# Analysis of Steel Space Frames

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**Abstract -** *The most common reason for analyzing a rigid-jointed frame is to work out the moments, shears, and axial forces acting on its members and joints. These could then be used with the design rules to work out if the members and joints are adequate. Frame analysis can be approached by variety of methods. First order elastic analysis is most common but least complete. When First order elastic analysis takes into account any important second order effects arising from the finite deflections of frame then it is known as Second order elastic analysis.*

*A less common reason for analysing a rigid-jointed frame is to work out whether or not the frame will reach an equilibrium position under the factored loads, in which case the frame is adequate. A first-order inelastic (plastic) analysis is also used for a frame with negligible second-order effects. When there are larger second-order effects, then a advanced analysis (second-order inelastic analysis) may be carried out in which account is taken of second-order effects, plastic behaviour, and residual stresses and geometrical imperfections, although this can be rarely done in practice.*

*As IS 800:1984 has changed recommend actions from working state to limit state design in year2007, this required modification in analysis from elastic to inelastic. Present commercial software's are not incorporating inelastic analysis i.e plastic analysis in the software hence the use of limit state design has been declined by structural designer. In this project study review of various methods of analysis of steel space frame is studied and attempt is made to establish relation between analysis results of first order elastic and second order inelastic analysis. This direct relation with the use of suitable multipliers which help structural designer to use codal provision of IS800:2007 in more convenient manner. It is necessary to make advanced analysis user friendly which results effective solution and economy for steel space frames.*

***Keywords****:* space frame, elastic analysis, plastic analysis, multi-bay-multi-story frame, steel structure etc.

## 1. GENERAL

The most common reason for analysing a rigid-jointed frame is to work out the moments, shears, and axial forces acting on its members and joints. These could then be used with the design rules to work out if the members and joints are adequate.

Such an analysis should allow for any important second-order moments arising from the finite deflections of the frame. Two methods may be applied to allow these second-order moments. Within the first of those, the moments determined by a first-order elastic analysis are amplified by using the results of an elastic buckling analysis. In the second and more correct method, a full second-order elastic analysis is carried out of the frame.

A less common reason for analysing a rigid-jointed frame is to work out whether or not the frame will reach an equilibrium position under the factored loads, in which case the frame is adequate. A first-order plastic analysis is also used for a frame with negligible second-order effects. When there are larger second-order effects, then a advanced analysis (second-order plastic analysis) may be carried out in which account is taken of second-order effects, plastic behaviour, and residual stresses and geometrical imperfections, although this can be rarely done in practice.

These various methods of analysis are mentioned in more detail within the following sub-sections.

### 1.1 First-order elastic analysis

A first-order (linear) elastic analysis of a rigid-jointed frame relies on the assumptions that:

a) the material behaves linearly, so all yielding effects are neglected,
b) the members behave linearly, with no member instability effects like those caused by axial compressions that reduce the members' flexural stiffness's (these are usually known as the P-δ effects), and
c) the frame beha ves linearly, with no frame instability effects like those caused by the moments of the vertical forces and also the horizontal frame deflections (these are usually known as the P-Δ effects).

For example, for the portal frame of Figure 2, a first-order elastic analysis ignores all second-order moments like $R_R(\delta + \Delta z/h)$ in the right-hand column, so that the bending moment distribution is linear in this case. First-order analyses predict linear behaviour in elastic frames, as shown in Figure 1.

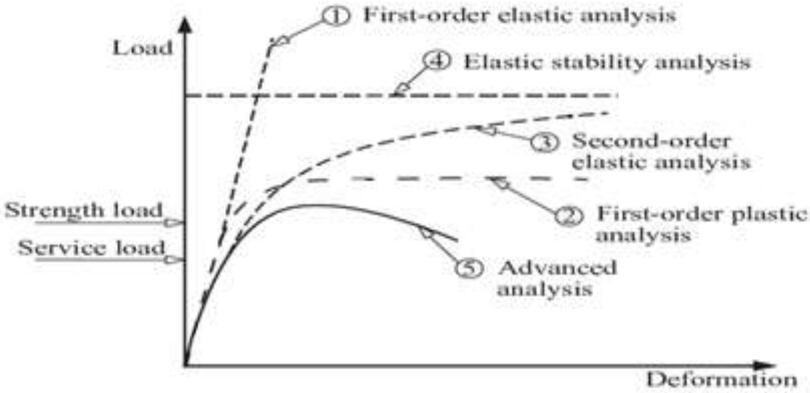


Fig 1: Predictions of structural analyses

Rigid-jointed frames are invariably statically indeterminate, and while there are number of manual methods of first-order elastic analysis available, these are labour-intensive and error-prone for all but the simplest frames. In the past, designers were usually forced to rely on approximate methods or available solutions for specific frames. However, computer methods of first-order elastic analysis have formed the basis of computer programs that are currently used extensively. These first-order elastic analysis programs need the geometry of the frame and its members to have been established (usually by a preliminary design), and then compute the first-order member moments, forces, and the joint deflections for every specified set of loads. Because these are proportional to the loads, the results of individual analyses may be combined by linear superposition.

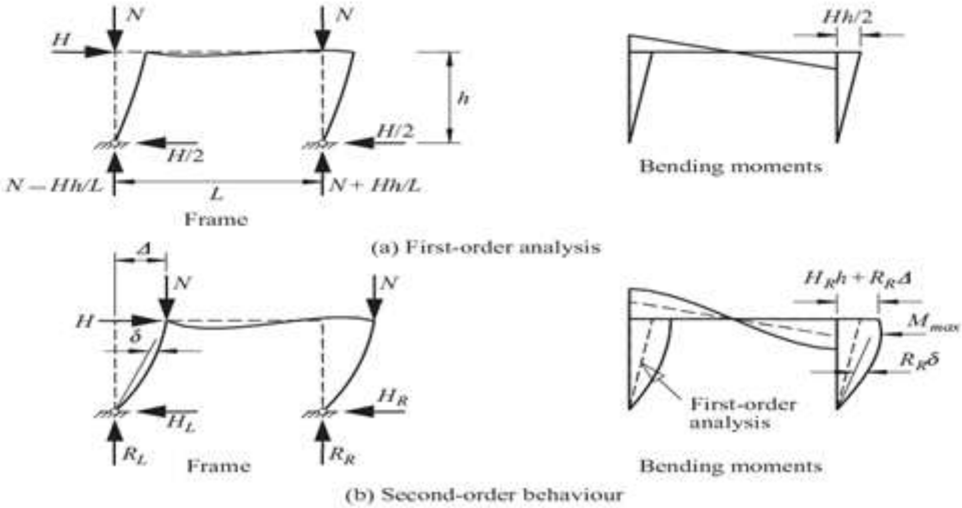


Fig 2: First-order and second-order behavior

### 1.2 Second-order elastic analysis

Second-order effects in elastic frames take account of additional moments like $R_R$ $(\delta + \Delta z/h)$ in the right-hand column of Figure 2 that results from the finite deflections δ and Δ of the frame. The second-order moments arising from the member deflections from the straight line connecting the member ends are usually known as the P-δ effects, whereas the second-order moments arising from the joint displacements Δ are usually known as the P-Δ effects. In braced frames, the joint displacements Δ are negligible, and only the P-δ effects are important. In un-braced frames, the P-Δ effects are important, and often much more so than the P-δ effects.

The P-δ and P- Δ second-order effects in elastic frames are most simply and accurately accounted for by using a computer second-order elastic analysis program. For the braced frame, the second-order moments are only slightly higher than the first-order moments. This is usually true for well-designed braced frames that have substantial bending effects and small axial compressions.

### 1.3 First Order Inelastic Analysis

This analysis takes account of inelastic (plastic) strength of member of the structure. Therefore it's also called as material non-linear analysis. In incremental loading and when elastic limit is crossed highly stressed section of the member yields entirely and the section behaves a hinge called plastic hinge. When it happens the particular section continues to resist plastic moment and undergoes large deformation. Incremental loading is sustained until sufficient numbers of plastic hinges are developed and structure no longer resists any further additional load because of transformation of structure into a mechanism and hence it's said to be a plastic collapse. In this analysis, member deformations and sway effect of structure are not considered therefore the analysis does not reflect buckling and stability evaluation.

### 1.4 Second Order Inelastic Analysis

This analysis is nothing but the addition of effects of member deformation and drift effect of the structure in first order inelastic analysis. This gives total, realistic and accurate analysis however makes the method complex. This analysis includes both geometric and material non-linearity's and referred to as "advanced analysis". This advanced analysis is further classified into following types.

#### 1.4.1 Elastic –Plastic hinge method:

It is simple, approximate and capable for representing inelasticity in frames. In this method, zero length plastic hinges are assumed to form at the ends of members, while other parts are assumed to remain elastic. Therefore, it accounts for inelasticity however disregards the spread of yielding and residual stress effects between the plastic hinges.

The elastic –plastic hinge method can be first-order or second-order plastic analysis. The first-order elastic–plastic hinge methods, in which the non-linear geometric effects are neglected, predict the same ultimate load as conventional rigid-plastic analysis. In second-order elastic –plastic hinge analysis the deformed structural geometry is considered for formulating the stiffness equation.

#### 1.4.2 Plastic zone method:

In this method the cross-section is subdivided into small sub-elements, the residual stresses are considered constant within every sub-element. The stress state at every sub-element can be derived clearly and hence the gradual spread of yielding can be predicted. The plastic zone method eliminates the necessity for individual member capacity check, therefore this method accepted to provide exact solution.

#### 1.4.3 Refined Plastic hinge method:

This approach is a advanced version of elastic-plastic hinge approach. This method takes account of gradual stiffness degradation of plastic hinge section as well as gradual stiffness degradation of member between two plastic hinge.

## 2. A CASE STUDY FOR VALIDATION OF MASTAN SOFTWARE

**Problem:**

In a warehouse, an area of 10mX40m is to be covered by rectangular portals to be placed at 5m c\c. If floor consist of 150 mm thick RCC slab with 50 mm thick finishing, design section of frame. The live load on frame is 5kN/m2.

Solution:

**The load calculation and analysis is explained in reference book [3]**

Elastic Analysis

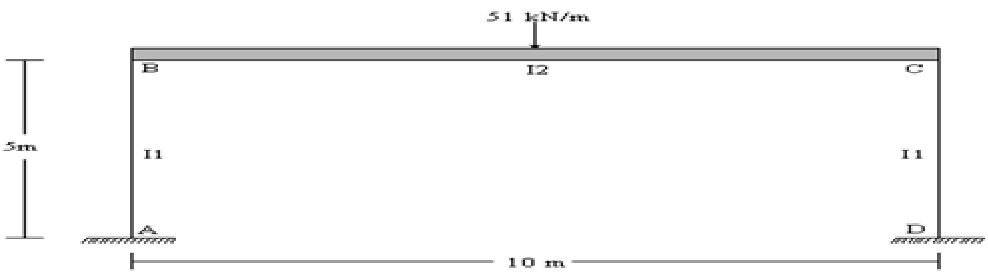


Fig 3: Elastic Analysis Portal frame with DL+LL

Results:
Manual calculated Elastic moment = 340kNm
Software calculated Elastic moment =342.3 kNm

Plastic analysis

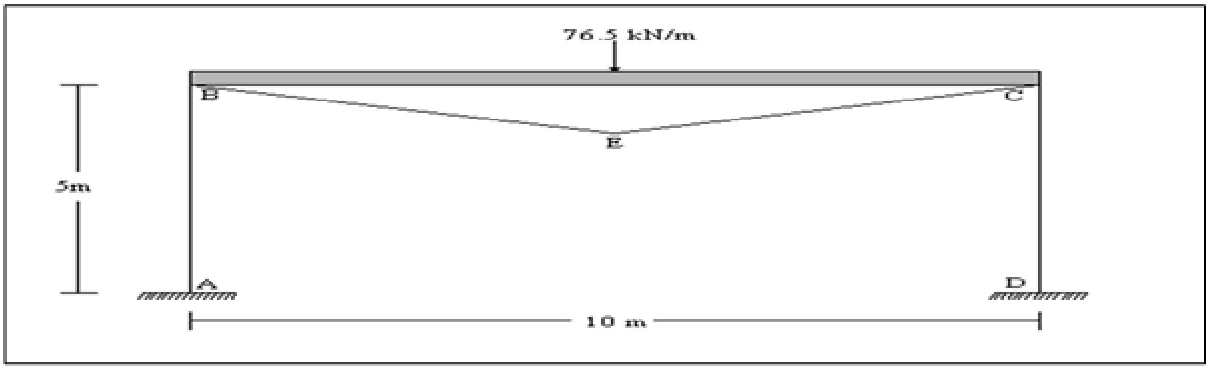


Fig 4: Inelastic Analysis Portal frame with DL+LL

Results:
Manual calculated Plastic moment = 478.12 kNm
Software calculated Plastic moment =480.4 kNm

**3. COMPARISON OF SPACE FRAME BY ALL ANALYTICAL METHODS:-**

Problem:
Public building of 3 bay 2 storey and 2 frames
Thickness of slab = 120mm

Live load intensity = 5 kN/m2
Floor finish intensity = 1.5 kN/m2

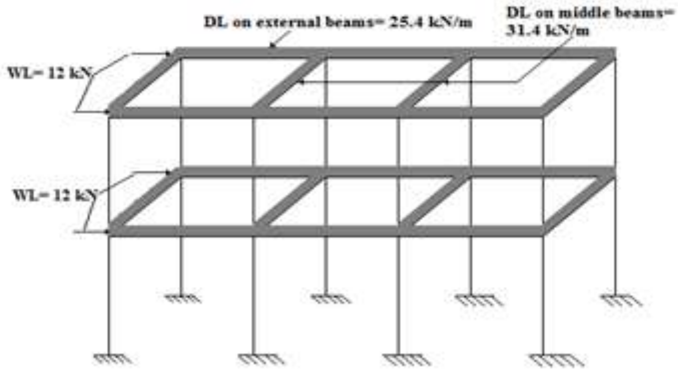


Figure 5: 3 bay, 2 story,2 frame space frame

Solution:

**DL+WL Combination:-**

Dead Load calculation:-

Self weight of slab = 0.120 X 25 kN/$m^3$ = 3 kN/$m^2$
Finishing: = 1.5 kN/$m^2$
DL Intensity = 3+1.5 = 4.5 kN/$m^2$
Equiv. triangular DL load on beam = $wl_x/3$
= 6 kN/m
Weight of brick wall per meter = 0.23m X 4m X 20 kN/$m^3$ = 18.4 kN/m
Self Weight of Beam per meter = 1 kN/m
Total DL intensity on external beam = 6 + 18.4 + 1 = 25.4 kN/m
Total DL intensity on middle beam = 6 X 2 + 18.4 + 1 = 31.4 kN/m

Wind load calculation:
Wind load = 12 kN at floor level on both frames

For inelastic analysis:
Factored DL on external beams =1.5 X DL =1.5X 24.8=37.2 kN/m
Factored DL on middle beams =1.5 X DL = 1.5X 30.3=45.45kN/m
Factored Wind load = 1.5 X WL= 1.5X 12 = 18 kN

FOEA (First order elastic analysis)

Properties

ISMB250---------Beam and Column

| | | |
|---|---|---|
| A | 4.68E-03 | $m^2$ |
| Izz (Ixx) | 5.07E-05 | $m^4$ |
| Iyy | 4.08E-06 | $m^4$ |
| J | 1.87E-07 | $m^4$ |
| Cw | 5.75E-08 | $m^6$ |
| Zzz(Zp = Zxx) | 4.05E-04 | $m^3$ |
| Zyy | 6.52E-05 | $m^3$ |

Material Properties

| | | |
|---|---|---|
| E | 200000000 | $kN/m^2$ |
| v | 0.3 | - |
| Fy | 250000 | $kN/m^2$ |
| wt. Density | 77.02 | $kN/m^3$ |

DL on exterior beams =25.4 kN/m & DL on middle beams =31.4 kN/m WL=12 kN

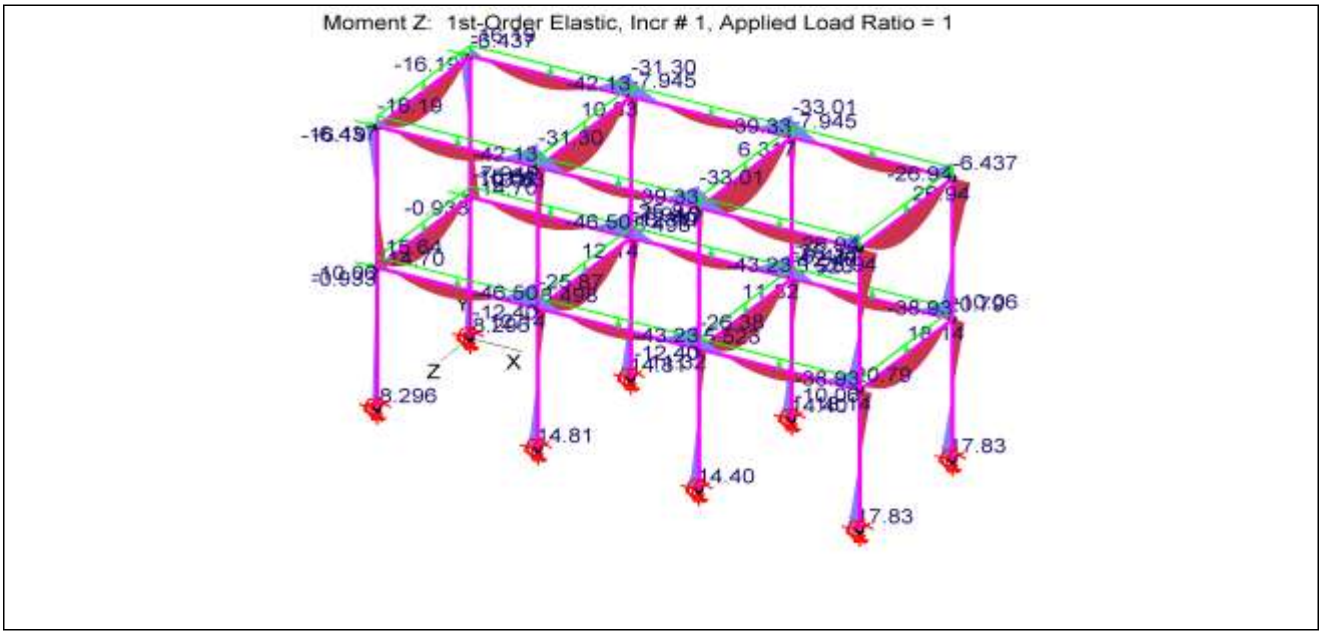


Figure 6: Output for First order elastic analysis of 3bay 2floor (DL+ WL)

Max B.M. =55.05 kNm

FOIA (First order inelastic analysis)

Properties

ISWB250---------Beam and Column

| | | |
|---|---|---|
| A | 5.15E-03 | $m^2$ |
| Izz (Ixx) | 5.93E-05 | $m^4$ |
| Iyy | 1.20E-05 | $m^4$ |
| J | 1.20E-07 | $m^4$ |
| Cw | 1.74E-07 | $m^6$ |
| Zzz(Zp = Zxx) | 5.45E-04 | $m^3$ |
| Zyy | 1.20E-04 | $m^3$ |

Material Properties

| | | |
|---|---|---|
| E | 200000000 | $kN/m^2$ |
| v | 0.3 | - |
| Fy | 250000 | $kN/m^2$ |
| wt. Density | 77.02 | $kN/m^3$ |

For inelastic analysis factored load is consider

DL on exterior beams =1.5 X 25.4 kN/m = 38.1 kN/m & DL on middle beams =1.5 X 31.4 kN/m= 47.1 kN/m WL= 18 kN

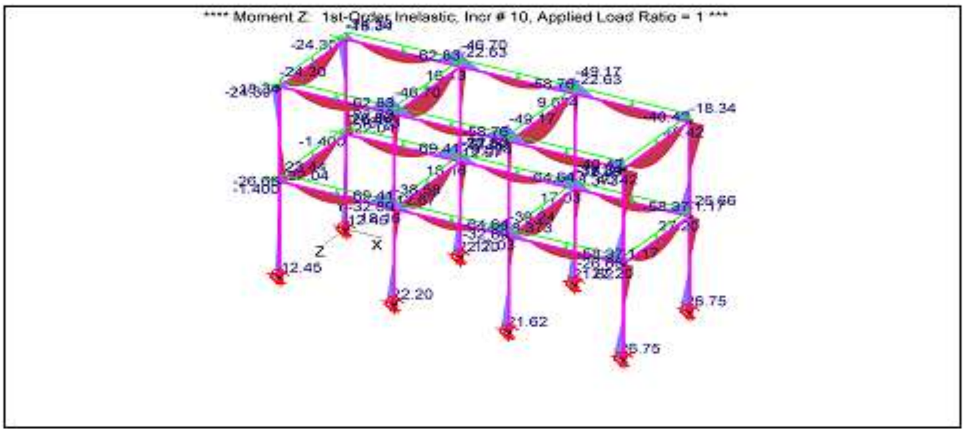


Figure 7: Output for First order inelastic analysis of 3bay 2floor (DL+ WL)

Max B.M. =71.58 kNm

SOEA (Second order elastic analysis)

Properties

ISMB250---------Beam and Column

| | | |
|---|---|---|
| A | 4.68E-03 | $m^2$ |
| Izz (Ixx) | 5.07E-05 | $m^4$ |
| Iyy | 4.08E-06 | $m^4$ |
| J | 1.87E-07 | $m^4$ |
| Cw | 5.75E-08 | $m^6$ |
| Zzz(Zp = Zxx) | 4.05E-04 | $m^3$ |
| Zyy | 6.52E-05 | $m^3$ |

Material Properties

| | | |
|---|---|---|
| E | 200000000 | $kN/m^2$ |
| v | 0.3 | - |
| Fy | 250000 | $kN/m^2$ |
| wt. Density | 77.02 | $kN/m^3$ |

DL on exterior beams =25.4 kN/m & DL on middle beams =31.4 kN/m

WL= 12 kN

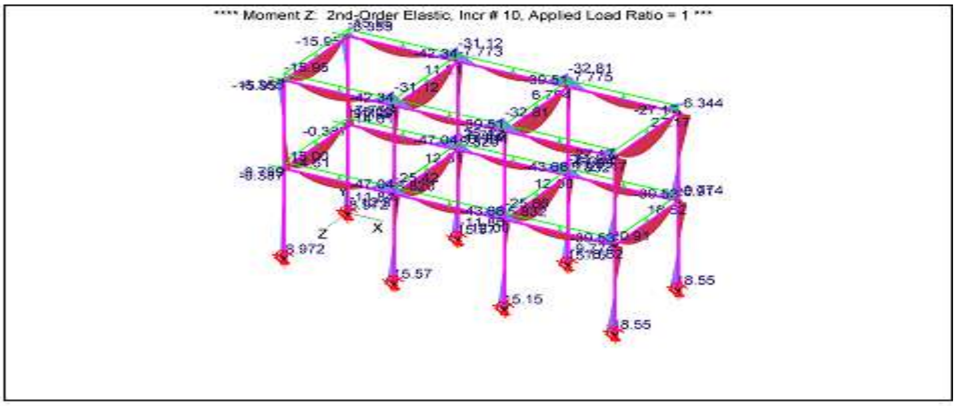


Figure 8 : Output for Second order elastic analysis of 3bay 2floor (DL+ WL)

Max B.M. = 55.22 kNm

SOIA (Second order inelastic analysis)

Properties

ISWB250---------Beam and Column

| | | |
|---|---|---|
| A | 5.15E-03 | $m^2$ |
| Izz (Ixx) | 5.93E-05 | $m^4$ |
| Iyy | 1.20E-05 | $m^4$ |
| J | 1.20E-07 | $m^4$ |
| Cw | 1.74E-07 | $m^6$ |
| Zzz(Zp = Zxx) | 5.45E-04 | $m^3$ |
| Zyy | 1.20E-04 | $m^3$ |

Material Properties

| | | |
|---|---|---|
| E | 200000000 | $kN/m^2$ |
| v | 0.3 | - |
| Fy | 250000 | $kN/m^2$ |
| wt. Density | 77.02 | $kN/m^3$ |

For inelastic analysis factored load is consider

DL on exterior beams =1.5 X 25.4 kN/m = 38.1 kN/m

DL on middle beams =1.5 X 31.4 kN/m = 47.1 kN/m

WL=18 kN

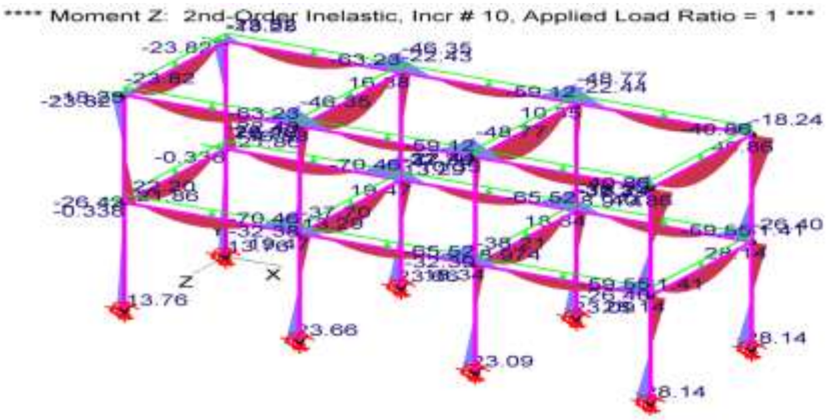


Figure 9 : Output for Second order inelastic analysis of 3bay 2floor (DL+ WL)

Max B.M. = 71.78 kNm

**4. SUMMARY OF RESULTS**

**4.1 DL+LL combination:**

| Model | Section used for FOEA | Section used for SOIA | FOEA Max B.M (Me) kNm | SOIA Max B.M (Mp) kNm | Ratio Mp/Me |
|---|---|---|---|---|---|
| 2 bay 1story 2frame | ISMB300 | ISWB250 | 85.92 | 113.2 | 1.318 |
| 2 bay 2story 2frame | ISMB300 | ISWB250 | 83.11 | 110.1 | 1.325 |
| 2 bay 3story 2frame | ISMB300 | ISWB250 | 83.25 | 110.1 | 1.323 |
| 2 bay 4story 2frame | ISMB300 | ISWB350 | 83.25 | 119.8 | 1.439 |
| 2 bay 5story 2frame | ISMB300 | ISWB350 | 83.25 | 119.8 | 1.439 |
| | | | | | |
| 3bay 1story 2frame | ISMB300 | ISWB250 | 85.89 | 113.2 | 1.318 |
| 3bay 2story 2frame | ISMB300 | ISWB250 | 83.11 | 110.1 | 1.324 |

| | | | | | |
|---|---|---|---|---|---|
| 3bay 3story 2frame | ISMB300 | ISWB300 | 83.26 | 114.5 | 1.375 |
| 3bay 4story 2frame | ISMB300 | ISWB350 | 83.25 | 119.8 | 1.439 |
| 3bay 5story 2frame | ISMB300 | ISWB400 | 83.26 | 123.8 | 1.487 |
| | | | | | |
| 4bay 1story 2frame | ISMB300 | ISWB250 | 85.89 | 113.2 | 1.318 |
| 4bay 2story 2frame | ISMB300 | ISWB250 | 83.12 | 110.1 | 1.325 |
| 4bay 3story 2frame | ISMB300 | ISWB300 | 83.27 | 114.5 | 1.375 |
| 4bay 4story 2frame | ISMB300 | ISWB350 | 83.26 | 119.8 | 1.439 |
| 4bay 5story 2frame | ISMB300 | ISWB400 | 83.26 | 123.8 | 1.487 |
| | | | | | **1.382** |

**4.2 DL+WL combination:**

| Model | Section use for FOEA | Section use for SOIA | FOEA Max B.M (Me) kNm | SOIA Max B.M (Mp) kNm | Ratio Mp/Me |
|---|---|---|---|---|---|
| 2 bay 1story 2frame | ISMB250 | ISMB175 + (140*12)P | 60.54 | 84.82 | 1.401 |
| 2 bay 2story 2frame | ISMB250 | ISMB175 + (140*12)P | 58.34 | 80.48 | 1.379 |
| 2 bay 3story 2frame | ISMB250 | C = ISWB250<br>B = ISMB175+(120*12)P | 61.07 | 94.19 | 1.542 |
| 2 bay 4story 2frame | ISMB300 | C = ISWB300<br>B = ISMB175+(120*12)P | 68.79 | 105.9 | 1.539 |
| 2 bay 5story 2frame | ISMB300 | C = ISWB350<br>B = ISMB200+(160*12)P | 80.98 | 125.9 | 1.555 |
| | | | | | |
| 3 bay 1story 2frame | ISMB250 | ISMB175+(120*10)P | 60.54 | 88.13 | 1.456 |
| 3 bay 2story 2frame | ISMB250 | ISMB175+(120*12)P | 58.34 | 83.68 | 1.434 |
| 3 bay 3story 2frame | ISMB250 | C = ISMB300<br>B = ISMB175+(120*12)P | 58.48 | 80.12 | 1.370 |
| 3 bay 4story 2frame | ISMB250 | C = ISMB400<br>B = ISMB175+(120*12)P | 60.65 | 82.99 | 1.368 |
| 3 bay 5story 2frame | ISMB300 | C = ISWB350<br>B = ISMB200+(120*12)P | 66.66 | 103.3 | 1.550 |
| | | | | | |
| 4 bay 1story 2frame | ISMB250 | ISMB175+(140*12)P | 60.55 | 84.81 | 1.400 |
| 4 bay 2story 2frame | ISMB250 | ISMB175+(140*12)P | 58.35 | 80.16 | 1.374 |
| 4 bay 3story 2frame | ISMB250 | ISMB300<br>ISMB175+(120*12)P | 57.23 | 80.12 | 1.400 |
| 4 bay 4story 2frame | ISMB250 | C = ISWB300<br>B = ISMB200+(120*12)P | 58.48 | 82.46 | 1.410 |
| 4 bay 5story 2frame | ISMB250 | C = ISWB350<br>B = ISMB200+(120*12)P | 60.10 | 92.02 | 1.531 |
| | | | | | **1.4473** |

**4.3 DL+LL+WL combination:**

| Model | Section used for FOEA | Section use for SOIA | FOEA Max B.M (Me) kNm | SOIA Max B.M (Mp) kNm | Ratio Mp/Me |
|---|---|---|---|---|---|
| 2 bay 1story 2frame | ISMB300 | ISMB175+(120*12)P | 84.18 | 98.51 | 1.170 |
| 2 bay 2story 2frame | ISMB300 | ISMB175+(120*12)P | 83.11 | 93.88 | 1.129 |
| 2 bay 3story 2frame | ISMB300 | ISMB175+(120*16)P | 83.25 | 94.12 | 1.130 |
| 2 bay 4story 2frame | ISMB300 | ISWB300 | 83.25 | 97.06 | 1.166 |
| 2 bay 5story 2frame | ISMB300 | ISWB300 | 90.59 | 123.4 | 1.362 |
| | | | | | |
| 3bay 1story 2frame | ISMB300 | ISMB175+(120*12)P | 85.89 | 98.5 | 1.147 |
| 3bay 2story 2frame | ISMB300 | ISMB175+(120*14)P | 83.11 | 93.65 | 1.127 |
| 3bay 3story 2frame | ISMB300 | ISMB175+(120*16)P | 83.26 | 94.13 | 1.131 |
| 3bay 4story 2frame | ISMB300 | ISWB400 | 83.25 | 98.2 | 1.179 |
| 3bay 5story 2frame | ISMB300 | ISWB350 | 83.25 | 100.1 | 1.202 |
| | | | | | |
| 4bay 1story 2frame | ISMB300 | ISMB175+(140*12)P | 85.89 | 94.94 | 1.105 |
| 4bay 2story 2frame | ISMB300 | ISMB175+(140*14)P | 83.12 | 89.65 | 1.056 |
| 4bay 3story 2frame | ISMB300 | ISMB175+(120*16)P | 83.27 | 94.13 | 1.130 |
| 4bay 4story 2frame | ISMB300 | ISWB300 | 83.26 | 91.67 | 1.101 |
| 4bay 5story 2frame | ISMB300 | ISWB350 | 83.26 | 95.99 | 1.153 |
| | | | | | **1.1525** |

**5. DISCUSSION AND CONCLUSIONS**

In this dissertation work attempt is made to establish relation between analysis results of first order elastic and second order inelastic analysis, which may help structural designers to use codal provision of IS800:2007 in a more convenient manner. The results obtained from various case studies, following conclusions are drawn:

1) Selection of section for columns and beams is on the basis of analysis performed by FOEA which also satisfy vertical displacement for beam and horizontal deflection of top storey of the frame well within permissible limits.
2) Software is validated and results of both FOEA and SOIA are found to be in close agreement with corresponding results obtained by manual calculations.
3) From analysis, it seen that there is less difference between the results obtained by First order elastic and Second order elastic analysis. Similarly the observation is seen in first order inelastic and second order inelastic analysis results.
4) As the degree of indeterminacy of structure increases the limit state of collapse by SOIA is due to formation of plastic hinges as well as buckling of member.
5) In 95% cases of DL+LL and DL+WL design moment by FOEA and SOIA are found to be below 1.5 and the ratio is 1.45 and 1.43 respectively. This is typically noticed as in SOIA the working loads are in multiple of 1.5, which is partial safety factor for the load combination as per IS 800:2007.
6) For DL+LL+WL combination design moment of 95% cases found to be below1.2 and the ratio is 1.12, which is partial safety factor for the load combination as per IS 800:2007
7) Observed values of ratio indicate economy in design.
8) In all three combinations as no. of bay increases the moments are found to be decreased. This is due to increased lateral stability of structure.

9) For DL+WL and DL+LL+WL loading combinations, as no. of story increases the displacement and moments are found to be increased especially in column members.
10) As the number of bay increases the stiffness of frame in lateral direction increases. Hence horizontal displacement of frame reduces from 2 bay to 3 bay by 9.55 % and that for 3 bay to 4 bay reduces by 5.15 %.

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