$

$\varphi_{dx} = \varphi_{\rho}$
[φ_{sx} from equation (5)] and the roller rolls on the left ramp and slips on the back plate.
(Where φ_a is the angle of sliding friction between the roller and the ramp).
And is given by the equation:-

$$\varphi_a = Tan^{-1}(\mu_s)$$

In the case of downshift, the frictional forces reverse their angles φ_{dx} and φ_{sx} and the directions change their sign in the above equations, but all the results on the roller motion remain the same.

Y_g is determined with the help of CAD drawing of the primary actuator by finding 3 different roller position from the rotational axis to the centre of the roller and also the angle tangent to the ramp (α). Then using polynomial equation the roller position along with tangent angle can be found at any instant.

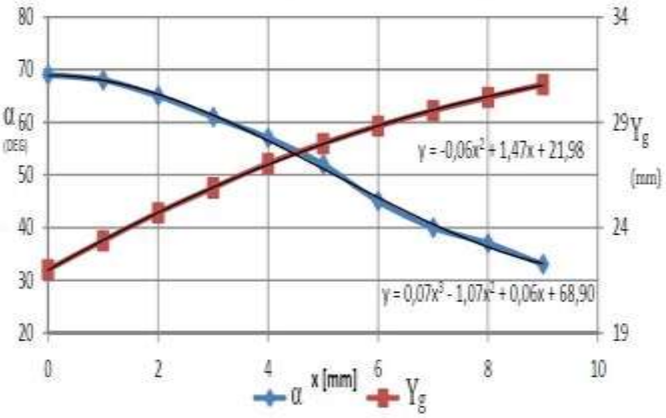


Chart -1: Determining the position and angle of roller

II. V-Belt

The V-belt is the linking member between the primary and secondary actuator. It is known that the belt is elastic in nature and due to this property of the belt, it penetrates into the side walls of the actuators and due to which there is a radial component of frictional force that comes into action and this plays an important role in the V-belt mechanics.

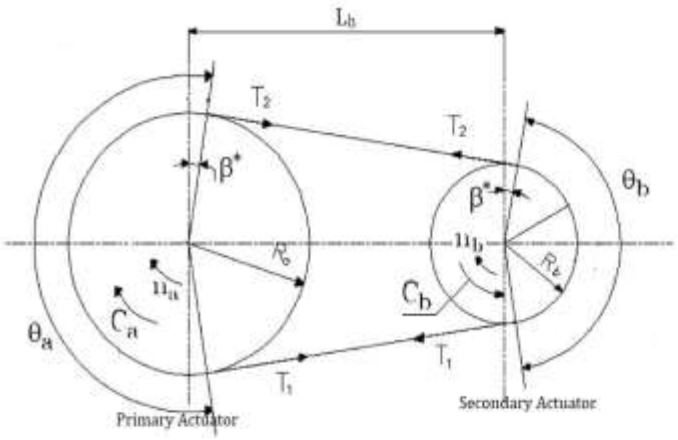


Fig -6: V-Belt Geometry

Using the Speed Ratio-Torque and Thrust Relationship
The axial force that is developed by the belt on the primary actuator can be given by the equation below

$$F_r = \frac{T_1}{2} \cdot \theta_r \left[\frac{1 - \mu \cdot Tan\left(\frac{\alpha}{2}\right)}{1 + Tan\left(\frac{\alpha}{2}\right)} \right] \qquad \text{(6)}$$

Also the axial force that is developed on the secondary actuator due to the V-belt is given by the equation

$$F_n = \frac{T_2}{2} \cdot \left(\theta_n - \theta_a\right) \cdot \left[\frac{1 - \mu \cdot Tan\left(\frac{\alpha}{2}\right)}{\mu + Tan\left(\frac{\alpha}{2}\right)} \right] + \frac{T_1 - T_2}{2\mu} \cdot Cos\left(\frac{\alpha}{2}\right) \qquad \text{(7)}$$

Where,

$$\theta_a = \frac{1}{\mu} \cdot Ln\left(\frac{T_1}{T_2}\right) \cdot Sin\left(\frac{\alpha}{2}\right) \qquad \text{(8)}$$

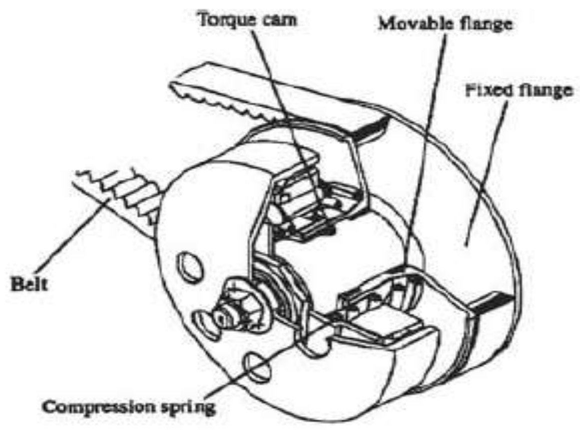
Using the value of F_r obtained in equation 4 and substituting it in the equation 6 to obtain the value of T_1. Then by using the belt equation given below

$$\frac{T_1}{T_2} = e^{\frac{\mu \cdot \theta_r}{Sin(\alpha)}} \qquad \text{(9)}$$

Finding the value of T_2 from the value obtained by equation 6. Then by substuting the value of T_1 and T_2 obtained from equations 6 and 8 respectively into equation 7 to find the value of the axial force developed by the V-belt on the secondary actuator.

III. Secondary Actuator

The axial force that is needed to assure the connection between the V-belt and the secondary actuator is generally produced by a helical spring compressed in between the half-secondary movable pulley and a fixed contrast wall. In addition , in motorcycle actuators , an extra force which is proportional to the output torque is generated by the slope of the sliding guides of a pre-defined constant angle ζ as shown in the figure below.



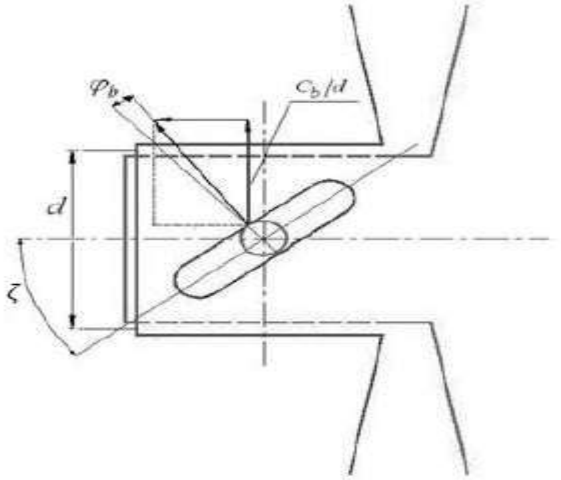


Fig -7: Secondary Actuator

The equation for axial force equilibrium of the movable half actuator is given by

$$F_n = \left(F_O + K\Delta x_b\right) + \frac{C_b}{d} Tan\left(\zeta \pm \phi_b\right) \quad \_\_\_(10)$$

Depending on the constraints on the spring, the half-actuator movement may cause some torsional effect on the spring. In such situation the reaction torque produces an extra axial thrust, which is the function of speed ratio. All together, the equation of the actuator axial force remains the same provided that the spring stiffness is given by

$$K_{eq} = \left[1 + (1 + V) \cdot \left(\frac{D \cdot Tan\ \zeta}{d}\right)^2\right] \cdot K \quad \_\_(11)$$

The torque that is generated at the secondary actuator (C_b) is obtained by substuting the value of F_n from equation 7 in equation 10. As the output of the dynamometer test is measured in terms of the wheel force, the torque obtained from the equation 10 need to be converted into wheel force, for which torque is multiplied by the reduction gear ratio and divided by the dynamic rolling radius of the wheel. Using all the formulas above an excel table is formulated in order to find various parameters at different speeds of the vehicle.

4. RESULTS

At first it is important to verify that the mathematical model hold true or not, for that a dyno test was conducted on the standard vehicle and results were compared with the mathematical model and a graph representing this comparison is shown below:-

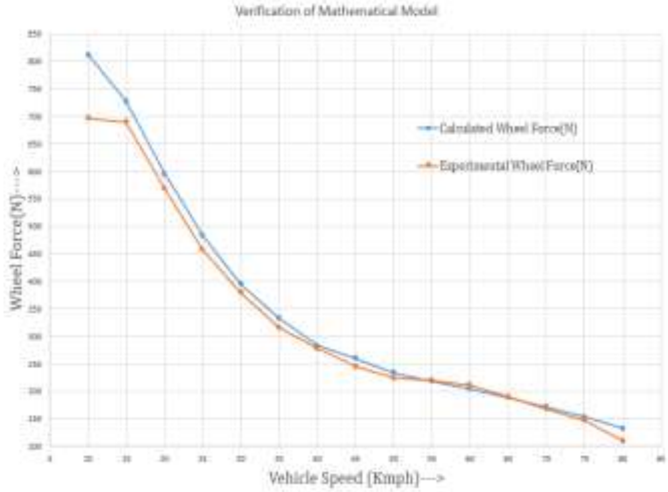


Chart -2: Verification of the Mathematical Model

As it can be seen the blue line represents the mathematical model values and orange line represents experimental values and both of them are comparable, for speeds upto 15Kmph the values divert a lot due to slippage of the belts on the actuators. Now we can compare various CVT parameters using this mathematical model.

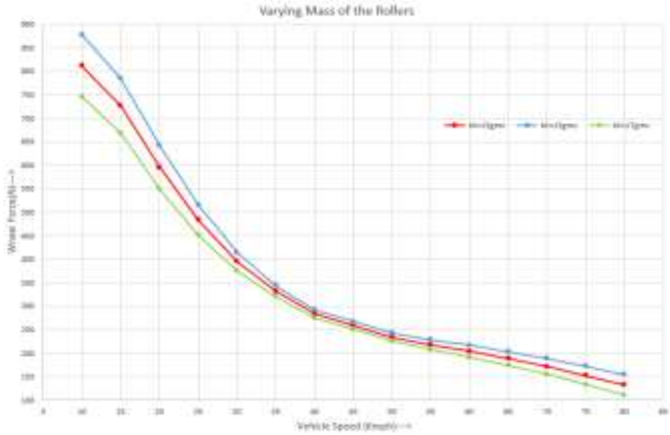


Chart -3: Varying the mass of the rollers

Similarly all other parameters such as Length of the V-belt, Pulley wedge angle, center distance between the pulley, spacer length and spring stiffness can varied in the mathematical model and can be studied for each and every combination of the parameter to obtain the best results in terms of power or torque transfer depending on the users need without conducting the actual test by varying each parameters.

5. CONCLUSIONS

After carrying out various test by varying the combination of the CVT parameters the following conclusion are drawn:-

1. Mass of the rollers: - The wheel force increases by decreasing the mass of the rollers and vice versa.
2. Length of Belt: - The wheel force decreases with increase in the length of V-Belt, in other words wheel force is inversely proportional to the length of belt.
3. The Wheel Force increases with increase in the wedge angle of the pulley, center distance of the pulley and spring stiffness.

ACKNOWLEDGEMENT

We would like to thank TVS Motor Company, Hosur for letting us carry out our research at their state of the art Research and Development center, and guiding us in using advanced equipments.

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