Validation of Results of Analytical Calculation of Steady State Heat Transfer in Nuclear Fuel Element using ANSYS APDL

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Abstract - *This research studied the analytical solution of the steady state analysis of heat conduction in a cylindrical Nuclear fuel element. The fuel element used for this modelling was Uranium Oxide fuel, the cladding material was Zircaloy-2. The model was a simple one, considering the fact that we excluded the effect of the gas gap in between the fuel pellet and the cladding material, we also excluded the effect of axial heating, this made us to assume an infinite length fuel element. After the analytical solution was obtained, a graph of temperature against the radial distance was plotted and compared the result with the one obtained using ANSYS APDL, the results were the same, hence our model was validated. The behavior of each of the contour along the radial direction depicts the four (4) boundary conditions and therefore validates the results of the Analytical solution. During the validation, it was observed that the boundary conditions taken, in reality actually affected the thermal flux and thermal gradient at the axial direction. From the Simulation results, there was an observation that the thermal gradient and thermal flux along the axial direction were fairly constant, except for some dents at edges due to the little flashes of heat during heat transfer along the radial direction. This is normal, as there is no perfect heat transfer medium. With this and other results obtained from the Simulation, the research can say that the aim of validating Steady State Heat Transfer of Nuclear Fuel Element was accomplished.*

Key Words: Analytical, steady, ANSYS, APDL, heat, conduction, simulation, cladding, nuclear fuel, element, calculations transfer, temperature, heat flux, thermal gradient.

1.INTRODUCTION

Heat removal from nuclear reactors involves the removal of heat from the cylindrical fuel elements, this occurs in the radial direction, through the principles of heat resistances by conduction. The thermal properties of fuel materials plays an important role in heat removal in nuclear reactors. Properties such as thermal conductivity, specific heat capacity and density depends on temperature. Hence materials with very bad thermal coductivity will definitely be a bad nuclear fuel element material, this is because of the important role played by heat transfer coefficient in removing heat from nuclear reactors

1.1 Formulation of Analytical result

Fourier's equation of heat conduction in cylindrical coordinate without the axial and azimuthal terms

$$\frac{1}{R_f}\frac{\partial}{\partial R_f}\left(k_f R_f \frac{\partial T}{\partial R_f}\right) + Q = \rho c_p \frac{\partial T}{\partial t} \quad (1)$$

Where ρ is the density, c_p is the heat capacity at constant pressure, k is the thermal conductivity and Q is the volumetric heat density in thefuel pellet.

Equation (1) is the transient equation of the fuel rod conduction.

If the conduction equation is time independent, then we have heat equation that is in steady state with internal heating (Q), hence we the poisson equation of heat conduction for the pellet and laplace equation of heat for the cladding material.

$$\frac{1}{R_f}\frac{d}{dR_f}\left(k_f R_f \frac{dT}{dR_f}\right) + Q = 0 \quad (2)$$

$$\frac{d}{dR_f}\left(R_f \frac{dT_{cl}}{dR_f}\right) = 0 \quad (3)$$

Where $k_f, T_f, and\ T_{cl}$ are the heat conductivity and temperature of the fuel pellet and temperature of the cladding.

By taking boundary conditions, we can solve the steady state case, analytically.

$$\left(\frac{dT_f}{dR_f}\right)_{R=0} = 0 \quad (4)$$

$$k_f\left(\frac{dT_f}{dR_f}\right)_{R=R_1} = k_{cl}\left(\frac{dT_{cl}}{dR_f}\right)_{R=R_1} \tag{5}$$

$$k_{cl}\left(-\frac{dT_{cl}}{dR_f}\right)_{R=R_2} = h\left(T_{cl}(R_2)-T_{cool}\right) \tag{6}$$

$$T_f(R_1)=T_{cl}(R_1) \tag{7}$$

The boundary conditions (4), (5), (6) and (7), shows that (a) temperature is constant at the innermost part of the fuel pellet, hence temperature gradient is zero, (b) at the layer between the pellet outer diameter and the cladding inner diameter, the heat flux is constant or the linear heat density is constant, (c) at the outer boundary between the cladding and the coolant, the thermal flux depends on the temperature difference of the cladding and the coolant, and the heat transfer coefficient of the coolant.

Solving equation (2)

$$\frac{1}{R_f}\frac{d}{dR_f}\left(k_f R_f \frac{dT}{dR_f}\right) = -Q \tag{8}$$

$$\frac{dT_f}{dR_f} = -\frac{QR_f}{2k_f} + \frac{A_1}{k_f R_f} \tag{9}$$

Applying boundary condition of equation (4), we have:

$$A_1 = 0$$

We therefore have:

$$\frac{dT_f}{dR_f} = -\frac{QR_f}{2k_f} \tag{10}$$

Solving (3)

$$\frac{d}{dR_f}\left(R_f\frac{dT_{cl}}{dR_f}\right) = 0$$

$$\frac{dT_{cl}}{dR_f} = -\frac{A_2}{R_f} \tag{11}$$

Applying boundary condition in (5) we have:

$$k_f\left(\frac{-QR_1}{2k_f}\right) = k_{cl}\left(\frac{A_2}{R_1}\right) \tag{12}$$

$$A_2 = -\frac{QR_1^2}{2k_{cl}} \tag{13}$$

$$\frac{dT_{cl}}{dR_f} = -\frac{1}{R_f}\frac{QR_1^2}{2k_{cl}} \tag{14}$$

$$T_{cl} = -\frac{QR_1^2}{2k_{cl}}\ln(R_f) + A_3 \tag{15}$$

Applying boundary condition in (6) we have:

$$k_{cl}\left(-\frac{QR_1^2}{2k_{cl}R_2}\right) = h\left(-\frac{QR_1^2}{2k_{cl}}\ln(R_2) + A_3 - T_{cool}\right) \tag{16}$$

$$A_3 = \frac{QR_1^2}{2hR_2} + \frac{QR_1^2}{2k_{cl}}\ln(R_2) + T_{cool} \tag{17}$$

$$T_{cl} = \frac{QR_1^2}{2hR_2} + \frac{QR_1^2}{2k_{cl}}\ln\left(\frac{R_2}{R_f}\right) + T_{cool} \tag{18}$$

Integrating (10), we have:

$$T_f(R_f) = -\frac{QR_f^2}{4k_f} + A_4 \tag{19}$$

Using the boundary condition (7)

$$-\frac{QR_1^2}{2k_{cl}}\ln(R_1) + \frac{QR_1^2}{2hR_2} + \frac{QR_1^2}{2k_{cl}}\ln(R_2) + T_{cool} = -\frac{QR_1^2}{4k_f} + A_4 \tag{20}$$

$$A_4 = -\frac{QR_1^2}{2k_{cl}}\ln(R_1) + \frac{QR_1^2}{2hR_2} + \frac{QR_1^2}{2k_{cl}}\ln(R_2) + T_{cool} + \frac{QR_1^2}{4k_f} \tag{21}$$

We can obtain the fuel pellet temperature distribution thus:

$$T_f(R_f) = -\frac{QR_f^2}{4k_f} + \frac{QR_1^2}{2hR_2} + \frac{QR_1^2}{2k_{cl}}\ln\left(\frac{R_2}{R_1}\right) + T_{cool} + \frac{QR_1^2}{4k_f} \tag{22}$$

Fuel geometrical and thermal parameters used for this validation exercise are as follows:

Table – 1: Table of parameters

Volumetric heat density of fuel (Q)	$4000000 W/m^3$
Heat transfer coefficient (h)	$100 W/m^2K$
Fuel pellet radius (R_1)	0.015m
cladding radius (R_2)	0.018m
Cladding thickness ($R_2 - R_1$)	0.003m
fuel element length (H)	0.01m
coolant tempearture (T_{cool})	300°C
density of the fuel (ρ_f)	$10950 kg/m^3$
density of cladding (ρ_{cl})	$6550 kg/m^3$
specific heat capacity of fuel (c_{pf})	236J/kgK
specific heat capacity of cladding (c_{pcl})	285.8J/kgK
thermal conductivity of fuel (k_f)	6W/mK
thermal conductivity of cladding (k_{cl})	21.5W/mK

1.2 Analytical Results

A graph of the temperature as a function of radius was plotted to observe how it changes within the fuel rod both in the pellet and the cladding, especially at the point where the pellet and the cladding overlap. In this work, our fuel rod is assumed to have infinite length, this is the essence of the boundary condition imposed to ease the analytical calculation. The resulting graph below showed a good behavior of the model, which will be validated using ANSYS APDL.

The PTC-MATHCAD toolbox was used to compute and plot the analytical solution it is user friendly computing environment with a lot of symbolic solution which provides an accurate analysis of result. As can be seen from the graph below, the plotting is quite simple with simple labeling system. Therefore, the research can say that while PTC-MATHCAD helped we to solve the Analytical solution, ANSYS APDL assisted with the numerical simulation result.

Using PTC-MATHCAD worksheet we plotted the analytical results

$q:=4000000 \quad k_f:=6 \quad h:=100 \quad k_{cl}:=21.5 \quad T_{cool}:=300 \quad R_1:=0.015 \quad R_2:=0.018$

$$T_f(R)=\frac{-q\cdot R^2}{4\,k_f}+\frac{q\cdot R_1^{\,2}}{2\,k_{cl}}\ln\left(\frac{R_2}{R_1}\right)+\frac{q\cdot R_1^{\,2}}{2\cdot h\cdot R_2}+T_{cool}+\frac{q\cdot R_1^{\,2}}{4\,k_f}$$

$$T_{cl}(R)=\frac{q\cdot R_1^{\,2}}{2\,k_{cl}}\ln\left(\frac{R_2}{R}\right)+\frac{q\cdot R_1^{\,2}}{2\,h\cdot R_2}+T_{cool}$$

$T_f(R_1):=591-1.66\cdot10^5\,R_1^{\,2} \qquad T_{cl}(R_2):=550+21\,\ln\left(\frac{0.018}{R_2}\right)$

$R_1:=0,0.001..0.015 \qquad R_2:=0.015,0.016..0.018$

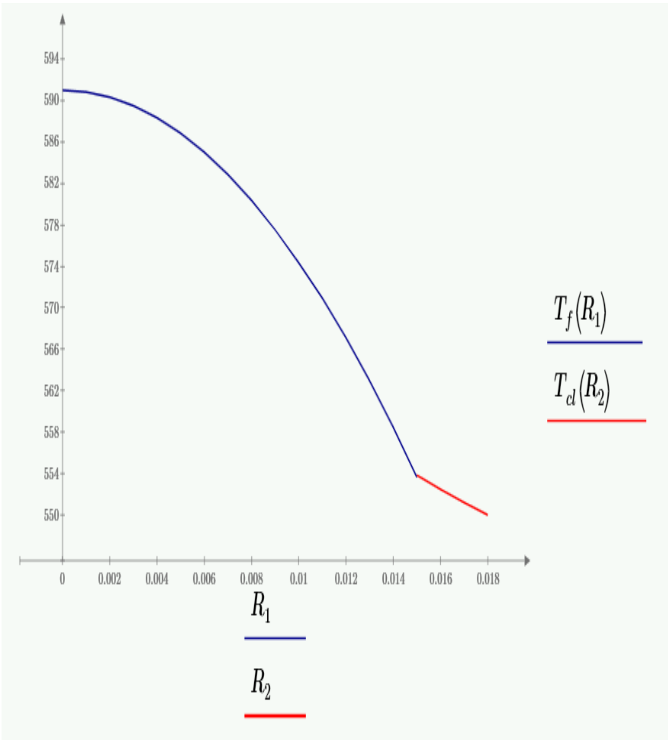

Chart -1: Temperature versus radius at steady state for Analytical solution

Numerical Simulation results for the steady state heat transfer using ANSYS APDL are as follows:

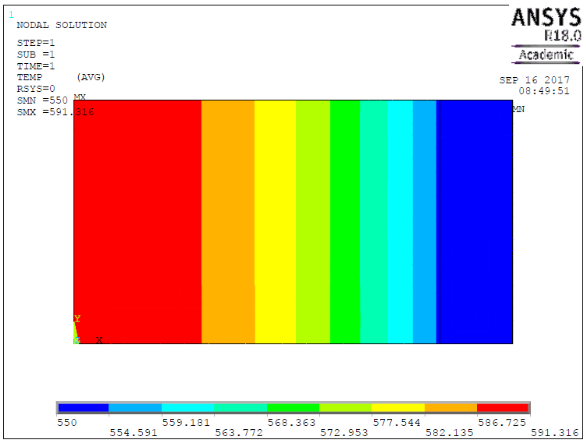

Chart -2: Radial Temperature distribution contour

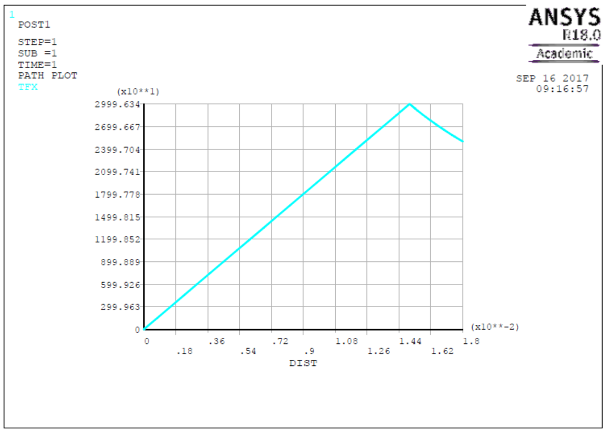

Chart – 5: Radial thermal flux distribution graph

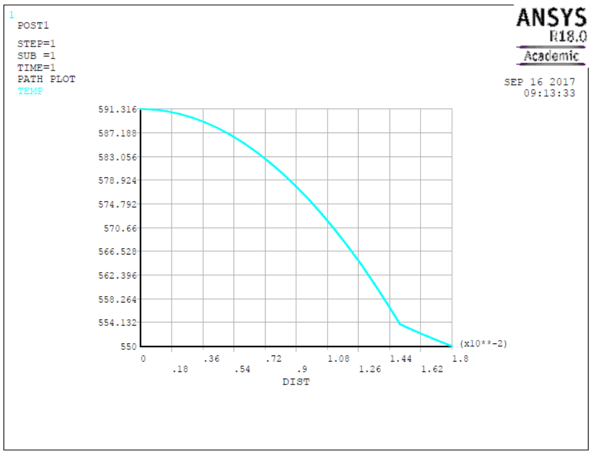

Chart – 3: Radial Temperature Distribution Graph

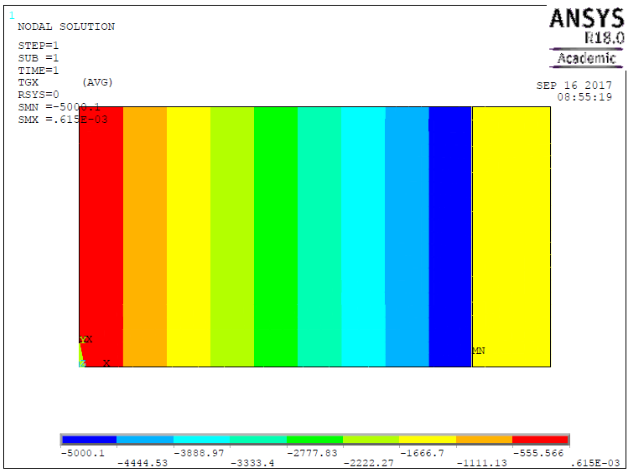

Chart – 6: Radial Thermal gradient distribution contour

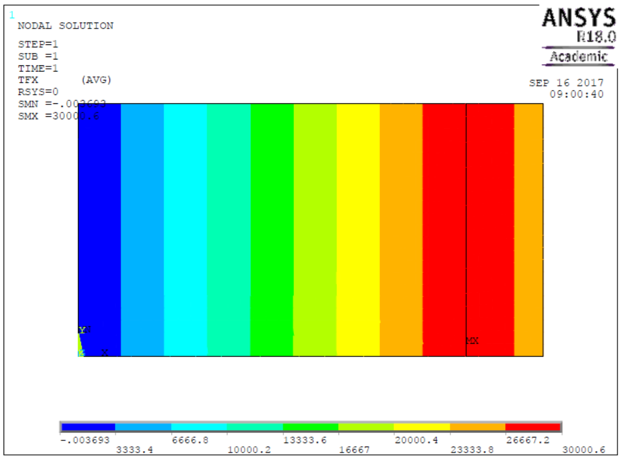

Chart – 4: Radial thermal flux distribution contour

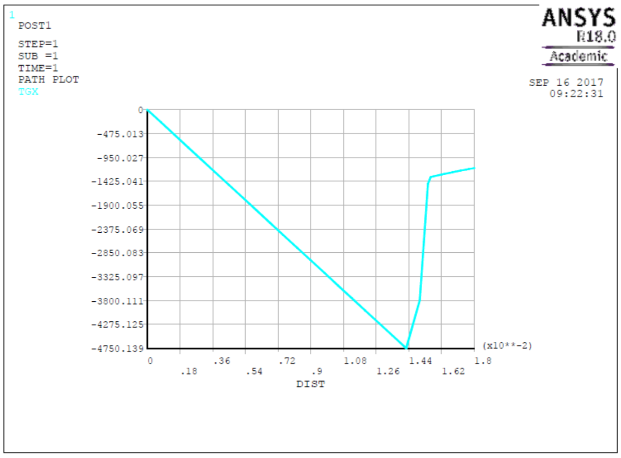

Chart – 7: Radial Thermal gradient distribution graph

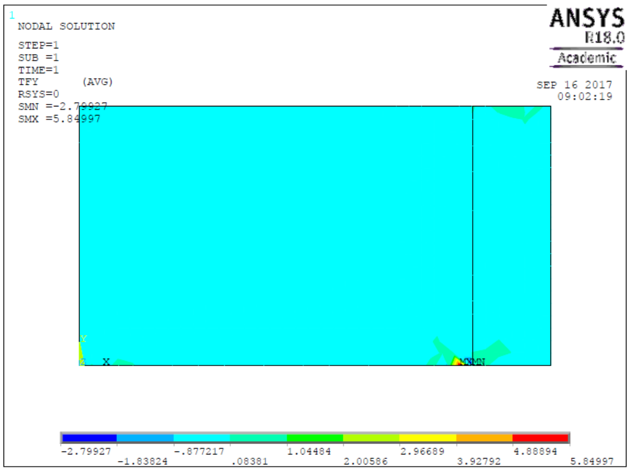

Chart – 8: Axial thermal flux distribution contour

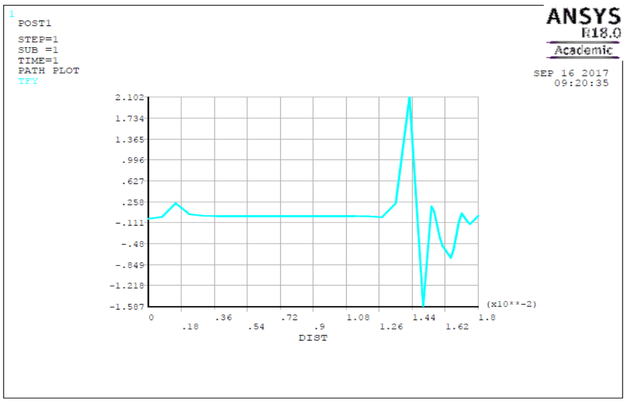

Chart – 9: Axial thermal flux distribution graph

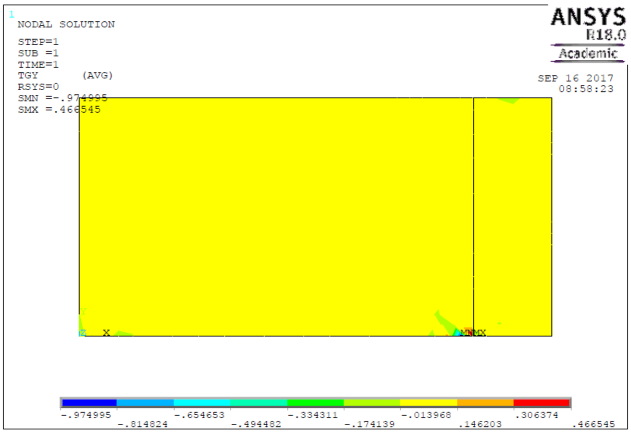

Chart – 10: Axial thermal gradient distribution contour

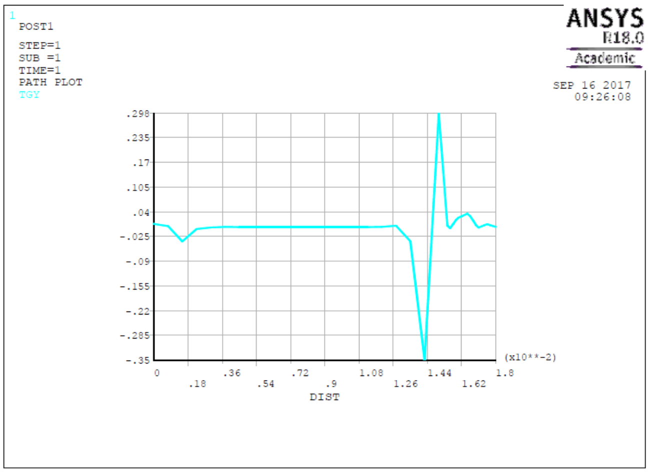

Chart – 11: Axial thermal gradient distribution graph

2. Conclusion:

The contour distributions and the corresponding graphical representations obtained from the Numerical simulation, corresponds to the boundary conditions taken in the analytical solution. The behavior of each of the contour along the radial direction depicts the four (4) boundary conditions and therefore validates the results of the Analytical solution. During the validation, it was observed that the boundary conditions taken in reality actually affected the thermal flux and thermal gradient at the axial direction. From the Simulation results, the research observed that the thermal gradient and thermal flux along the axial direction were fairly constant, except for some dents at edges due to the little flashes of heat during heat transfer along the radial direction. This is normal, as there is no perfect heat transfer medium. With this and other results obtained from the Simulation, the research can conclude that the aim of validating Steady State Heat Transfer of Nuclear Fuel Element was accomplished.

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