A STUDY OF PROPERTIES OF SOFT SET AND ITS APPLICATIONS

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Abstract - *In this paper the authors study the theory of soft sets initiated by Molodtsov. Equality of two soft sets, subset, superset of a soft set. Complement of soft set, null soft set, and absolute of soft set and examples. Soft set operation like OR, AND, union, intersection relative intersection, relative union, symmetric difference, relative symmetric difference and examples. Properties of soft sets, De Morgan's law (with proof by particular example) associative, commutative, distributive, etc.*

Key Words: **– Soft set, subset and superset of soft set, approximation, complement, relative Complement, NOT set , Null set , Soft set operations,**

1. INTRODUCTION

Most of the traditional tools for formal modelling, and computing are crisp, deterministic, and precise in character. However, there are many complicated problems in economics, engineering environment, social science, medical science, etc. that involve data which are not always crisp. We cannot successfully use classical methods because of various types of uncertainties present in these problems. There are theories theory of probability theory, fuzzy set theory, intuitionistic fuzzy sets theory, vague set theory, interval mathematics theory, rough set theory, which can considered mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties as pointed out .the reason for these difficulties is, possibly, the inadequacy of the parameterization tools of theories. Consequently, Molodtsov initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties.

(We are aware of the soft sets defined by Pawlak, which is different concept and useful to solve some other type of problems). Soft set theory has a rich potential for application in several directions, few of which had been shown by Molodtsov in his pioneer work in the present paper, we make a theoretical study of the **Soft Set Theory** in more detail.

2. SOFT SET

Let U be an initial universe set E is the set of parameters or attributes with respect to U . Let $P(U)$ Denote the power set of U and $A \subseteq E$, then pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \to P(U)$.

In other words:- A soft set (F, A) over U is parameterized family of subsets U for $e \in A, F(e)$, may be consider as the soft set of $e-$ elements or $e-$ approximate Elements of the soft set. (F, A) , thus (F, A) is defined as

$$(F, A) = \{F(e) \in P(U) : e \in E, F(e) = \phi \ \ if \ \ e \notin A\}$$

Example 2.1- Assume that $U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ be universal set constricting of a set of Six **Cars** under sale. Now $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of parameters with respect to U .

Where each parameters $e_i, \forall i = 1,2,3,4,5$ stands for {Expensive, Good design, Mileage, Modern, Space capacity} respectively and $A = \{e_1, e_2, e_3, e_4, e_5\} \in E$ suppose a soft set (F, A) .

Describe the attraction by costumer for cars.

$F(e_1) = \{c_1, c_4\}$, $F(e_2) = \{c_1, c_3, c_5\}$, $F(e_3) = \{c_3, c_4, c_5\}$, $F(e_4) = \{c_2, c_3\}$, $F(e_5) = \{c_2, c_4, c_6\}$

Then the soft set (F, A) is a parameters family $F(e_i), \forall i = 1,2,3,4,5$ of a subset U defined as $(F, A) = \{F(e_1), F(e_2), F(e_3), F(e_4), F(e_5)\}$ i.e.

$$\Rightarrow (F, A) = \{(c_1, c_2), (c_1, c_3, c_4), (c_3, c_4, c_5), (c_2, c_3), (c_2, c_4, c_6)\}$$

The soft set (F, A) can also represented as a set ordered pair as follows

$(F, A) = \{(e_1, F(e_1)), (e_2, F(e_2)), (e_3, F(e_3)), (e_4, F(e_4)), (e_5, F(e_5))\} \Rightarrow$
$(F, A) = \{(e_1, (c_1, c_2)), (e_2, (c_1, c_3, c_4)), (e_3, (c_3, c_4, c_5)), (e_4, (c_2, c_3)), (e_5, (c_2, c_4, c_6))\}$

Notation- (F, A) or (F_A) and (F_A, E)

By Tabular form (Table1)

U	e_1 Expensive	e_2 Good Design	e_3 Mileage	e_4 Modern	e_5 Space
c_1	1	1	0	0	0
c_2	0	0	0	1	1
c_3	0	1	1	1	0
c_4	1	0	1	0	1
c_5	0	1	1	0	0
c_6	0	0	0	0	1

For the purpose of storing a soft set in computer, we could represent a soft set in the form of table 1 (corresponding to the soft set in the above example).

2.1 Soft Sub-set

Definition- For two soft set (F, A) and (G, B) over a common universe U we say that (F, A) is a soft subset of (G, B) if

(a) $A \subset B$ and
(b) $\forall \varepsilon \in A, F(\varepsilon)$ and $G(\varepsilon)$ are identical approximation, we write $(F, A) \tilde{\subset} (G, B)$.

If (F, A) is said to be a **soft supper set** of (G, B) ,if (G, B) is a soft subset of (F, A). We Denote if $(F, A) \tilde{\supset} (G, B)$.

2.2 Equality of two Sets

Definition – Two soft set (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is soft subset of (G, B) and (G, B) is soft subset of (F, A).

Example2.2 - Previous example 2.1

$E = \{e_1, e_2, e_3, e_4, e_5\}$, be the set of parameters with respect to U and $A = \{e_1, e_3, e_5\} \subset E$, $B = \{e_1, e_2, e_3, e_5\} \subset E$ clearly $A \subset B$

Let (F, A) and (G, B) be two soft sets over the same universe $U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ such that

$G(e_1) = \{c_2, c_4\}$, $G(e_2) = \{c_1, c_3\}$, $G(e_3) = \{c_3, c_4, c_5\}$, $G(e_4) = \{c_2, c_4, c_6\}$ and
$F(e_1) = \{c_2, c_4\}$, $F(e_3) = \{c_3, c_4, c_5\}$, $F(e_5) = \{c_2, c_4, c_6\}$

Therefore $(F, A) \tilde{\subset} (G, B)$ $A \subseteq B$ and $F(e) \subset G(e) \forall e \in A$ but $(G, B) \not\subset (F, A)$ i.e. $(F, A) \neq (G, B)$

Remark 2.1 – let soft set (F, A) and (G, B) over a common universe U. $(F, A) \tilde{\subset} (G, B)$ Does not imply that every element of (F, A) is an element of (G, B) therefore definition of classical Subset does not hold for soft subset.

Example 2.3 – let $U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ be universal and $E = \{e_1, e_2, e_3\}$ be the set of parameters with respect to U. If $A = \{e_1\}$ and $B = \{e_1, e_3\}$

$F(e_1) = (c_2, c_4)$ for A. $(F, A) = \{e_1, (c_2, c_4)\}$

$G(e_1) = (c_2, c_3, c_4)$, $G(e_3) = (c_1, c_5)$ for B.

$(G, B) = \{(e_1, (c_2, c_3, c_4)), (e_3, (c_1, c_5))\}$

Then $\forall e \in A, F(e) \subset G(e)$ and $A \subset B$ Hence $(F, A) \tilde{\subset} (G, B)$.

Clearly, $(e_1, F(e_1)) \subset F(A)$ and $(e_1, F(e_1)) \not\subset G(B)$

2.3 Not set of a set of parameters

Let $E = \{e_1, e_2, e_3 ...., e_n\}$ be the set of Parameters the **Not set** of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, \neg e_3 ...., \neg e_n\}$ where $\neg e_i = Not(e_i) \forall i = 1,2,3,4,........ ..n$.

Proposition 2.1

(a) $\neg(\neg A) = A$
(b) $\neg(A \cup B) = \neg A \cup \neg B$
(c) $\neg(A \cap B) = \neg A \cap \neg B$

Remark 2.2 it has been proved that $\neg A \neq A^c$ and the $\neg A \not\subset E$ and so proposition hold but new assumption that $\neg A \subset E$ and came up with the following proposition

(a) $\neg(A \cup B) = \neg A \cap \neg B$
(b) $\neg(A \cap B) = \neg A \cup \neg B$ **(De Morgan's law)**

Example 2.4 – Consider the above example here

$E =$ {Expensive, Good design, Mileage, Modern, Space capacity} then the **Not set** of this is

$\neg E =$ {Not Expensive, Not Good design, No Mileage, No Modern, No Space capacity} i.e. $E = \{e_1, e_2, e_3, e_4, e_5\}$, then **Not set** is $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4, \neg e_5\}$.

2.4 Compliment of soft set

The complement of a soft set (F, A) is denoted by $(F, A)^C$ and defined as

$(F, A)^C = (F^C, \neg A)$, where $F^C : \neg A \to P(U)$ is a mapping given by

$F^C(\alpha) = U - F(\neg\alpha) \forall \alpha \in A$.

Example- 2.5 by Example 2.1

Then the soft set $(F, A)^C$ is a parameters family $F(e_i)^C, \forall i = 1,2,3,4,5$ of a subset U defined as $F(e_1)^C = F(\neg e_1) = \{c_1, c_3, c_5, c_6\}$, $F(e_2)^C = F(\neg e_2) = \{c_2, c_4, c_6\}$

$F(e_3)^C = F(\neg e_3) = \{c_1, c_2, c_6\}$, $F(e_4)^C = F(\neg e_4) = \{c_1, c_4, c_5, c_6\}$

$F(e_5)^C = F(\neg e_5) = \{c_1, c_3, c_5\}$

$(F,A)^C = \{F(e_1)^C, F(e_2)^C, F(e_3)^C, F(e_4)^C, F(e_5)^C\}$

$\Rightarrow$

$(F,A)^C = \{(c_1,c_3,c_5,c_6),(c_2,c_4,c_6),(c_1,c_2,c_6),(c_1,c_{4,},c_5,c_6),(c_1,c_3,c_5)\}$

The soft set $(F,A)^C$ can also represented as a set ordered pair as follows

$(F,A)^C = \{(\neg e_1, F(e_1)^C), (\neg e_2, F(e_2)^C), (\neg e_3, F(e_3)^C), (\neg e_4, F(e_4)^C),$
$(\neg e_5, F(e_5)^C)\}$

$\Rightarrow (F,A)^C = \{(\neg e_1,(c_1,c_3,c_5,c_6)),(\neg e_2,(c_2,c_4,c_6)),(\neg e_3,(c_1,c_2,c_6)),$
$(\neg e_4,(c_1,c_{4,},c_5,c_6)),(\neg e_5,(c_1,c_3,c_5))\}$

2.5 Relative complement

The relative complement of a soft set (F,A) denoted by $(F,A)^r$ and defined as $(F,A)^r = (F^r, A)$, where $F^r : A \to P(U)$, is a mapping given by $F^r(\alpha) = U - F(\alpha) \forall \alpha \in A$.

Example 2.6 –by example 2.1

Then the soft set $(F,A)^r$ is a parameters family $F(e_i)^r, \forall i = 1,2,3,4,5$ of a subset U defined as $F(e_1)^r = \{c_1,c_3,c_5,c_6\}$, $F(e_2)^r = \{c_2,c_4,c_6\}$, $F(e_3)^r = \{c_1,c_2,c_6\}$, $F(e_4)^r = \{c_1,c_4,c_5,c_6\}$, $F(e_5)^r = \{c_1,c_3,c_5\}$

$(F,A)^r = \{F(e_1)^r, F(e_2)^r, F(e_3)^r, F(e_4)^r, F(e_5)^r\}$

$\Rightarrow$

$(F,A)^r = \{(c_1,c_3,c_5,c_6),(c_2,c_4,c_6),(c_1,c_2,c_6),(c_1,c_{4,},c_5,c_6),(c_1,c_3,c_5)\}$

The soft set $(F,A)^r$ can also represented as a set ordered pair as follows

$(F,A)^r = \{(e_1, F(e_1)^r), (e_2, F(e_2)^r), (e_3, F(e_3)^r), (e_4, F(e_4)^r), (e_5, F(e_5)^r)\}$

$\Rightarrow (F,A)^r = \{(e_1,(c_1,c_3,c_5,c_6)),(e_2,(c_2,c_4,c_6)),(e_3,(c_1,c_2,c_6))$
$(e_4,(c_1,c_{4,},c_5,c_6)),(e_5,(c_1,c_3,c_5))\}$.

Proposition 2.1- let U be the universe, (F,A) is soft set over U. Then

(a) $((F,A)^C)^C = (F,A)$
(b) $((F,A)^r)^r = (F,A)$
(c) $(\tilde{U}_A)^C = \tilde{\phi}_A = (\tilde{U}_A)^r$
(d) $(\tilde{\phi}_A)^C = \tilde{U}_A = (\tilde{\phi}_A)^r$

2.6 Null set

A soft set (F,A) over universal set U is said to be a null soft set defined by ϕ if $F(e) = \phi, \forall e \in A$.

Example – 2.7 suppose that,

U is the set of wooden houses under the consideration

A is the set of parameters

Let there be five houses in the universe U given by

$U = \{h_1,h_2,h_3,h_4,h_5\}$ and A ={brick, muddy, steel, stone}.

The soft set (F,A) describe the "construction of the houses". The soft set (F,A) is

Defined, as

$F(brick)$ =means the brick built houses

$F(muddy)$ = means the muddy houses

$F(steel)$ = means the steel built houses

$F(stone)$ = means the stone built houses

The soft set (F,A) is the collection of approximations as below:

(F,A) = {brick built houses = ϕ , muddy houses= ϕ steel built houses= ϕ stone built houses= ϕ }.

2.7 Absolute soft set

A soft set (F,A) over universal set U is said to be a absolute soft set denoted by $\tilde{A}$, if $F(e) = U, \forall e \in A$ .Clearly $\tilde{A}^C = \phi$ and $\tilde{A} = \phi^C$

Example 2.8 – Suppose that,

U is the set of wooden houses under the consideration.

B is the set of parameters.

Let there be five houses in the universe U given by

$U = \{h_1,h_2,h_3,h_4,h_5\}$ And B = {not brick, not muddy, not steel, not stone}.

The soft set (G,B) describe the "construction of the houses". The soft set (G,B) is defined as

$F(brick)$ =means the houses not built by brick

$F(muddy)$ = means the houses not by muddy

$F(steel)$ = means the houses not built by steel

$F(stone)$ = means the houses not built by stone

The soft set (G,B) is the collection of approximations as below:

(G,B) = {not brick built houses $=\{h_1,h_2,h_3,h_4,h_5\}$, not muddy houses= $\{h_1,h_2,h_3,h_4,h_5\}$, not steel built houses= $\{h_1,h_2,h_3,h_4,h_5\}$, not stone built houses= $\{h_1,h_2,h_3,h_4,h_5\}$ }.

The soft set (G,B) is the absolute soft set

2.8 Relative null soft set

Let U be a universe, E be a set of parameters and $A \subseteq E$. (F,A) is called a relative Null soft set with respect to A denoted by $\tilde{\phi}_A$ if $F(e)=\phi, \forall e \in A$.

2.9 Relative whole soft set

Let U be a universe, E be a set of parameters and $A \subseteq E$. (F,A) is called relative whole soft set or $A-$ universal with respect to A denoted by $\tilde{U}_A$, if $F(e)=U, \forall e \in A$

2.10 Relative Absolute soft set

The relative whole soft set with respect to E denoted $\tilde{U}_E$ b is called the relative absolute soft set over U.

Example 2.9 – let $E=\{e_1,e_2,e_3,e_4\}$ if $A=\{e_2,e_3,e_4\}$ such that $F(e_2)=\{c_2 c_4\}$, $F(e_3)=\phi$,

$F(e_4)=U$, then soft set $(F,A)=\{(e_2,(c_2,c_4),(e_4,U)\}$.

if $B=\{e_1,e_3\}$ such that the soft set $(G,B)=\{(e_1,\phi),(e_3,\phi)\}$ then the soft set (G,B) is a relative null soft set $(G,B)=\tilde{\phi}_B$.

if $C=\{e_1,e_2\}$ such that $H(e_1)=U, H(e_2)=U$ then the soft set (H,C) is a relative whole soft set $(H,C)=\tilde{U}_c$.

if $D=E$ such that $F(e_1)=U, \forall e_i \in E, i=1,2,3,4$ then the soft set $(F,D)=\tilde{U}_E$ is an absolute soft set.

Proposition 2.2- let U be the universe, E a set of parameters, $A,B,C \subset E$ .if (F,A), (G,B) and (H,C) are soft set over U . Then

(e) $(F,A)\tilde{\subset}\tilde{U}_A$

(f) $\tilde{\phi}\tilde{\subset}(F,A)$

(g) $(F,A)\tilde{\subset}(F,A)$

(h) $(F,A)\tilde{\subset}(G,B)$ and $(G,B)\tilde{\subset}(H,C)$ implies $(F,A)\tilde{\subset}(H,C)$

$(F,A)=(G,B)$ and $(G,B)=(H,C)$ implies $(F,A)=(H,C)$

3 SOFT SET OPERATIONS

3.1 Union

Let (F,A) and (G,B) be two soft sets over a common universe U .then the union of (F,A) and (G,B), denoted $(F,A)\tilde{\cup}(G,B)$ by is a soft set (H,C), where $C=A\cup B$ and $\forall e \in C$

$$H(e)=\begin{cases} F(e), & e \in A-B \\ G(e), & e \in B-A \\ F(e)\cup G(e), & e \in A \cap B \end{cases}$$

Example 3.1 – Consider the soft set (F,A) which describes the "cost of the houses" and the soft set (G,B) which describes the "attractiveness of the houses".

Suppose that $U=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8,h_9,h_{10}\}$,

$A=$ {Very costly, Costly, Cheap} And

$B=$ {Cheap, Beautiful, in the green surroundings}

$\Rightarrow$ $A=\{e_1,e_2,e_3\}$ And $A=\{e_3,e_4,e_5\}$ respectively.

Let $F(e_1)=\{h_2,h_4,h_7,h_8\}$, $F(e_2)=\{h_1,h_3,h_5\}$, $F(e_3)=\{h_6,h_9,h_{10}\}$, And

$G(e_3)=\{h_6,h_9,h_{10}\}$ $G(e_4)=\{h_2,h_3,h_7\}$, $G(e_5)=\{h_5,h_6,h_8\}$, then

$(F,A)\tilde{\cup}(G,B)=(H,C)$ $\forall$ $C=A\cup B$, where

$H(e_1)=\{h_2,h_4,h_7,h_8\}$, $H(e_2)=\{h_1,h_3,h_5\}$,

$H(e_3)$ $=\{h_6,h_9,h_{10}\}$,

$H(e_4)=\{h_2,h_3,h_7\}$, $H(e_5)=\{h_5,h_6,h_8\}$

3.2 Intersection

Let (F,A) and (G,B) be two soft sets over a common universe U .then the intersection of (F,A) And (G,B), denoted $(F,A)\tilde{\cap}(G,B)$ by is a soft set (H,C) where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)\, or\, G(e)$ (as both are same set).

Example 3.2 by example **3.1**

$\Rightarrow$ $(F,A)\tilde{\cap}(G,B)=(H,C)$ $\forall$ $C=A\cap B$, Where

$H(e_1)=\phi, H(e_2)=\phi, H(e_3)=\{h_6,h_9,h_{10}\}$,

$H(e_4)=\phi, H(e_5)=\phi$

Proposition 3.1 -let U be the Universe, (F,A) is soft set over U . Then

(a) $(F,A)\tilde{\cup}(F,A)=(F,A)$

(b) $(F,A)\tilde{\cap}(F,A)=(F,A)$

(c) $(F,A)\tilde{\cup}\phi=\phi$, where ϕ is a null set

(d) $(F,A)\tilde{\cap}\phi=\phi$ where ϕ is a null set

(e) $(F,A)\tilde{\cup}\tilde{A}=A$, where $\tilde{A}$ is absolute set

(f) $(F,A)\tilde{\cap}\tilde{A}=(F,A)$

Proposition 3.2

(a) $((F,A)\tilde{\cup}(G,B))^C=(F,A)^C\tilde{\cup}(G,B)^C$

(b) $((F,A)\tilde{\cap}(G,B))^C=(F,A)^C\tilde{\cap}(G,B)^C$

Proof-

(a) Suppose that $(F,A)\tilde{\cup}(G,B)=(H,A\cup B)$, where

$$H(\alpha)=F(\alpha),\quad if\ \alpha\in A-B,$$
$$=G(\alpha),\quad if\ \alpha\in B-A$$
$$=F(\alpha)\cup G(\alpha), if\ \alpha\in A\cap B.$$

Therefore, $((F,A)\tilde{\cup}(G,B))^C=(H,A\cup B)^C$

$=(H^C,\neg A\cup\neg B)$

Now, $H^C(\neg\alpha)=U-H(\alpha),\forall\neg\alpha\in\neg A\cup\neg B$

Therefore,
$$H^C(\neg\alpha)=F^C(\neg\alpha),\quad if\ \neg\alpha\in\neg A-\neg B,$$
$$=G^C(\neg\alpha),\quad if\ \neg\alpha\in\neg B-\neg A$$
$$=F^C(\neg\alpha)\cup G^C(\neg\alpha), if\ \neg\alpha\in\neg A\cap\neg B.$$

Again, $(F,A)^C\tilde{\cup}(G,B)^C=(F^C,\neg A)\tilde{\cup}(G^C,\neg B)$,

$=(K,\neg A\cup\neg B)$ (Say)

Where,
$$K(\neg\alpha)=F^C(\neg\alpha),\quad if\ \neg\alpha\in\neg A-\neg B,$$
$$=G^C(\neg\alpha),\quad if\ \neg\alpha\in\neg B-\neg A$$
$$=F^C(\neg\alpha)\cup G^C(\neg\alpha), if\ \neg\alpha\in\neg A\cap\neg B.$$

$\Rightarrow H^C$ and K are same function . Hence proved.

(b) Suppose that, $(F,A)\tilde{\cap}(G,B)=(H,A\cap B)$

Therefore $((F,A)\tilde{\cap}(G,B))^C=(H,A\cap B)^C$
$=(H^C,\neg A\cap\neg B)$
Again

$(F,A)^C\tilde{\cap}(G,B)^C=(F^C,\neg A)\tilde{\cap}(G^C,\neg B)$,

$=(K,\neg A\cap\neg B)$ (Say)

Where, $\forall\neg\alpha\in\neg A\cap\neg B$, we have

$K(\neg\alpha)=F^C(\neg\alpha)$ or $G^C(\neg\alpha)$
= $F(\alpha)$ or $G(\alpha)$, Where $\forall\alpha\in A\cap B$
= $H(\alpha)$ = $H^C(\neg\alpha)$

$\Rightarrow H^C$ And K are same function. Hence proved.

Proposition 3.3 if (F,A), (G,B) and (H,C) are three soft sets over U . Then

(a) $((F,A)\tilde{\cup}(G,B))\tilde{\cup}(H,C)=(F,A)\tilde{\cup}((G,B)\tilde{\cup}(H,C))$

(b) $((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)=(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C))$

(c) $((F,A)\tilde{\cup}(G,B))\tilde{\cap}(H,C)=((F,A)\tilde{\cup}(G,B))\tilde{\cap}((F,A)\tilde{\cup}(H,C))$

(d) $((F,A)\tilde{\cap}(G,B))\tilde{\cup}(H,C)=((F,A)\tilde{\cap}(G,B))\tilde{\cup}((F,A)\tilde{\cap}(H,C))$

3.3 AND

Let (F,A) and (G,B) be two soft sets over a common universe U .then the AND operation of (F,A) and (G,B), denoted by (F,A) AND (G,B) or $(F,A)\wedge(G,B)$ by is a soft set defined by $(F,A)\wedge(G,B)=(H,A\times B)$, where $H(a,b)=F(a)\cap G(b)\forall(a,b)\in A\times B$

Example 3.3 –by example **3.1**

$(F,A)\wedge(G,B)=(H,A\times B)$, where, $H(e_1,e_3)=\phi$, $H(e_1,e_4)=\{h_2,h_7\}$, $H(e_1,e_5)=\{h_8\}$, $H(e_2,e_3)=\phi$,

$H(e_2,e_4)=\{h_3\}$, $H(e_2,e_5)=\{h_5\}$, $H(e_3,e_3)=\{h_6,h_9,h_{10}\}$
$H(e_3,e_4)=\phi$ $H(e_3,e_5)=\{h_6\}$

3.4 OR

Let (F,A) and (G,B) be two soft sets over a common universe U .then the AND operation of (F,A) and (G,B), denoted by (F,A)]OR (G,B), therefore $(F,A)\vee(G,B)$ by is a soft set defined by $(F,A)\wedge(G,B)=(H,A\times B)$,where $H(a,b)=F(a)\cup G(b)\forall(a,b)\in A\times B$.

Example 3.4 –by example **3.1**

Then, $(F,A)\vee(G,B)=(H,A\times B)$, where

$H(e_1,e_3)=\{h_2,h_4,h_6,h_7,h_8,h_9,h_{10}\}$, $H(e_1,e_4)$ $=\{h_2,h_3,h_4,h_5,h_7,h_8\}$, $H(e_1,e_5)=\{h_2,h_4,h_5,h_6,h_7,h_8\}$, $H(e_2,e_3)$ $=\{h_1,h_3,h_5,h_6,h_9,h_{10}\}$, $H(e_2,e_4)=\{h_1,h_2,h_3,h_5,h_7,h_8\}$, $H(e_2,e_5)$ $=\{h_1,h_3,h_5,h_6,h_8\}$, $H(e_3,e_3)=\{h_6,h_9,h_{10}\}$, $H(e_3,e_4)$ $=\{h_2,h_3,h_6,h_7,h_9\}$, $H(e_3,e_5)=\{h_5,h_6,h_8,h_9,h_{10}\}$.

Remark 3.1 We also use the tabular form for AND, OR solving the examples.

Proposition 3.4

(c) $((F,A)\vee(G,B))^C=(F,A)^C\tilde{\vee}(G,B)^C$
(d) $((F,A)\wedge(G,B))^C=(F,A)^C\wedge(G,B)^C$

Proof-

(c) Suppose that $(F,A)\vee(G,B)=(H,A\times B)$,

Therefore,

$((F,A)\vee(G,B))^C=(H,A\times B)^C$

$=(H^C,\neg(A\times B))$,now

$(F,A)^C\wedge(G,B)^C=(F^C,\neg A)\wedge(G^C,\neg B)$,

$=(K,\neg A\times\neg B)$,where $K(x,y)=F^C(x)\cap G^C(y)$,

$=(K,\neg(A\times B))$

Now take $=(\neg\alpha,\neg\beta)\in\neg(A\times B))$

Therefore,

$$\begin{aligned}K(\neg\alpha,\neg\beta)&=U-H(\alpha,\beta),\\&=U-[F(\alpha)\cup G(\beta),\\&=F^C(\neg\alpha)\cap G^C(\neg\beta),\\&=K(\neg\alpha,\neg\beta)\end{aligned}$$

$\Rightarrow H^C$ And K are same function. Hence proved.

(d) Suppose that $(F,A)\wedge(G,B)=(H,A\times B)$,

Therefore, $((F,A)\tilde{\wedge}(G,B))^C=(H,A\times B)^C$

$=(H^C,\neg(A\times B))$

Now, $(F,A)^C\vee(G,B)^C=(F^C,\neg A)\vee(G^C,\neg B)$,

$=(K,\neg A\times\neg B)$, where $K(x,y)=F^C(x)\cup G^C(y)$, $=(K,\neg(A\times B))$

Now take $=(\neg\alpha,\neg\beta)\in\neg(A\times B))$

Therefore,
$$\begin{aligned}K(\neg\alpha,\neg\beta)&=U-H(\alpha,\beta),\\&=U-[F(\alpha)\cap G(\beta),\\&=[U-F(\alpha)]\cup[U-G(\beta)],\\&=F^C(\neg\alpha)\cup G^C(\neg\beta),\\&=K(\neg\alpha,\neg\beta)\end{aligned}$$

$\Rightarrow H^C$ And K are same function. Hence proved

3.5 Extended intersection

Let (F,A) and (G,B) be two soft sets over a common universe U .then the extended intersection of (F,A) and (G,B), denoted by $(F,A)\cap_\varepsilon(G,B)$ is a soft set (H,C), where $C=A\cup B$ and $\forall e\in C$

$$H(e)=\begin{Bmatrix}F(e), & e\in A-B\\G(e), & e\in B-A\\F(e)\cap G(e), & e\in A\cap B\end{Bmatrix}$$

Example 3.5- Consider the soft set (F,A) which describes the "cost of the houses" and the soft set (G,B) which describes the "attractiveness of the houses".

Suppose that $U=\{h_1,h_2,h_3,h_4,h_5,h_6\}$, $E=\{e_1,e_2,e_3,e_3,e_4,e_5\}$ respectively.

$E=$ {Very costly, Costly, Cheap, beautiful, in the green surroundings}

$A=$ {Very costly, Costly, Cheap}, $B=$ {cheap, beautiful, in the green surroundings}

$\Rightarrow$ $A=\{e_1,e_2,e_3\}\subset E$, $B=\{e_3,e_4,e_5\}\subset E$ respectively.

Let $F(e_1)=\{h_2,h_4\}$, $F(e_2)=\{h_1,h_3,h_5\}$, $F(e_3)=\{h_3,h_4,h_5\}$ and

$G(e_3)=\{h_1,h_2,h_3\}$ $G(e_4)=\{h_2,h_3,h_6\}$, $G(e_5)=\{h_2,h_3,h_4\}$, then

$(F,A)\cap_{\varepsilon}(G,B)=(H,C)$, where $C=A\cup B$ $\forall e\in C$

$$H(e)=\begin{cases}F(e), & e\in A-B\\ G(e), & e\in B-A\\ F(e)\cap G(e), & e\in A\cap B\end{cases}.$$

$H(e_1)=\{h_2,h_4\}$, $H(e_2)=\{h_1,h_3,h_5\}$,

$H(e_3)=\{h_3\}$, $H(e_4)=\{h_2,h_3,h_6\}$, $H(e_5)=\{h_2,h_3,h_4\}$

3.6 Restricted Intersection

Let (F,A) and (G,B) be two soft sets over a common universe U .then the restricted intersection of (F,A) and (G,B), denoted by $(F,A)\cap_R(G,B)$ is a soft set (H,C) where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)\cap G(e)$

Example 3.6 –by example **3.5**

Then, $(F,A)\cap_R(G,B)=(H,C)$, where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)\cap G(e)$.

$H(e_1)=\phi, H(e_2)=\phi, H(e_3)=\{h_3\}$,

$H(e_4)=\phi, H(e_5)=\phi$

3.7 Restricted Union

Let (F,A) and (G,B) be two soft sets over a common universe U .then the restricted union of (F,A) and (G,B), denoted by $(F,A)\cup_R(G,B)$ is a soft set (H,C) where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)\cup G(e)$

Example3.7 – by example **3.5**

Then, $(F,A)\cup_R(G,B)=(H,C)$, where

$C=A\cap B$ $\forall e\in C$, $H(e)=F(e)\cup G(e)$.

$H(e_1)=\phi, H(e_2)=\phi, H(e_3)=\{h_1,h_2,h_3,h_4,h_5\}$,

$H(e_4)=\phi, H(e_5)=\phi$

3.8 Restricted difference

Let (F,A) and (G,B) be two soft sets over a common universe U .then the restricted difference of (F,A) and (G,B), denoted by $(F,A)-_R(G,B)$ is a soft set (H,C) where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)-G(e)$

Example 3.8 –by example **3.5**

Then, $(F,A)-_R(G,B)=(H,C)$, where

$C=A\cap B$ $\forall e\in C$, $H(e)=F(e)-G(e)$.

$H(e_1)=\phi, H(e_2)=\phi, H(e_3)=\{h_4,h_5\}$,

$H(e_4)=\phi, H(e_5)=\phi$

3.9 Restricted symmetric difference

Let (F,A) and (G,B) be two soft sets over a common universe U .then the restricted symmetric difference of (F,A) and (G,B), denoted by $(F,A)\tilde{\Delta}(G,B)$ is a soft set (H,C) where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)\Delta G(e)$

In other words

Let (F,A) and (G,B) be two soft sets over a common universe U .then the restricted symmetric difference of (F,A) and (G,B), denoted $(F,A)\tilde{\Delta}(G,B)$ is a soft set defined by

$$(F,A)\tilde{\Delta}(G,B)=((F,A)\cup_R(G,B))-_R((F,A)\tilde{\cap}(G,B))$$

$$(F,A)\tilde{\Delta}(G,B)=((F,A)-_R(G,B))\cup_R((G,B)-_R(F,A))$$

Example 3.9 –by example **3.5**

Then,

$(F,A)\tilde{\Delta}(G,B)=((F,A)-_R(G,B))\cup_R((G,B)-_R(F,A))$.

now, $(F,A)-_R(G,B)=(H,C)$, where $C=A\cap B$ $\forall e\in C$, $H(e)=F(e)-G(e)$.

$H(e_1)=\phi, H(e_2)=\phi, H(e_3)=\{h_4,h_5\}$,

$H(e_4)=\phi, H(e_5)=\phi$

$(G,B)-_R(F,A)=(K,D)$, where $D=A\cap B$ $\forall e\in D$, $K(e)=G(e)-H(e)$

$K(e_1)=\phi, K(e_2)=\phi$,

$K(e_3)=\{h_1,h_2\}, K(e_4)=\phi, K(e_5)=\phi$, now the value of

$(F,A)\tilde{\Delta}(G,B)=((F,A)-_R(G,B))\cup_R((G,B)-_R(F,A))$

$= (H,C) \cup_R (K,D)$ where, $Z = C \cap D$ $\forall e \in Z$,
$J(e) = H(e) \cup K(e)$,

$J(e_1) = \phi$, $J(e_2) = \phi$, $J(e_3) = \{h_1, h_2, h_4, h_5\}$, $J(e_4) = \phi$,
$J(e_5) = \phi$

4. PROPERTIES OF SOFT SET OPERATION

4.1 Idempotent properties

(a) $(F,A)\tilde{\cup}(F,A) = (F,A) = (F,A) \cup_R (F,A)$

(b) $(F,A)\tilde{\cap}(F,A) = (F,A) = (F,A) \cap_\varepsilon (F,A)$

4.2 Identity properties

(a) $(F,A)\tilde{\cup}\tilde{\phi} = (F,A) = (F,A) \cup_R \tilde{\phi}$

(b) $(F,A)\tilde{\cap}\tilde{U} = (F,A) = (F,A) \cap_\varepsilon \tilde{U}$

(a) $(F,A) -_R \tilde{\phi} = (F,A) = (F,A)\tilde{\Delta}\tilde{\phi}$

(b) $(F,A) -_R (F,A) = \tilde{\phi} = (F,A)\tilde{\Delta}(F,A)$

4.3 Domination properties

(a) $(F,A)\tilde{\cup}\tilde{U} = \tilde{U} = (F,A) \cup_R \tilde{U}$

(b) $(F,A)\tilde{\cap}\tilde{\phi} = \tilde{\phi} = (F,A) \cap_\varepsilon \tilde{\phi}$

4.4 Complementation properties

(a) $\tilde{\phi}^C = \tilde{U} = \tilde{\phi}^r$

(b) $\tilde{U}^C = \tilde{\phi} = \tilde{U}^r$

4.5 Double Complementation (involution) property

(a) $((F,A)^C)^C = (F,A) = ((F,A)^r)^r$

4.6 Exclusion property

(a) $(F,A)\tilde{\cup}(F,A)^r = \tilde{U} = (F,A) \cup_R (F,A)^r$

4.7 Contradiction property

(a) $(F,A)\tilde{\cap}(F,A)^r = \tilde{\phi} = (F,A) \cap_\varepsilon (F,A)^r$

Remark 4.1– Exclusion and contradiction properties do not hold with respect to complement in definition 2.6.

4.8 De Morgan's properties

(a) $((F,A)\tilde{\cup}(G,B))^C = (F,A)^C \cap_\varepsilon (G,B)^C$

(b) $((F,A) \cap_\varepsilon (G,B))^C = (F,A)^C \tilde{\cup}(G,B)^C$

(c) $((F,A) \cup_R (G,B))^r = (F,A)^r \tilde{\cap}(G,B)^r$

(d) $((F,A)\tilde{\cap}(G,B))^r = (F,A)^r \cup_R (G,B)^r$

(e) $((F,A)\tilde{\cup}(G,B))^r = (F,A)^r \cap_\varepsilon (G,B)^r$

(f) $((F,A) \cap_\varepsilon (G,B))^r = (F,A)^r \tilde{\cup}(G,B)^r$

(g) $((F,A) \wedge (G,B))^C = (F,A)^C \vee (G,B)^C$

(h) $((F,A) \wedge (G,B))^C = (F,A)^C \vee (G,B)^C$

(i) $((F,A) \wedge (G,B))^r = (F,A)^r \vee (G,B)^r$

(j) $((F,A) \wedge (G,B))^r = (F,A)^r \vee (G,B)^r$

Remark 4.2 De Morgan's property does not hold for restricted union and restricted intersection with respect to complement in definition that means

(a) $((F,A) \cup_R (G,B))^C \neq (F,A)^C \tilde{\cap}(G,B)^C$

(b) $((F,A)\tilde{\cap}(G,B))^C = (F,A)^C \cup_R (G,B)^C$

4.9 Absorption properties

(a) $(F,A)\tilde{\cup}((F,A)\tilde{\cap}(G,B)) = (F,A)$

(b) $(F,A)\tilde{\cap}((F,A)\tilde{\cup}(G,B)) = (F,A)$

(c) $(F,A) \cup_R ((F,A) \cap_\varepsilon (G,B)) = (F,A)$

(d) $(F,A) \cap_\varepsilon ((F,A) \cup_R (G,B)) = (F,A)$

Remark 4.3

(a) $\tilde{\cup}$ And $\cap_\varepsilon$ do not absorb over each other

(b) $\tilde{\cap}$ And $\cup_R$ do not absorb over each other

4.10 Commutative properties

(a) $(F,A)\tilde{\cup}(G,B) = (G,B)\tilde{\cup}(F,A)$

(b) $(F,A) \cup_R (G,B) = (G,B) \cup_R (F,A)$

(c) $(F,A)\tilde{\cap}(G,B) = (G,B)\tilde{\cap}(F,A)$

(d) $(F,A) \cap_\varepsilon (G,B) = (G,B) \cap_\varepsilon (F,A)$

(e) $(F,A)\tilde{\Delta}(G,B) = (G,B)\tilde{\Delta}(F,A)$

Remark 4.4

(a) $\wedge$ And $\vee$ do not commute over each other

4.11 Associative properties

(a) $(F,A)\tilde{\cup}((G,B)\tilde{\cup}(H,C))=((F,A)\tilde{\cup}(G,B))\tilde{\cup}(H,C)$

(b) $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C))=((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)$

(c) $(F,A)\cup_R((G,B)\cup_R(H,C))=((F,A)\cup_R(G,B))\cup_R(H,C)$

(d) $(F,A)\cap_\varepsilon((G,B)\cap_\varepsilon(H,C))=((F,A)\cap_\varepsilon(G,B))\cap_\varepsilon(H,C)$

(e) $(F,A)\wedge((G,B)\wedge(H,C))=((F,A)\wedge(G,B))\wedge(H,C)$

(f) $(F,A)\vee((G,B)\vee(H,C))=((F,A)\vee(G,B))\vee(H,C)$

4.12 Complementation properties

(a) $(F,A)\tilde{\cup}((G,B)\tilde{\cap}(H,C))=(F,A)\tilde{\cup}(G,B)\tilde{\cap}(F,A)\tilde{\cup}(H,C)$

(b) $(F,A)\tilde{\cap}((G,B)\tilde{\cup}(H,C))=(F,A)\tilde{\cap}(G,B)\tilde{\cup}(F,A)\tilde{\cap}(H,C)$

(c) $(F,A)\cup_R((G,B)\cap_\varepsilon(H,C))=(F,A)\cup_R(G,B)\cap_\varepsilon(F,A)\cup_R(H,C)$

(d) $(F,A)\cap_\varepsilon((G,B)\cup_R(H,C))=(F,A)\cap_\varepsilon(G,B)\cup_R(F,A)\cap_\varepsilon(H,C)$

(e) $(F,A)\cup_R((G,B)\tilde{\cap}(H,C))=(F,A)\cup_R(G,B)\tilde{\cap}(F,A)\cup_R(H,C)$

(f) $(F,A)\tilde{\cap}((G,B)\cup_R(H,C))=(F,A)\tilde{\cap}(G,B)\cup_R(F,A)\tilde{\cap}(H,C)$

(g) $(F,A)-_R((G,B)\tilde{\cup}(H,C))=(F,A)-_R(G,B)\tilde{\cup}(F,A)-_R(H,C)$

(h) $(F,A)-_R((G,B)\cap_\varepsilon(H,C))=(F,A)-_R(G,B)\cap_\varepsilon(F,A)-_R(H,C)$

(i) $(F,A)-_R((G,B)\cup_R(H,C))=(F,A)-_R(G,B)\cup_R(F,A)-_R(H,C)$

(j) $(F,A)-_R((G,B)\tilde{\cap}(H,C))=(F,A)-_R(G,B)\tilde{\cap}(F,A)-_R(H,C)$

(k) $(F,A)\tilde{\cap}((G,B)-_R(H,C))=(F,A)\tilde{\cap}(G,B)-_R(F,A)\tilde{\cap}(H,C)$

(l) $(F,A)\tilde{\cap}((G,B)\tilde{\Delta}(H,C))=(F,A)\tilde{\cap}(G,B)\tilde{\Delta}(F,A)\tilde{\cap}(H,C)$

Remark 4.5

$\tilde{\cup}$ And $\cap_\varepsilon$ do not distribute over each other

$\wedge$ And $\vee$ do not distribute over each other

$\tilde{\cup}$, $\cup_R$ and $\cap_\varepsilon$ do not distribute over $-_R$

$\tilde{\cup},\cup_R,\cap_\varepsilon,\tilde{\cap}$ And $-_R$ do not distribute over $\wedge$ and $\vee$

$\cup_R$ Distribute over $\tilde{\cup}$ but converse is false

$\tilde{\cap}$ Distribute over $\cap_\varepsilon$ but the converse is false

5. CONCLUSIONS

In this paper, we have discussed in detail the fundamentals of soft set theory such as equality of soft set, subset, Complement with examples. Its properties with solved examples. It was observe that some properties does not hold some classic result and soft set operation. In this paper we discussed the De Morgan's law. And all operation union, intersection, AND, OR, and also relative operations and its examples. Same as we proof the all properties in future.

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