INDEPENDENT MIDDLE DOMINATION NUMBER IN JUMP GRAPH

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ABSTRACT - The middle graph of a jump graph J(G) denoted by M(J(G)) is a graph whose vert4x set is V(J(G)) ∪E(J(G)) and two vertices are adjacent if they are adjacent edges of J(G) or one is a vertex and other is a edge incident with it.

A set of vertices of jump graph M(J(G)) is an independent set and every vertex not in S is adjacent to a vertex in S. The independent middle domination number of J(G) denoted by iM(J(G)) is the minimum cardinality of an independent dominating set of M(J(G)).

In this paper many bounds on iM(J(G)) were obtained in terms of the vertices, edges and other different parameters of J(G) and not interims of the elements of M(J(G)). Further its relation with other different domination parameters are also obtained.

INTRODUCTION

In this paper all the graphs considered here are simple, finite, nontrivial,undirect4d and connectd.The vert4x set and edge set of graph J(G) are denoted by V(J(G))=p and E(J(G))=q respectively. Terms are not defined here are used in the sense of Harary [1].

The degree, neighborhood and closed neighborhood of a vertex v in a jump graph J(G) are denoted by deg(v), N(u0, and N[v]=N(u) ∪ {v} respectively For a subset S of V(J(G))the graph induced by S⊆ V is denoted by <S>.

As usual the maximum degree of a vertex (edge) in J(G) is denoted by Δ (J(G))

(Δ ‘(J(G))). For any real number x . $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greater integer not greater than x.

A vertex cover in a jump graph J(G) is a set of vertices that covers all the edges of J(G).The vertex covering number $^{\gamma}0$(J(G) is the minimum cardinality of a vertex cover in J(G).

A set of vertices /edges in a graph J(G) is called independent set if no two vertices/edges in the set are adjacent. The vertex independent number β_0(J(G)) is the minimum cardinality of an independent set of vertices. The edge independence number β_1(J(G)) of a graph J(G) is the maximum cardinality of an independent set of edges.

A line graph L(J(G)) is a graph whose vertices corresponds to the edges of J(G) and two vertices in L(J(G)) are adjacent if and only if the corresponding edges in J(G) are adjacent.

A subdivision of an edge e=uv of a graph J(G) is the replaced of the edge e by apath(u,v,w). The graph obtained from a graph J(G) by subdividing each edges of J(G) exactly once is called the subdivision of graph of J(G) and is denoted by S(J(G)).

A set D of a graph J(G)=(V,E) is called dominating set if every vertex in V-D is adjacent to sime vertex in D. .The domination number $\sqrt{}$(J(G)) of J(G) is the minimum cardinality taken over all dominating set of J(G).

A set F of edges in a graph J(G) is called an edge dominatingset of J(G)if every edge E-F is adjacent to at least one edge in F. The edge domination number $\sqrt{}'$(J(G)) of jump graph J(G).Edge domination number was studied by S.L Mitchell and Hedetniemi in [2].

A dominating set D of a jump graph J(G) is a strong split dominating set, if the induced subgraph<V-D> is totally disconnectee with only two vertices. The strong split domination number $\sqrt{}_{ss}$(J(G)) of a graph J(G)is the minimum cardinality of a strong split dominating set of J(G) see[6]

A dominating set D of a jump graph J(G)=(V,E) is a paired dominating set if the induced subgraph<D> contained atleast one perfect matching. The paired domination number $\surd_p(J(G))$ of a graph J(G) is the minimum cardinality of the paired dominating set.

A dominating set D of a graph J(G)=(V, E) is an independent dominating set, if the induced sub graph <D> has no edges.. The independent domination number i(J(G)) of a graph J(G) is the minimum cardinality of a independent dominating set.

Analogously, we define independent middle domination number in Jump graphs as follows.

A set of vertices of jump gaph M(J(G)) is an independent dominating set of M(J(G)). If S is an independent set and every vertex not in S is adjacent to a vertex in S. The independent domination number of middle graph M(J(G)) denoted by $i_M(J(G))$ is the minimum cardinaluity of an independent dominating set of M(J(G))

In this paper many bounds in $i_M(J(G))$.we obtained and expressed in terms of the vertices edges and other different parameters of J(GO but not interms of members of M(J(G)). Also we establish independent middle domination number in jump graph and express the results with other different domination parameters of J(G)..

We need the following Theorems:

Theorem A [4] For any connected graph G with $p \geq 4$ $G \neq K_p$ $\surd_{ss}(G) = \alpha_0(G)$
Theorem B [4] For any connected ngraph $\lceil \frac{p}{1+\Delta} \rceil \leq \surd(G)$.
Theorem c [5] For any graph $i[L(G)] \leq \beta_1(G)$.
Theorem D [4] For any graph G with no isolated vertices $\surd_p(G) \leq 2\beta_1(G)$.
Theorem E [3] For any gaph G, $\surd(M(G)) = i_M(G)$ where M(G) denotes the middle graph of G.

Results We list out the exact values $i_M J(G))$ for some standard jump graphs

Theorem1.

a) For any path P_p with $p \leq 2$ vertices
$i_M(J(G)) = \frac{p}{2}$ if p is even
$i_M(J(G)) = \lceil \frac{p}{2} \rceil$ if p is odd
b) For any cycle C_n
$i_M(J(G)) = \frac{n}{2}$ if n is even
$i_M(J(G)) = \lceil \frac{n}{2} \rceil$ if n is odd
c) For any star $K_{1,n}$
$i_M(J(G)) = n-1$
d) For any wheel W_p
e) $i_M(J(G)) = \frac{p}{2}$ if p is even
$i_M(J(G)) = \lceil \frac{p}{2} \rceil$ if p is odd.

Theorem2: Let J(G) be a connected jump graph of order $p \geq 2$ then $i_{ss}(J(G)) \geq \lceil \frac{p}{2} \rceil$

Proof: let $V(J(G)) = \{v_1, v_2, v_3, \ldots\ldots v_p\} = p$ and $E(J(G)) = \{e_1, e_2, e^3 \ldots\ldots e_q\} = q$. Now by the definition of middle graph $\surd(M(G)) = p+q$. Let D be the minimal independent dominating set of M(J(G)). Let $D_1 = D \cap V(J(G))$ and $D_2 = D \cap [V(M(J(G)) - V(G)$. Now we consider the following cases.

Case i) Suppose $D_2 = \phi$ then $D_1 = V(J(G))$ and hence $p+q-|D| \leq 2|D|$ so that $|D| \geq \lceil \frac{p}{2} \rceil$
Case ii) Suppose$D_1 = \phi$, since each vertex of D dominates exactly two vertices of $[V[M(J(G))]-D$, then it follows that $|V[M(J(G))-D| = 2|D|$.Hence $p+q-|D| \leq 2|D|$ so that $|D| \geq \lceil \frac{p}{2} \rceil$

Case iii) Suppose D_1 and D_2 are non empty, since each vertex of D_2 dominates exactly two vertices of M(J(G)) then it follows that p - $|D_1| \le 2\ |D_2|$, Hence $p \le |D_1| + 2\ |D_2| \le 2|D|$ Thus
$I_M(J(G)) = |D| \ge \lceil \frac{p}{2} \rceil$

The next theorem gives the relationship between domination number of J(G) ad $i_{M(J(G))}$.

Theorem 3 : For any connected jump graph J(G), $\surd(J(G)) \le i_M(J(G))$

Proof: Let $V_1 = \{ v_1, v_2, v_3,......v_n \} \subset V(J(G))$ be the set of all the vertices with $deg(v_i) \ge 2$
$\forall\ v_i \in J(G)\quad 1 \le I \le n$. Then there exists a minimal dominating set of J(G) with [S] = $\surd(J(G))$. Now by the definition of M(J(G)). V[M(J(G))]= V(J(G)) ∪ E(J(G). Consider a minimal set of vertices $D_1 \subseteq V[M(J(G))]$ such that $|D_1| = \surd[M(J(G))]$. If <D_1> is totally disconnected then D_1 itself forms the independent dominating set of M(J(G)),otherwise we consider a set
$D_2 = D'_1 \cup D'_2$ where $D'_1 \subseteq D_1$ and $D'_2 \subseteq V[M(J(G))] - D_1$ such that for all $v_i \in$ < $D'_1 \cup D'_2$> deg v_i = 0 Thus $D'_1 \cup D'_2$ is the minimal independent dominating set of J(G)
Since V(J(G))) ⊆ V[M(J(G))] we have $|\ D'_1 \cup D'_2\ | \ge |S|$
Hence $\surd(J(G)) \le i_M(J(G))$

Theorem 4: For any non-trivial connected jump graph J(G)
$i_M(J(G)) + \surd_{ss}[\ M(J(G))] \le p + q$

Proof: Since $i_M(J(G)) \le \beta_0[M(J(G))]$
Also from theorem A $\surd_{ss}[\ M(J(G))] = \alpha_0[M(J(G))]$

Further $i_M(J(G)) + \surd_{ss}[\ M(J(G))] \le \beta_0[M(J(G))] + \alpha_0[M(J(G))]$
= V[MJ(G)]
= V(J(G)) + E(J(G))
= p + q
Hence $i_M(J(G)) + \surd_{ss}[\ M(J(G))] \le p + q$

Theorem 5: Let J(G) be any connect4d jump graph then $i_M(J(G)) = \alpha_1(J(G))$

Proof: Let $E_1 = \{ e_1, e_2, e_3......e_n \} \subseteq E(J(G)$ be the minimal edges in J(G) such that
$|E_1| = \alpha_1(J(G))$. Since V[M(J(G))] = V(J(G)) ∪ E(J(G)),let $S=\{s_1, s_2, s_3......s_k\}$ be the set of vertices subdividing the edges of J(G) in M(J(G)). Now let $S_1 = \{ s_1, s_2,s_3,....s_i \} \subseteq S\ 1 \le i \le k$ be the vertices subdividing each edge $e_i \in E_1(J(G))\quad 1 \le i \le n$. Thus $N[S_1]$ = V(J(G)) and also in M(J(G)) $N(S_1)$= V(S-S_1) Thus $N[S_1]$ = V(J(G)) ∪ V(S – S1) =V(M(J(G))]. Hence clearly <S_1> forms the minimal dominating set of M(J(G)) such that $|S_1| = \surd(M(J(G)))$. Further by theorem E we have $\surd(M(J(G)))$. = $i_M(J(G))$. Therefore $i_M(J(G))$ = $|S_1| = |E_1|$
which gives $i_M(J(G)) = \alpha_1(J(G))$

Theorem 6: For any complete bipartite graphs $K_{m,n}$ $i_M(J(K_{m,n})) = n$ for n≥ m

Proof: Let (X, Y) be a partition of $K_{m,n}$ n≥ m with |X| =m and |Y|=n let X= $\{ x_1, x_2, x_3,......x_m\}$ and Y=$\{ y_1, y_2, y_3,......y_n \}$ let $E_1 = \{x_iy_j$ / 1≤ I ≤m 1≤ j ≤ n} be the independent edges n $K_{m,n}$ Clearly $|E_1| = \min(m,n) = m$ In M(J(G)), let S= $\{v_1, v_2, v_3,.....v_k\}$ be the vertices subdividing each edge of J(G) in M(J(G)). Consider a set $S_1 = \{ v_i / 1 \le I \le k\} \subseteq S$ be the vertices subdiving the edges of . Clearly S_1 is an independent set of vertices in M(J(G)). Now let Y= $\{ y_i / y_i = N(v_1)$ for each $v_1 \in S_1 \}$ Clearly $|Y_1| = m$ with pot loss of generality $Y_2= Y - Y_1$ is an independent set of vertices in M(J(G)) Now $N(S_1)= X \cup V(S-S_1) \cup Y_1$ and hence $N[S_1 \cup Y_2]$= V[M(J(G))] Since $(S_1 \cup Y_2)$ is independent thus the induced subgraph < $S_1 \cup Y_2$> is a minimal independent dominating set in M(J(G)). Clearly $|S_1| = |E_1|$=m and $|Y-Y_1|$=n-m. Therefore $|S_1 \cup Y_2| = |S_1| + |Y_2|$= m+n-m = m Hence $i_M(J(K_{m,n})) = n$ where n≥m.

Theorem7: For any connected (p,q) jump graph $| \frac{p}{1+\Delta(J(G))} | \le i_M(J(G))$

Proof: By theorem B and also by theoem 3 we have the required result.
The following theorem relates $i_M(J(G))$ and the independent domination number in line jump graph i[L(J(G))] in terms of the vertices of J(G)

Theorem 8: For any connected non trivial jump graph J(G)

$i_M(J(G)) + i[L(J(G))] \leq p$

Proof: Let G be a connected graph, By theorem C, we have $i[L(J(G))] \leq \beta_1 (J(G))$
Also by theorem 5, $i_M(J(G)) = \alpha_1(J(G))$
Hence $i_M(J(G)) + i(L(J(G)) \leq \alpha_1(J(G)) + \beta_1 (J(G))$
$= V(J(G)) = p$
Therefore $i_M(J(G)) + i[L(J(G))] \leq p$

The following theorem relates $i_M(J(G))$ and $\surd'(J(G))$ in terms of the vertices of J(G)

Theorem 9: For any connected graph J(G) , $i_M(J(G)) \leq p - \surd'(J(G))$.

Proof: Let $E_1 = \{e_1, e_2, e_3, ...e_n\} \subseteq E(J(G))$ be the minimal set of edgs such that for each $e_i \in E_1$
$i=1,2,3,...q$. $N(e_{1)} \cap E_1 = \phi$ the $|E_1| = \surd'(J(G))$ in M(J(G)),
$V(M(J(G))) = V(J(G)) \cup E(J(G))$.
Let $D = \{v_1, v_2, v_3,.....v_i\}$ be the set of vertices subdividing the edges of J(G) in M(J(G)).
Let $D_1 \subseteq D$ such that each $v_i \in D_1$ sub divides the edges $e_i \in E_1$ in M(J(G)). Thus $|E_1| = |D_1|$ Now if $N[D_1] = V[M(J(G))]$ and for each $v_i \in D_1$ is an isolate, the D_1 forms an independent dominating set of M(J(G)). Otherwise consider a set $D_1 = D_2 \cup D'_2$ where $D_2 \subseteq D_1$ and
$D'_2 \subseteq V(M(J(G))) - D_1$ such that $\forall v \in V[M(J(G)] - D_2 \cup D'_2$ $N(v) \cap (D_2 \cup D'_2) \neq \phi$ and also $(D_2 \cup D'_2)$ is totally disconnected. Then $| D_2 \cup D'_2|$ forms a minimal independent dominating set of M(J(G)). Hence clearly $| D_2 \cup D'_2| \leq |V(J(G)| - \surd'(J(G))$
Which gives $i_M(J(G)) \leq p - \surd'(J(G))$.

Theorem 10: Lw=et J(G) be a connected graph then $i_M (J(G)))) + \surd_P(J(G) \leq p + \beta_1(J(G))$ where $\surd_p(J(G)$ is the paired domination in J(G).

Proof: By theorem 5, $i_M(J(G)) = \alpha_1(J(G))$
Also by theorem D $\surd_p(J(G)) \leq 2 \beta_1 (J(G))$
Further $i_M(J(G)) + \surd_p (J(G)) \leq \alpha_1(J(G)) + 2 \beta_1 (J(G))$
$= p - \beta_1 (J(G)) + 2 \beta_1 (J(G))$
$= p + \beta_1 (J(G))$
Hence $i_M (J(G)))) + \surd_P(J(G) \leq p + \beta_1(J(G))$.

Theorem 11: For any connected graph J(G) $i_M(J(G)) \leq i[S(J(G))]$

Proof: Let $V(J(G)) = [v_1, v_2, v_3......vi\}$ and $E(J(G)) = \{ e_1, e_2\ e_3......e_j\}$
let $S = \{ u_1, u_2,.........u_k\} \subseteq V|S(J(G))|$ be the minimum number of vertices subdividing the edges of J(G) in S(J(G)).
If $N[S] = V[S(J(G))]$ then <S> $\forall u_iu_j \in S$ $1 \leq I, j \leq k$ $N(u_i) \cap \{u_j\} = \phi$ hence <S> itself forms the independent dominating set of S(J(G)). In case if $N[S] \neq V[S(J(G))]$ we consider a set $I = D_1 \cup D_2$ where $D_1 \subseteq S$ and $D_2 = V[S(J(G))] - D_1$ such that $N[I] = V[S(J(G))]$ and hence $u_i \in$ <I>. Since $degu_i = 0$ then < I > forms a minimal independent dominating set of S(J(G)) Further without loss of generality $S \subseteq V[M(J(G)]$. Since for each $u_iu_j \in S$, $N(u_i) \cap \{u_j\} = \phi$ then <n S_1 > forms a minimal independent dominating set of M(J(G)) otherwise let $S_2 \subseteq S'_1 \cup S'_2$
where $S'_1 \subseteq V[M(J(G))] \cap S$ and $S'_2 \subseteq V[M(J(G))] \cap V(J(G))$ and also no two vertices in <S_2> are adjacent Hence <S_2> forms the minimal independent dominating set of M(J(G)) Clearly ,
$| I | \geq | S_2 |$ which gives $i_M(J(G)) \leq i\ |S(J(G))|$

The followingtheorem gives Norduss-Gaddum type of result.

Theorem 12: Let J(G) be a graph such that both J(G) and $J(\bar{G})$ have no isolated edges then

$I_M(J(G)) + i_M(J(\bar{G})) \leq 2 \lceil \frac{p}{2} \rceil$

$I_M(J(G)) * i_M(J(\bar{G})) \leq \lceil \frac{p}{2} \rceil^2$.

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