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TOTAL DOMATIC NUMBER OF A JUMP GRAPH

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ABSTRACT - Let d_t and $\bar{d}_t$ denotes the total domatic number of a jump graph J(G) and its complement $J(\bar{G})$. In this paper wecharactrize the class of all regular jump graphs for which $d_t + \bar{d}_t$ =p-2 where p is the order of J(G).

Key words: Total domination, connected domination, Total domatic number, connected domatic number
Mathematics subject-Classification:05C

1. Introduction

By a graph we mean a finite undirected graph without loops or multiple edges. Terms not defined here are used in Harary [1]

Let J(G)+(v,E) be a jump graph of order p and size q. a subset S of V(J(G)) is dominating set of G if every vertex in V(J(G))-S is adjacent to some vertex in S. Let J(G) be a jump graph without isolated vertices. A subset S of V(J(G)) is called a total dominating set of J(G) is called the total domination number of J(G) and is denoted by $\sqrt{}_t$(J(G)). The maximum order of a partition of V(J(G)) into total dominating sets of J(G) is called the total domatic number of J(G) and is denoted by d_t(J(G)).

Let J(G) be a connected jump graph. a dominating set S of J(G) is called a connected dominating , if <S> is connected. The cardinality of a minimum connected dominating set is called the connected domination number of J(G) and is denoted by $\sqrt{}_c$(J(G))..The minimum order of a partition of V into connected dominating set is called the domatic number of J(G) and is denoted by d_c. The total connected domatic number of complement $J(\bar{G})$ is denoted by $\bar{\sqrt{}}_t$ ($\bar{\sqrt{}}_c$).The total(connected) domatic number of $J(\bar{G})$. is denoted by $\bar{d}_t$ ($\bar{d}_c$). The maximum and minimum degrees of a vertex in jump graph J(G) are denoted by Δ and δ respectively. For any real number x, $\llcorner x \lrcorner$ denotes the largest integer less than or equal to x.

Cockayne, Dawes and Hedetniem i[2] have proved the following .

Theorem1.1 [2] if g has p vertices no isolates and Δ < p-1 then v ≤ p-1 then $d_t + \bar{d}_t$ ≤ p-1 with inequality if and only if G or $\bar{G}$ is isomorphic to C_4.

Hence it follows that if G is a graph of order p > 4 then $d_t + \bar{d}_t$ ≤ p - 2 we give an independent proof of this inequality which enables us to obtain a characterization of all regular graphs for which $d_t + \bar{d}_t$ = p-2. The characterization of non-regular graph for which $d_t + \bar{d}_t$ = p - 2 will be repeated in a subsequent paper.

We need the following theorems

Theorem 1.2 [2] For any graph without isolate $d_t \leq \delta$
Theorem 1.3 [2] Let G be a regular graph of order p such that both G and $\bar{G}$ are connected. Then
$d_c + \bar{d}_c$ =p-2if and only if G or $\bar{G}$ is isomorphic to C_6 or G_1 or G_2 Where G_1 and G_2 are given in Figure 1

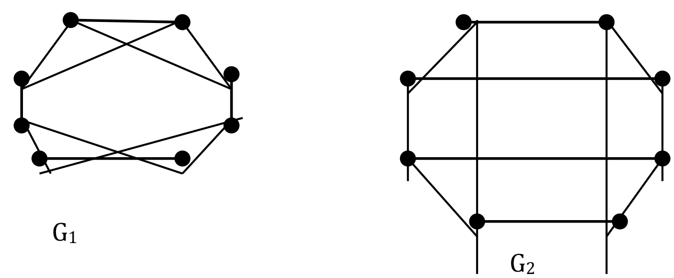

Figure 1

2. Main Results

Theorem 2.1 Let J(G) be any jump graph with at least 5 vertices no isolates and Δ ≤p-2 then $d_t + \bar{d}_t$ ≤ p-2

Proof; Since $\sqrt{}_t \geq 2$ $d_t \leq \llcorner p/2 \lrcorner$. If G disconnected jump graph with n components, n≥2, we have $\sqrt{}_t \geq 2n$ so that $d_t \leq \llcorner p/2n \lrcorner$ and $d_t + \bar{d}_t \leq \llcorner p/2 \lrcorner \leq p - 2$.Hence. we may assume that both G and $\bar{G}$ are connected.

If $d_t \leq \llcorner p/2 \lrcorner - 1$ and $\bar{d}_t \leq \llcorner p/2 \lrcorner - 1$ then the result is trivial. Hence we may assume without loss of generality that $d_t = \llcorner p/2 \lrcorner$. We consider the following cases

Case i) p is even and J(G) is r-regular. Then J ($\bar{G}$) is $\bar{r}$ - regular. Where $\bar{r}$ = p – 1 – r. Since $d_t \leq \delta$ we have r ≥ p/2 so that $\bar{r} \leq$ (p/2) – 1. Hence $\bar{r}_t \geq 3$ and $\bar{d}_t \leq \llcorner P/3 \lrcorner$

Thus $d_t + \bar{d}_t \leq \llcorner p/2 \lrcorner + \llcorner p/3 \lrcorner \leq$ p-2 provided p >6 when p=6 r=3 and $\bar{r}$ = 2 so that $\bar{G}$ is isomorphic to G hence $d_t + \bar{d}_t$ = 3 + 1 = 4 = p – 2.

Case ii) p is even and J(G) is non regular Then $\delta \geq p/2$, $\Delta \geq (p/2) + 1$ so that $\bar{\delta} \geq (p/2) - 2$ Hence $\bar{d}_t \leq (p/2) - 2$ and $d_t + \bar{d}_t \leq p-2$

Case iii): p is odd Then $\Delta \geq \lfloor \frac{p}{2} \rfloor + 1$ and henc it follows that $\bar{r}_t \geq 3$ Then $\bar{d}_t \leq \lfloor P/3 \rfloor$

So that $d_t + \bar{d}_t \leq \lfloor p/2 \rfloor + \lfloor p/3 \rfloor \leq p-2$ we now proceed to characterize the class of all regular graphs for which $d_t + \bar{d}_t = p-2$.

Theorem 2.2: Let J(G) be disconnect4ed r-regular jump graph with at least 5 vertices.

Then $d_t + \bar{d}_t = p-2$. If and only if J(G) is isomorphic to $2C_3$, $2C_4$, $3K_2$ or 2K

proof; Suppose J(G) has n components. Then it follows from Theorem2.1 that $d_t + \bar{d}_t = p-2$. If and only if $\lfloor p/2n \rfloor + \lfloor p/2 \rfloor = p - 2$ and hence $n \leq 3$ when n=2 , p=6 or 8 when p=6 J(G) is isomorphic to $2C_3$ and p=8 J(G) is isomorphic to $2K_2$.

Converse is trivial.

Theorem 2.3 Let J(G) be connected 2-regular jump graph with atleast 5 vertices

Then $d_t + \bar{d}_t +$ p-2. If and only if J(G) is isomorphic to C_6 or C_8

Proof; Trivial

Theorem 2.4 Let J(G) be a r-regular jump graph such that J(G) and its complement

J($\bar{G}$) are c onnected and $r, \bar{r} \geq 3$ Then $d_t + \bar{d}_t +$ p-2. If J(G) and J($\bar{G}$) is isomorphic if

G_1 or G_2 where G_1 and G_2 are the jump graphs given in Figure 1.

Proof; Suppose $d_t + \bar{d}_t =$ p-2 it follows from theorem 2.1 that

$\lfloor p/2 \rfloor + \lfloor p/3 \rfloor =$ p-2 hence $5 \leq p \leq 12$
$p \neq 6,11$ Since $r, \bar{r} \geq 3$ we have $p \geq 8$ so that p= 8,9,10 or 12. Now we claim that

d_t or $\bar{d}_t = \lfloor p/2 \rfloor$, otherwise we have $d_t = \bar{d}_t = \lfloor p/2 \rfloor$. – 1 and hence p is even so that $d_t \leq r$ and $\bar{d}_t \leq \bar{r}$.Hence either r or $\bar{r}$ is (p/2)-1 suppose $\bar{r}$ =(p/2)-1. Then $\bar{d}_t \leq \lfloor p/3 \rfloor$ and hence $d_t + \bar{d}_t <$ p-2, which is a contradiction. Thus $d_{t\,b}$ or $\bar{d}_t = \lfloor p/2 \rfloor$ suppose $d_t = \lfloor p/2 \rfloor$. If p=9 it follows that r=5 and $\bar{r}$ = 3which is impossible. Hence p=8,10, 12 suppose p=10 V= $U_{i=1} V_i$ and $|V_i| = 2$ and $|V_i|$ is total dominating set in J(G). Then $r \geq 5$ and $\bar{r} \leq 4$ and $\sqrt{t} \geq 3$ we claim that $\bar{r}_t \geq 4$ suppose { v_1, v_2 , } is a total dominating set in J($\bar{G}$) and let v_2v_1, $v_2v_3 \in E$ (J($\bar{G}$)) we assume that vi $\in Vi$ I = 1, 2, 3, let w_2 be the other vertex of V_2 Since v_1v_2 , $v_2v_3 \in E$ (J($\bar{G}$)) it follows that $v_1w_1 \notin E$ (J($\bar{G}$)) Similarly
$v_2w_{2\ 2} \notin E$ (J($\bar{G}$)) and w2 is not dominated by any vertex of S in J. Then $\sqrt{t} \geq 4$ and $\overline{dt} \leq 2$ so that $d_t + \bar{d}_t <$ p-2 which is a contradiction. Then $p \neq 10$ by a similar argument $p \neq 12$ Then p = 8 since $d_t = 4$ it follows that $d_c = 4$. Also r-4 and and $\bar{r}$ =3, $\overline{dt}$=2. Hence by Theporem 1.3 J(G) is isomorphic to $J(G_1)$ or $J(G_2)$.

REFERECES

[1] F. Harary Graph Theory Addison Wesley Reading Mass (1972)

[2] E.J Cockayne, R.M Dawes and S.T. Hedetniemi, Networks 10 (1980)21-219

[3] J. Paulraj Joseph and S. Armugam J. Ramanujan math.Sco,9 (19940 No.1 69-77.

[4] S.Armugam and A. T Thuraiswamy, Indian.J.pure appl. Math.29(5) (1998) 513-515