Cantilever Beam Crack Detection using FEA and FFT Analyser

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Abstract - *In this project establish a proper approach to analyze and study the cantilever beam crack. This work gives approach to find the crack size and crack location in beams. This study is based on natural frequency measurement, which is a global parameter, measured at a point on structure. In theoretical analysis crack is simulated by a spring, connecting the two segments of beam. To get the correlation between the stiffness & location of crack, the catachrestic equation obtained from the vibration analysis of Eluer-Bernoulli beam is manipulated. Using finite element method, model of beam is generated. Initially, natural frequencies for un-cracked beam are got by finite element analysis, then developed a crack of known dimensions at known location. The newly created crack causes natural frequency reduction. Using the derived equation plots of spring stiffness versus crack location got for three lowest transverse mode. There common intersection point gives crack location & corresponding stiffness. Crack size can be calculated from the standard relation between stiffness & crack size. After analysis of number of cases, error in the prediction of crack size & location is less than 4%.*

***Key Words*: Crack location,Euler-Bernoulli Beam, Finite element analysis, Natural frequency, Stiffness , Vibration**

1. INTRODUCTION

Cracks in structures develop due to certain reasons such as mechanical defects, faults from the manufacturing process etc. Occurrence of cracks in the Material may lead to change in the whole behaviourr of the element and also may decrease its safety. Single machine component failure in the process industries like power stations or petrochemicals, can result into big loss. So the detection or identification of cracks in Machine component is an important issue.

1.1 Problem Statement

As given above, the failure of machine component is big loss by of time, money and life. Because of crack, most of the machine component failures. So there is a need to predict such failures in advance, so that loss due to such failures are restricted or minimized. For plant maintenance condition monitoring is used as preventive maintenance method. So it is necessary to develop methodology to predict the crack in the machine component. Which can be done using vibration data.

1.2 Objective

Objective is to find a method for finding the crack location and crack depth in cantilever beam using vibration data. The scope of this work limited to cantilever beam with single crack.

1.3 Scope Of The Project

In Engineering design and construction, beams are important elements. In this research, cantilever beam is considered for vibration and FEA analysis. This method can be used as condition based monitoring, which can reduce the loss of time and money.

2. LITERATURE REVIEW

S. K. Maiti et al [1-2] He given a method based on measurement of natural frequencies, is presented for detection of the location and size of a crack in a stepped cantilever beam. Crack increases the damping and reduces stiffness, which results in reduction of natural frequency and change in mode shape.

J.M. Montaliao et al. [3] As per his study, due to the crack, there is local influence which results from reduction and second moment of area of cross section where it is located.

S. Christides et al. [4] had studied one dimensional theories of cracked Bernoulli¬ Euler beams. As per him, to develop crack detection procedure, the first important step is modeling of the crack. The crack is simulated by an equivalent spring which is connecting these two parts of beam.

Ostachowicz W.M. et al. [5] had studied the effect of cracks on the natural frequencies of cantilever beam. As per his reserch, crack detection can be done by using change in the natural frequencies / change in mode shapes / change in structural parameters.

R. D. Adams et al. [6] had studied vibration technique for non-destructively analyzing the integration of structures. As per his study. The crack is modeled as a mass-less linear spring. These divide the beam into two sections and are defined as B and C having reacceptances β and γ respectively. If Kx is the stiffness of the bar, then natural frequencies of cracked bar will satisfy following equation.

$$\beta x + \gamma x + {}^{1}/_{Kx} = 0$$

Crack location and natural frequency is given by:

$$EA/Kx = 1/\lambda \;[\cot\lambda x + \cot\lambda(1-x)]$$

Where E, A, and l are the modulus of elasticity, cross-sectional area and length of the beam respectively, and $\lambda = \omega \sqrt{1/x}$ is the frequency parameter and ω is natural frequency of a vibrations. The position of crack is given by plotting Kx against x for the available modes. The possible crack sites are given by the point of intersection curves for three available modes of vibration. This method has been tested experimentally for different type of damages and component geometries like taper bar camshaft etc. This relationship is in terms of ratios,

$$K = -\lambda \, \Delta_2 / \Delta_1$$

Where Δ is the frequency parameter and Δ_1 and Δ_2 are obtained from the characteristic equation of the system. The vibration of stiffness can be plotted against the crack location for various modes. Since, In real there is only one crack and its stiffness is mode independent, the intersection of all such curves gives the crack location. A minimum three modes are needed for an efficient prediction.

Samer Masoud Al-Said [7] had studied crack detection in a steeped beam having rigid disk. The beam centre-line is assumed to have only lateral deformation in the Y-direction. It showed that, The error in forecast crack dept and location using algorithm was 10%.

P. Cawley et al. [8] had studied the due to crack and slots natural frequencies. As per his study the larger frequency reactions predicted for slots compared with cracks of same depth are not due to removal of mass from beam, indeed the reduction in mass would cause increase in natural frequency. This results in further local decrease in stiffness of structure which gives larger reduction in natural frequency. This is significant reason.

3. THEORETICAL ANALYSIS

Euler-Bernoulli Beam's Equation of Motion is as given below:

$$\frac{\partial^4 y}{\partial \xi^4} - \lambda^4 Y = 0$$

Where,

Using linear differential equation, solution for above equation is:

$$Y(x) = a1 \sin\lambda\beta + a2 \cos\lambda\beta + a3 \sinh\lambda\beta + a4 \cosh\lambda\beta$$

For available one crack beam, it is considered that, the two beam are connected by spring.

$$Y_1(x) = a_1 \sin\lambda\beta + a_2 \cos\lambda\beta + a_3 \sinh\lambda\beta + a_4 \cosh\lambda\beta$$

...(1A)

$$Y_2(x) = a_5 \sin\lambda\beta + a_6 \cos\lambda\beta + a_7 \sinh\lambda\beta + a_8 \cosh\lambda\beta$$

...(1B)

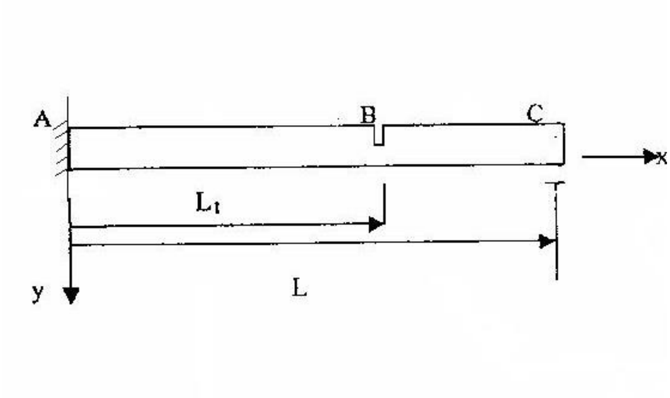

Figure 1: Cracked Cantilever Beam Model

For free vibrations of the given beam, there is no external excitation, no displacement and no moments at fixed supports.

$Y_{1A} = Y'_{1A} = 0, \quad \beta = 0$ and

$Y''_{2C} = Y'''_{2C} = 0, \quad \beta = 1$

The continuity conditions at the crack position and the moments , displacement and shear forces are,

$$Y_{1B} = Y_{2B}, \; Y''_{1B} = Y''_{2B}, \quad Y'''_{1B} = Y'''_{2B}, \quad Y'_{2B} = Y'_{1B} + (\lambda/K)\, Y''_{2B}$$

putting all above boundary conditions in above equation (1A) and (1B), For eight unknown coefficients, set of eight homogeneous linear algebraic equations are obtained.

$$|\Delta| = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\lambda & \cosh\lambda & -\sin\lambda & \sinh\lambda \\ 0 & 0 & 0 & 0 & \sin\lambda & \sinh\lambda & -\cos\lambda & \cosh\lambda \\ \cos\alpha & \cosh\alpha & \sin\alpha & \sinh\alpha & -\cos\alpha & -\cosh\alpha & -\sin\alpha & -\sinh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & \cos\alpha & -\cosh\alpha & \sin\alpha & -\sinh\alpha \\ \sin\alpha & \sinh\alpha & -\cos\alpha & \cosh\alpha & -\sin\alpha & -\sinh\alpha & \cos\alpha & -\cosh\alpha \\ -\frac{K}{\lambda}\sin\alpha - \cos\alpha & \frac{K}{\lambda}\sinh\alpha + \cosh\alpha & \frac{K}{\lambda}\cos\alpha - \sin\alpha & \frac{K}{\lambda}\cosh\alpha + \cosh\alpha & \frac{K}{\lambda}\sin\alpha & -\frac{K}{\lambda}\sinh\alpha & -\frac{K}{\lambda}\cos\alpha & -\frac{K}{\lambda}\cosh\alpha \end{vmatrix} = 0$$

Where, $\alpha = \lambda\beta$

This above formula can be presented in the form,

$$\frac{K}{\lambda}|\Delta_1(\lambda,\beta)| + |\Delta_2(\lambda,\beta)| = 0$$

Alternatively it can be written as,

$$K = -\lambda \, |\Delta_2| / |\Delta_1| \qquad (2)$$

Where the explicit expression for $|\Delta_2|$ and $|\Delta_1|$ are,

$$|\Delta_1| = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\lambda & \cosh\lambda & -\sin\lambda & \sinh\lambda \\ 0 & 0 & 0 & 0 & \sin\lambda & \sinh\lambda & -\cos\lambda & \cosh\lambda \\ \cos\alpha & \cosh\alpha & \sin\alpha & \sinh\alpha & -\cos\alpha & -\cosh\alpha & -\sin\alpha & -\sinh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & \cos\alpha & -\cosh\alpha & \sin\alpha & -\sinh\alpha \\ \sin\alpha & \sinh\alpha & -\cos\alpha & \cosh\alpha & -\sin\alpha & -\sinh\alpha & \cos\alpha & -\cosh\alpha \\ -\sin\alpha & \sinh\alpha & \cos\alpha & \cosh\alpha & \sin\alpha & -\sinh\alpha & -\cos\alpha & -\cosh\alpha \end{vmatrix}$$

$$|\Delta_2| = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\lambda & \cosh\lambda & -\sin\lambda & \sinh\lambda \\ 0 & 0 & 0 & 0 & \sin\lambda & \sinh\lambda & -\cos\lambda & \cosh\lambda \\ \cos\alpha & \cosh\alpha & \sin\alpha & \sinh\alpha & -\cos\alpha & -\cosh\alpha & -\sin\alpha & -\sinh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & \cos\alpha & -\cosh\alpha & \sin\alpha & -\sinh\alpha \\ \sin\alpha & \sinh\alpha & -\cos\alpha & \cosh\alpha & -\sin\alpha & -\sinh\alpha & \cos\alpha & -\cosh\alpha \\ -\cos\alpha & \cosh\alpha & -\sin\alpha & \sinh\alpha & 0 & 0 & 0 & 0 \end{vmatrix}$$

For the beam, the available initial three natural frequencies are measured. Using one frequency & considering the crack locations (e), the non dimensionalized stiffness K is found out from equation (2). Hence stiffness variation with crack location is found out. Similarly curves are plotted for other two natural frequencies. In real we have only one crack, we can find out using intersection point of curves of location of crack & stiffness of spring K. Then crack size calculated using following standard relation between stiffness and crack size,

$$K = \frac{b\ h^2\ E}{72\pi I (a/h)^2\ f\ (a/h)} \qquad (3)$$

$f\ (a/h)$ is obtained from the following equation,

$$f\ (a/h) = 0.6384 - 1.035\ (a/h) + 3.7201\ (a/h)^2 - 5.1773\ (a/h)^3 + 7.553\ (a/h)^4 - 7.332\ (a/h)^5 + 6.799\ (a/h)^6$$

Where b = width of beam, h = height of beam, a = crack depth

4. CASE STUDY

A cantilever beam with below details is considered for the study.

As given in Fig 2.

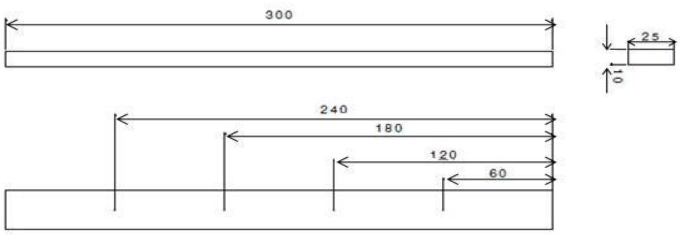

Figure 2 : Cantilever Beam Dimensions

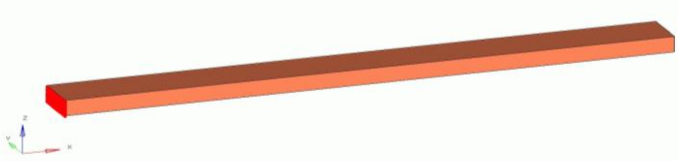

Figure 3 : Un-cracked Cantilever Beam Finite Element Model

Table -1: Material Details

Material and geometrical data	Mild steel
Young's modulus (E)	2.1 e11 N/m²
Density (ρ)	7860 kg/m³
Poisson's Ratio	0.29
Cross-section area (b x h)	0.025 x 0.010 m
Length of the beam (L)	0.3m

Finite elements analysis of cracked aswellas un-cracked beam is carried out. Normal mode analysis result of un-cracked aswellas cracked beam is tabulated in table2. Mode shapes of un-cracked beam are shown in figure 4 to 9. As discussed above, using equation (2), using the lowest three transverse natural frequencies the change in stiffness with respect to crack location is found out. Spring stiffness and crack location is obtained from the intersection of three curves. Find the details of it in the fig 11 to 15. Using eqn (3) crack size is calculated. Using MATLAB program change in stiffness with the location of crack is found out.

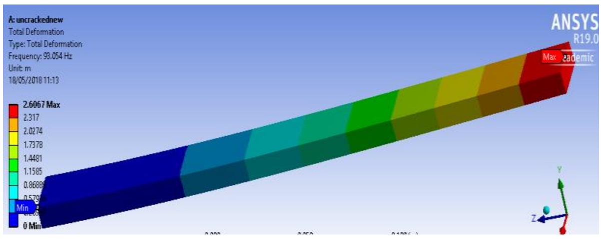

Figure 4 : Mode Shape 1 of Un-cracked Beam

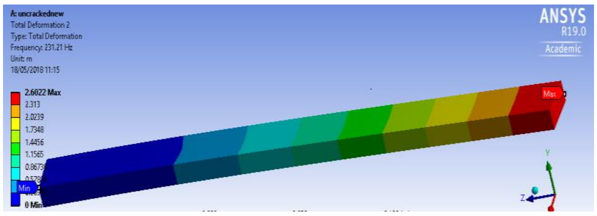

Figure 5 :Mode Shape 2 of Un-cracked Beam

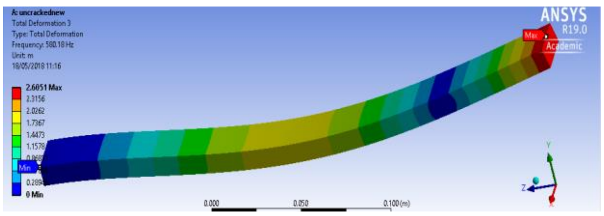

Figure 6 :Mode Shape 3 of Un-cracked Beam

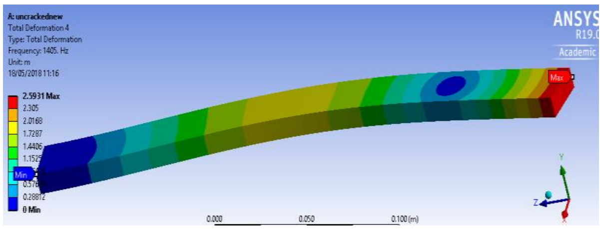

Figure 7 : Mode Shape 4 of Un-cracked Beam

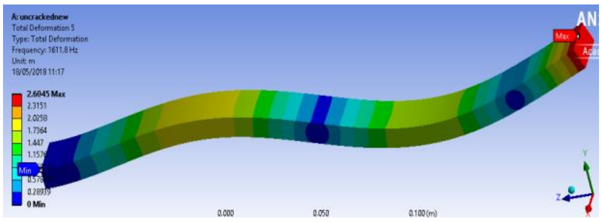

Figure 8 : Mode Shape 5 of Un-cracked Beam

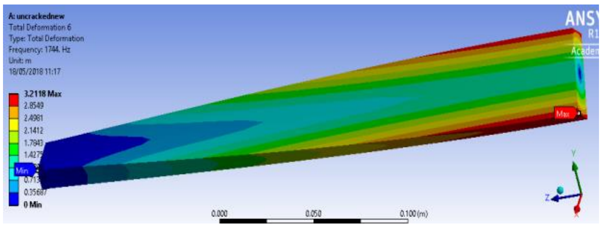

Figure 9 : Mode Shape 6 of Un-cracked Beam

Tab-2. Frequency Results

CASE NO.	Natural Frequencies from CAE (rad/s)		
	W1	W2	W3
Uncracked Beam	584.65	3645.37	10127.23
1	564.62	3644.31	10031.10
2	583.99	3637.42	10114.53
3	568.94	3499.92	9888.47
4	583.83	3597.31	10048.06
5	580.88	3445.07	9815.59

Experimental Setup

The experimental set up consists of the test instruments FFT Analyser, the test specimens and a clamping. A clamping is done with help of vice. One end of the beam is clamped while the other end is free. This was attempting to simulate the cantilever boundary condition. The schematic diagram of experimental setup is shown in figure 10.

Figure 10 : Experimental Setup Using FFT Analyzer

Experimental Procedure

The aim of the experimental work is to determine the three lowest natural frequencies of cantilever beam by easiest possible method. The test method to use in industrial applications would be the transient test, in which the response of the structure to an impulse is recorded and natural frequency is computed by Fourier transformation technique. To achieve this, following procedure is adopted.

The consistent impulse response of structure is obtained by tapping the beam by hammer. The change induced in accelerometer is amplified by charge amplifier before feeding it to FFT analyzer. Data collection is done with method of accelerometer at one location and excitation at multiple locations. The accelerometer with magnetic base is placed at one specified location and the excitation is four specified locations along the length of the beam. collection is done on the NVgate and Modal software.

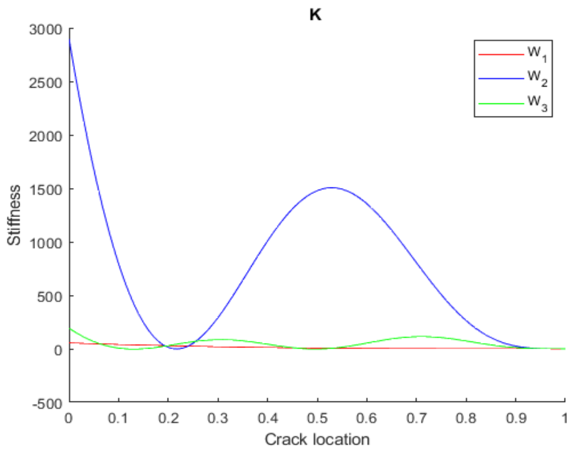

Figure 11 : Graph of Case-1- Stiffness Vs Location of Crack

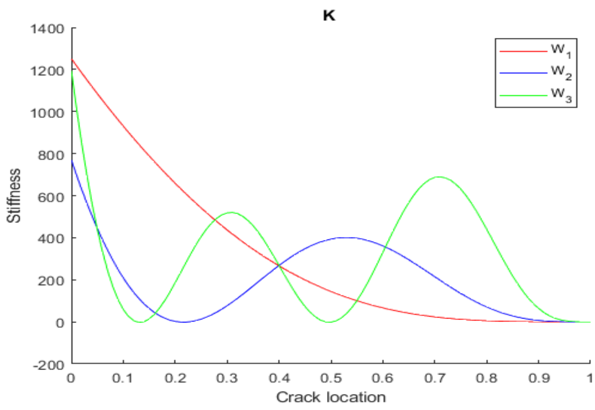

Figure 12 : Graph of Case-2- Stiffness Vs Location of Crack

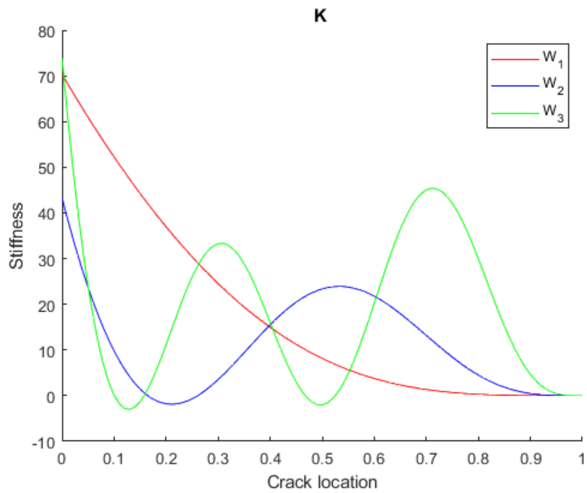

Figure 13 : Graph of Case-3- Stiffness Vs Location of Crack

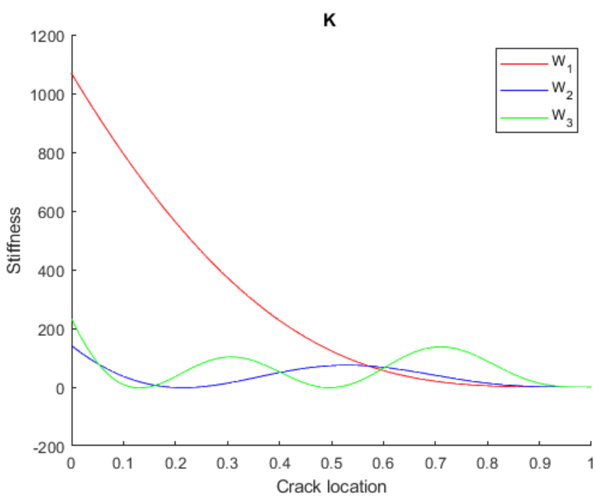

Figure 14 : Graph of Case-4- Stiffness Vs Crack Location

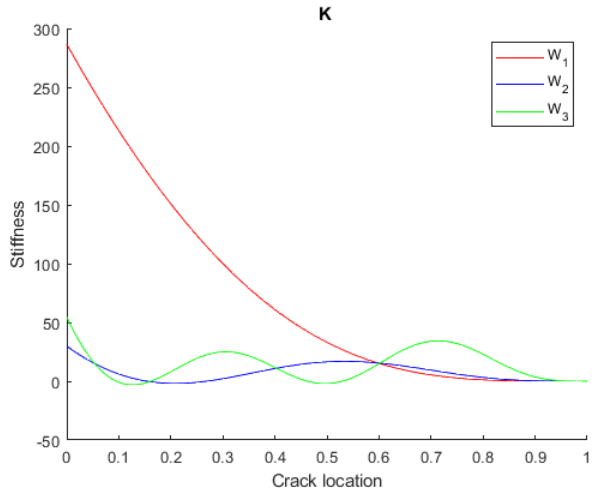

Figure 15 : Graph of Case-5- Stiffness Vs Location of Crack

Tab-3. Crack Results

Case No	Actual		Predicted result				
	Location β	Size a/h	Location β	Error for location %	Stiffness K	Size a/h	Error for Size %
Uncracked beam							
1	0.2	0.3	0.195	-2.257	32.920	0.292	-2.535
2	0.4	0.1	0.414	3.457	267.940	0.102	2.387
3	0.4	0.4	0.387	-3.189	17.197	0.389	-2.659
4	0.6	0.2	0.614	2.389	68.703	0.205	2.723
5	0.6	0.4	0.582	-2.938	16.824	0.393	-1.755

5. CONCLUSION

Vibration measurement based technique for analyzing the integrity of structure has certain advantages over the common non-destructive techniques, like condition monitoring etc. It's tedious and very time consuming job to use common NDTs to big structures like rail tracks, long pipelines, engineering components, aircraft structures etc. Analytical method has been found out for uniform beams, which provides the theoretical basis for crack detection using three obtained natural frequencies of un-cracked and cracked beam. The crack has been modeled as a torsional spring and is at the root of the crack. Few important assumptions are made in deriving these theories which are: the structural member behaves linearly, the structural properties are time invariant. The error in forecasting of crack location size is up to 4%. The given method is by comparing it with results of FEM results. proposed method is found to be easy and accurate.

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NOMENCLATURE

a	Crack Depth
b	Width of the Beam
h	Beam Height
a/h	Crack Depth Ratio
L	Beam Length
A	Beam Cross Sectional Area
ρ	Material Density
E	Material Young's Modulus
V	Material Poisson's Ratio
I	Moment of Inertia
ω	Natural Frequency
β	Crack Location
λ	Frequency Parameter
K	Non-Dimensional Stiffness of the Rotational Spring Representing the Crack

REFERENCES

[1] B. P. NANDWANA and S. K. MAITI, "Detection of the location and size of a crack in stepped cantilever beams based on measurements of natural frequencies", Journal of Sound and Vibration (1997) 203(3), 435-446

[2] B. P. NANDWANA and S. K. MAITI, "Modelling of vibration of beam in presence of inclined edge or internal crack for it's possible detection based on frequency measurements", Engineering Fracture Mechanics Vol. 58, No.3, pp. 193-205, 1997

[3] M. MONTALIAO, E.SILVA AND A.J.M. ARANJO GOMES "Experimental Dynamic Analysis of cracked free-free beams" Experimental Mechanics, March 1990, pp 20-25.

[4] S.CHRISTIDES and A.D.S. BARR, "One dimensional theory of cracked Bernoulli¬ Euler beams." International Journal of Mechanical Science, 1984 Vol. (26), No. (11/12), pp.639-648.

[5] OSTACHOWICZ W.M. and KRAWCZOK M, "Analysis of the effect of cracks on the natural frequencies of cantilever beam" Journal of Sound and Vibration 1991 Vol. (150), pp.191-210.

[6] R.D.ADAMS and P. CAWLEY, "A vibration technique for non-destructively assessing the integrating of structures." Journal of Mechanical Engineering Science, 1978 Vol.20 (2), pp.93-100.

[7] SAMER MASOUD AL-SAID, "Crack identification in a steeped beam carrying a rigid disk", Journal of Sound and Vibration 300 (2007) 863–876

[8] 'P. CAWLEY, R. RAY, "A comparison of natural frequency changes produced by cracks and slots" Journal of vibration, Acoustic, stress and reliability in design. July 1988, Vol.110, pp-366-370.